LESSON 11

Understanding Polynomials

Warm Up

1. **Vocabulary** A ________ is a letter that is used to represent a number.
(2)
2. Distribute. $-5(6x - 2)$
(1)
3. Collect like terms. $6 + 3x - 5x + 4y + 17 - 2y$
(2)
4. Let $g(x) = 2x + 7$ and $f(x) = x - 5$. Find $f(g(x))$.
(6)
5. True/False The function $f(x) = 3x^2 + 4$ is linear.
(4)

New Concepts

A **monomial** is a number, a variable, or the product of a number and one or more variables with whole-number exponents. A **polynomial** is a monomial or sum of monomials. The terms of a polynomial are its monomials. The **degree of a monomial** is the sum of the exponents of its variable factors. The **degree of a polynomial** is the degree of its monomial with the greatest degree.

> **Math Language**
>
> The suffix *nomial* means a name or a term.

The monomial $5ab^3$ has degree 4.

$$5a^1b^3 \longleftarrow 1 + 3 = 4$$

The polynomial $-xy^3 - 3x^2y^2 + x^2y^3$ has degree 5.

$$-x^1y^3 - 3x^2y^2 + x^2y^3 \longleftarrow 2 + 3 = 5$$

A one-variable polynomial is in **standard form** when all like terms have been combined and its terms are in descending order by degree.

The polynomial $-x^4 + x^3 - 3x + 2$ is in standard form.

$$-x^4 + x^3 - 3x^1 + 2x^0 \longleftarrow \text{4, 3, 1, 0 are in descending order.}$$

A **polynomial function** is a function of the form

$$f(x) = a_nx^n + a_{n-1}x^{n-1} + \ldots + a_1x + a_0,$$

where n is a nonnegative integer and the coefficients $a_n, \ldots, a_0$ are real numbers. The **leading coefficient** is a_n, the **constant term** is a_0, and the degree is n.

Example 1 Writing a Polynomial in Standard Form

Write the polynomial $x - 5 + 4x^2 + 2x^5 - x^2$ in standard form. Then identify the leading coefficient and the constant term.

SOLUTION

$$x - 5 + 4x^2 + 2x^5 - x^2 = 2x^5 + 4x^2 - x^2 + x - 5$$
$$= 2x^5 + 3x^2 + x - 5$$

Write terms in descending order by degree. Then combine like terms.

The leading coefficient is 2 and the constant term is -5.

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Hint

The prefix *quart-* indicates 4, as in *quarter* and *quartet*. The prefix *quint-* indicates 5, as in *quintuplets*.

Polynomials can be classified by degree and by number of terms. Some examples are given below.

Polynomial	Degree	Classification by Degree	Number of Terms	Classification by Number of Terms
-4	0	constant	1	monomial
$x + 2$	1	linear	2	binomial
$x^2 + x - 6$	2	quadratic	3	trinomial
x^3	3	cubic	1	monomial
$-3x^4 + 2x^3 - x^2$	4	quartic	3	trinomial
$x^5 - 4x^2$	5	quintic	2	binomial

Example 2 Classifying Polynomials

Classify each polynomial by degree and by number of terms.

a. $x^2 - 9$

SOLUTION The degree is 2, so it is quadratic. It has 2 terms, so it is a binomial.

b. $x^3 + x - 2x^3 - \frac{1}{2}x^5$

SOLUTION Combine like terms and write in descending order by degree to write the polynomial in standard form.

$$x^3 + x - 2x^3 - \frac{1}{2}x^5 = -\frac{1}{2}x^5 - x^3 + x$$

The degree is 5, so it is quintic. The standard form has 3 terms, so it is a trinomial.

Example 3 Adding Polynomials

Add: $(2x^3 - 3x^2 + x - 5) + (5 - x^2)$.

SOLUTION To add polynomials, combine like terms.

$$\begin{aligned}(2x^3 - 3x^2 + x - 5) + (5 - x^2) &= 2x^3 - 3x^2 + x - 5 + 5 - x^2 \\ &= 2x^3 - 3x^2 - x^2 + x - 5 + 5 \\ &= 2x^3 - 4x^2 + x\end{aligned}$$

Example 4 Subtracting Polynomials

Subtract: $(5x^3 - 3x^2) - (2x^3 - x^2 + 4)$.

SOLUTION To subtract a polynomial, add the opposite of each term.

$$\begin{aligned}(5x^3 - 3x^2) - (2x^3 - x^2 + 4) &= 5x^3 - 3x^2 - 2x^3 + x^2 - 4 \\ &= 5x^3 - 2x^3 - 3x^2 + x^2 - 4 \\ &= 3x^3 - 2x^2 - 4\end{aligned}$$

Example 5 Application: Revenue from Sales of Electricity

Revenue (in billions of dollars) from retail sales of electricity in the U.S. for the years 2000 through 2005, is modeled by these polynomial functions. The variable x represents the number of years since 2000.

Residential $R = 0.75x^2 + 1.73x + 99.32$

Commercial $C = 0.44x^2 + 3.91x + 79.26$

Industrial $I = 0.63x^2 - 1.49x + 49.89$

Write a model for total revenue from sales in all three sectors combined.

SOLUTION

1. **Understand** Sales in each sector is modeled by a polynomial function.

2. **Plan** To find a model for all three sectors combined, add the polynomials.

3. **Solve**

$$\begin{array}{r} 0.75x^2 + 1.73x + 99.32 \\ 0.44x^2 + 3.91x + 79.26 \\ +\ 0.63x^2 - 1.49x + 49.89 \\ \hline 1.82x^2 + 4.15x + 228.47 \end{array}$$

$f(x) = 1.82x^2 + 4.15x + 228.47$ is a model for all three sectors combined.

4. **Check** The answer makes sense because the polynomial $1.82x^2 + 4.15x + 228.47$ is the sum of the three given polynomials.

> **Math Reasoning**
>
> **Estimate** Use the model in Step **3** to estimate the revenue for the year 2002.

Lesson Practice

a. (Ex 1) Write the polynomial $3x - 1 + 4x^4 + x$ in standard form. Then identify the leading coefficient and the constant term.

b. (Ex 2) Classify $-x^3$ by degree and by number of terms.

c. (Ex 2) Classify $x^2 + 40x + 10 - 9x^2 - 1$ by degree and by number of terms.

d. (Ex 3) Add: $(2x^2 - 3x^5) + (2x^5 - x^2 + 1)$.

e. (Ex 4) Subtract: $(1 - x - x^3) - (x^3 + 2x^2 - x)$.

f. (Ex 5) The average retail price (in cents per kilowatt hour) of electricity for the years 2000 through 2005, is modeled by these polynomial functions. The variable x represents the number of years since 2000.

Residential $R = 0.041x^2 + 0.009x + 8.334$ (highest price)

Commercial $C = 0.013x^2 + 0.137x + 7.555$

Industrial $I = 0.028x^2 + 0.037x + 4.756$ (lowest price)

What polynomial function is a model that describes how much higher the average retail price was in the residential sector than in the industrial sector?

Practice Distributed and Integrated

2005 City Population	
City	**Population**
Oklahoma City	x
Chicago	$5.354x$
Arlington	$0.683x$
Total of 3 cities	3,736,275

***1.** *(7)* **Geography** The table shows the relationship of the annual estimated population for three cities in the year 2005. The population in Oklahoma City, Oklahoma, is represented by the variable x.

a. Formulate Write a linear equation to approximate the population in Oklahoma City in 2005.

b. Estimate Find the approximate population in Oklahoma City in 2005. Round to the nearest hundred.

2. *(7)* **Multi-Step** Twice a number is decreased by 9, and then multiplied by 4. The result is 8 less than 10 times the number. What is the number?

Simplify the expressions.

3. *(3)* $x^6(2x^2)$

4. *(3)* $2(2x^2)^6$

5. *(3)* $\frac{x^2}{2x^6}$

6. *(7)* **Geometry** Let x represent the first angle in a triangle. The second angle is twice the measure of the first. The third angle is triple the measure of the first. Write and solve a linear equation to find the measure of all three angles.

Solve the inequalities.

***7.** *(10)* $-7 \leq -2x + 3 < 9$

***8.** *(10)* $\frac{3}{2}x + 2 < \frac{1}{2}x + 5$

Evaluate.

9. *(2)* $(a - b) - a(-b)$ if $a = -5$ and $b = 3$

10. *(2)* $(a - x)(x - a)$ if $a = -2$ and $x = 4$

11. *(2)* $a^2 - y^3(a - y^2)y$ if $a = -2$ and $y = -3$

Write each fraction or decimal as a percent.

12. *(6)* 0.05

13. *(6)* $\frac{3}{10}$

14. *(6)* $\frac{4}{5}$

15. *(6)* 0.37

***16.** *(8)* **Error Analysis** Two scientists solved the following problem:

Sulfur is mixed with other chemicals to make sulfuric acid. If it takes 16 tons of sulfur to make 49 tons of sulfuric acid, how many tons of other constituents are needed to make 294 tons of acid? (Hint: Sulfuric acid is the total.) Which scientist solved the problem using the correct steps? Explain your reasoning.

Scientist A	Scientist B
$\frac{16}{49} = \frac{x}{294}$	$\frac{33}{49} = \frac{x}{294}$
$(16)(294) = 49x$	$(33)(294) = 49x$
$x = 96$ tons	$x = 198$ tons

17. (8) **Chemistry** A formula requires that 20 kilograms (kg) of carbon be used to produce 160 kg of a compound. How many kilograms of other components are needed to make 640 kg of the compound?

Justify **Determine if these statements are a tautology, a contradiction, or neither. Justify your answer by constructing truth tables.**

18. (Inv. 1) $\neg p \rightarrow q$

19. (Inv. 1) $\neg p \vee q$

20. (10) **Consumer** Nerissa's car can travel between 380 and 410 miles on a full tank of gas. She filled her gas tank and drove 45 miles. How many miles m can she drive without running out of gas?

21. (3) **Astronomy** The speed of light is about 3×10^8 meters/second. If Saturn's distance from Earth is about 1.321×10^{12} meters, about how many seconds would it take light to reach Saturn from Earth? Round the answer to the nearest second.

22. (3) **Earth Science** The garter snake is a small snake that is typically no more than 6×10^{-4} km in length. About how many garter snakes would it take to equal the circumference of the earth at the equator, if its circumference measures about 4.0076×10^4 km? Write the answer in scientific notation rounded to the nearest ten thousandth.

Analyze **Determine if the expressions are polynomials. If the expression is a polynomial, rewrite it in standard form. If the expression is not a polynomial, explain why.**

***23.** (11) $5x^2 + 3x^4 + 2$

***24.** (11) $\sqrt{3x + 5x^3}$

Classify the polynomial expressions by their degree and by the number of terms.

***25.** (11) $7x^3 + 3x^2 + 5x$

***26.** (11) $x^1 + 5x^0$

Simplify the expression.

***27.** (11) $(7x^3 + 3x^2 + 5x) - (3x^3 + x)$

28. (4) **Multiple Choice** Choose a letter that represents a set of x- and y-values that makes the equation $y = \frac{1}{2}x + 4$ true.

A $x = 2,\ y = 4$

B $x = 6, y = 6$

C $x = 4, y = 6$

D $x = 4, y = 8$

Graphing Calculator **Multiply the matrices. Check using a graphing calculator.**

***29.** (9) $\begin{bmatrix} 2 & -3 & 4 \\ 5 & 8 & -1 \\ 7 & 0 & 9 \end{bmatrix} \cdot \begin{bmatrix} -4 & 7 \\ 0 & 1 \\ 3 & 6 \end{bmatrix}$

***30.** (9) $\begin{bmatrix} 4 & -1 \\ 6 & -2 \end{bmatrix} \cdot \begin{bmatrix} -1 & \frac{1}{2} & 0 \\ -2 & 4 & -3 \end{bmatrix}$

LESSON 12 Solving Inverse Variation Problems

Warm Up

1. (8) **Vocabulary** Of the terms x, z, and 3, only the number 3 is a __________.
2. (7) Is $(3, -3)$ a solution to the equation $3x - 7y = 15$?
3. (5) Graph $5x + 4y = 20$.

New Concepts

If the product of two variables is a constant, then the equation is an **inverse variation.**

$$xy = k \text{ or } y = \frac{y}{x}$$

Both x and y are variables and k is a nonzero constant. The term k is also referred to as the constant of variation. You have previously learned about direct variations. Here are some differences between direct and inverse variations.

Math Reasoning

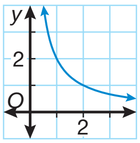
Generalize Compare the relationship of the variables in a direct and inverse variation.

Direct Variation	Inverse Variation
As x increases, y increases.	As x increases, y decreases.
As x decreases, y decreases.	As x decreases, y increases.
The ratio $\frac{y}{x}$ is a constant.	The product yx is a constant.

Example 1 Modeling Inverse Variation

The values in the table and the graph represent the equation $y = \frac{2}{x}$.

x	1	2	4
y	2	1	0.5



a. Is this equation an inverse variation? If so, find the constant of variation.

SOLUTION Multiply the x and y values in the data table.

x	1	2	4
y	2	1	0.5
xy	$(1)(2) = 2$	$(2)(1) = 2$	$(4)(0.5) = 2$

The equation is an inverse variation with a constant of variation equal to 2.

b. **Predict** What will the y value be when the x value is 8?

SOLUTION The product of x and y will be 2.

$$8y = 2$$

$$y = \frac{1}{4}$$

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Given a set of coordinates, it is possible to determine if they represent an inverse variation. By finding the constant of variation, it is possible to find the equation for the inverse variation.

Example 2 Testing for an Inverse Variation

a. Determine if the data set shown represents an inverse variation. If so, find the constant of variation and the equation.

x	1	2	3	4
y	3	1.5	1	0.75

SOLUTION Multiply the x and y coordinates to see if the products equal the same constant.

x	1	2	3	4
y	3	1.5	1	0.75
xy	$(1)(3) = 3$	$(2)(1.5) = 3$	$(3)(1) = 3$	$(4)(0.75) = 3$

Yes, each product is equal to 3, which is the constant of variation for the inverse variation $y = \frac{3}{x}$.

b. Determine if this data set represents a direct or an inverse variation. If so, find the constant of variation and the equation.

x	1	2	3	4
y	25	10	3	1

SOLUTION

Step 1: Before performing any calculations with the data, look at the basic relationship between x and y.

As x increases, does the value of y decrease? Yes. This could be an inverse variation and is definitely not a direct variation.

Step 2: Multiply the x and y coordinates to see if the products equal the same constant.

x	1	2	3	4
y	25	10	3	1
xy	25	20	9	4

The product xy is not the same from one set of coordinates to another, so the data set is not an inverse variation.

Caution

Check carefully to make sure that each product of the variables equals the same constant.

A **joint variation** involves three variables, one of which depends on changes in the other two variables. A joint variation can be written as shown below.

$$\frac{z}{xy} = k$$

The variables are x, y, and z, and k is the nonzero constant of variation.

Reading Math

The expression $\frac{z}{xy} = k$ is read "z varies jointly as x and y." This means that z varies directly as the product of x and y.

Example 3 Joint Variation

a. The force (F) on an object of mass m that causes it to move with acceleration a can be found using the equation $F = ma$. Is this an example of a joint variation? Explain.

SOLUTION In the equation shown, there are three variables, F, m, and a. The equation can be rewritten as $\frac{F}{ma} = 1$.

Since 1 is a constant, the equation $F = ma$ is a joint variation.

b. The kinetic energy (KE) of a moving object is based on its mass (m) and the square of its velocity (v^2). Use the data set below to see if this is a joint variation.

KE	2	9	24	50
m	1	2	3	4
v^2	4	9	16	25

SOLUTION Calculate the ratio $\frac{KE}{mv^2}$ for each set of data points to see if they equal a constant value.

KE	2	9	24	50
m	1	2	3	4
v^2	4	9	16	25
$\frac{KE}{mv^2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$

In each case the ratio is the same value: $\frac{1}{2}$. So this is a joint variation.

c. In an electric circuit, the electric power (P) is the product of the square of the current (I) and the electrical resistance (R). Write an equation that represents this relationship. Is it a joint variation?

SOLUTION The equation can be written as $P = I^2R$.

It is a joint variation with a constant of variation equal to 1.

Many real-world phenomena can be explained through direct, inverse, or joint variation. To determine which, if any, variation applies, look for the constant of variation.

Example 4 Determining the Type of Variation

a. Two objects of different masses experience a gravitational force of attraction. Isaac Newton found that this force is proportional to the product of the two masses. What kind of variation is this? Write the equation.

SOLUTION Define the variables m_1 and m_2 to represent the masses, and let F_g be the gravitational force of attraction. If F_g is proportional to the product of m_1 and m_2, then the equation can be written as $F_g = km_1m_2$.

Since there are two variables for mass, this represents a joint variation.

b. Newton also found that the gravitational force of attraction, F_g, is inversely proportional to the square of the distance between the two masses. What kind of variation is this? Write the equation.

SOLUTION

Step 1: Define the variable d to represent the distance between the two masses.

Step 2: If F_g is inversely proportional to d^2, then the equation can be written as $F_g = \frac{k'}{d^2}$.

(The constant of variation k' is used to distinguish it from the constant of variation, k, in part **a.**) This equation represents an inverse variation with constant of variation k'.

c. Newton's Universal Law of Gravitation shows that the gravitational force of attraction is both a joint variation and an inverse variation. Use the results from parts **a** and **b** to show how this equation is derived.

Math Reasoning

Connect Use your understanding of scientific notation to judge how strong the force of gravitation is.

SOLUTION

Step 1: Define the constant of variation g.

Step 2: F_g is inversely proportional to d^2 and directly proportional to m_1m_2. So the combined equation can be written as $F_g = g\frac{m_1m_2}{d^2}$.

This is Newton's Universal Law of Gravitation and is made up of both an inverse and a joint variation. The accepted numerical value for g is approximately 6.67×10^{-11}.

Lesson Practice

a. *(Ex 1)* Look at the data set below. How can you tell if it represents an inverse variation?

x	1	2	3	4
y	4	2	$1\frac{1}{3}$	1

b. *(Ex 1)* Is $k = 3.5$ the constant of variation for this data set? Explain.

x	3	4	5	6
y	0.5	0.375	0.3	0.25

c. *(Ex 2)* Determine if this data set is an inverse variation. If so, find the constant of variation and the equation.

x	$-\frac{1}{2}$	3	5	9
y	$-\frac{1}{2}$	$\frac{1}{12}$	$\frac{1}{20}$	$\frac{1}{36}$

d. *(Ex 2)* Determine the kind of variation represented by this data. Find the constant of variation and the equation for the data.

x	–2	8	10	20
y	–9	36	45	90

e. *(Ex 2)* Determine whether this is a direct or an inverse variation. If so, find the constant of variation and the equation.

x	1	2	3	4
y	2	6	8	10

f. *(Ex 3)* Is the equation $V = CR$, where V is voltage, C is current, and R is resistance, a joint variation? How do you know?

g. *(Ex 3)* **Economics** In economics, the graph of the *supply curve* has this behavior: As the supply (S) item decreases, the price (P) for the item increases. Likewise, when the supply of the item increases, the price decreases. The product of P and S is a constant. Name the type of variation that describes this economic phenomenon and write a generic equation.

h. *(Ex 3)* The potential energy (PE) of an object of mass m depends on the height (h) of the mass above Earth's surface. Use the data set at right to see if this is a joint variation. If so, find the constant of variation and the equation.

PE	m	h
196	2	10
588	3	20
1176	4	30
1960	5	40

i. *(Ex 4)* The intensity (I) of light is inversely proportional to the distance (d) from the light squared. Write an equation for this phenomenon.

j. *(Ex 4)* Two electrically charged objects have a charge of q_1 and q_2. The electric force (F_e) between the two charges is directly proportional to the product of the two charges. Write an equation for this phenomenon.

k. *(Ex 4)* Coulomb's Law states that the electric force (F_e) between two objects of charge q_1 and q_2 is directly proportional to the product of the charges and inversely proportional to the square of the distance (d) between the charges. Write an equation for this phenomenon.

Practice Distributed and Integrated

Analyze **Determine if each expression is a polynomial. If the expression is a polynomial, write it in standard form. If the expression is not a polynomial, explain why not.**

*1. (11) $\dfrac{3x + 7x^2}{10x^5}$

*2. (11) $28x^2 - x^9 + 12x^7$

*3. (8) **Data Analysis** Two scientists solved the following problem:

It takes 900 kg of acetylene to make 2400 kg of a compound. How many kilograms of other components will be required to make 3600 kg of the compound? Which scientist solved the problem using the correct steps? Explain your reasoning.

Scientist A	Scientist B
$\dfrac{1500}{2400} = \dfrac{x}{3600}$	$\dfrac{900}{2400} = \dfrac{x}{3600}$
$\dfrac{5}{8} = \dfrac{x}{3600}$	$\dfrac{3}{8} = \dfrac{x}{3600}$
$8x = 5(3600)$	$8x = 3(3600)$
$x = 2250$ kg	$x = 1350$ kg

*4. (7) **Geometry** An isosceles triangle has two angles that each measure $x°$. The third angle is 1.75 times the measure of either angle. Write and solve a linear equation to find the measure of each angle.

Determine the constant k, then solve the problem.

*5. (12) The number of revolutions per minute (RPM) varies inversely as the number of teeth in the gear. If 40 teeth result in 120 RPM, what would the RPM be if the gear had 30 teeth?

*6. (12) The number of students in every class varies directly with the number of basketballs. For one class there are 8 students and 2 basketballs. If in another class there are 7 basketballs, how many students are in this class?

*7. (12) The number of runners varies jointly as bicyclists and walkers. If 100 runners went with 4 bicyclists and 5 walkers, how many bicyclists would go with 20 runners and 2 walkers?

Simplify the expressions.

8. (3) $5x^7(3^3xy^2)$

9. (3) $\left(\dfrac{5xy^3}{2x^3y^2z^{-4}}\right)^2$

10. (3) $\dfrac{2^{-2}}{x^{-6}}$

11. (3) **Earth Science** The typical length of a green frog is about 5 to 10 cm. If a green frog is about 7.5×10^{-5} km, about how many green frogs would it take to equal the diameter of the Earth? The diameter is about 1.2753×10^4 km. Round the answer to the nearest ten thousandth.

Evaluate.

12. (1) $-8 - 3^2 - (-2)^2 - 3(-2) + 2$

13. (1) $-\{-[-5(-3 + 2)\ 7]\}$

14. (2) $-p^2 - p(a - p^2)$ if $a = 4$ and $p = -3$

Solve the inequalities.

15. (10) $-9 \leq 2x - 5 < 9$

16. (10) $-\dfrac{1}{2}x + 7 \geq 15$

***17.** (7) **Geography** The table shows the relationship of the annual estimated population for three cities in the year 2005. The population in Phoenix, Arizona, is represented by the variable x.

2005 City Population

City	Population
Albuquerque	$0.151x$
Madison	$0.336x$
Phoenix	x
Total of 3 cities	2,185,816

a. **Formulate** Write a linear equation to approximate the population in Phoenix in 2005.

b. **Estimate** Find the approximate population of Phoenix in 2005. Round to the nearest thousand.

18. (7) **Multi-Step** The number of ducks on the pond tripled when the new flock landed. Then, 11 more ducks came. The resulting number of ducks was 13 less than 4 times the original number. How many ducks were there to begin with?

Write each fraction or decimal as a percent.

19. (6) 1.3 **20.** (6) $\frac{5}{8}$ **21.** (6) $\frac{13}{20}$ **22.** (6) 0.4

23. (8) **Chemistry** A formula required that 500 kilograms (kg) of sulfur be used to produce 3000 kg of a compound. How many kilograms of other materials are needed to make 6000 kg of the new compound?

24. (4) **Multiple Choice** Which set of x- and y-values would make the equation $y = \frac{3}{4}x - 1$ true?

A $x = 8, y = -5$ **B** $x = -8, y = -7$

C $x = 4, y = -2$ **D** $x = -8, y = 7$

Graphing Calculator **Multiply the matrices. Check using a graphing calculator.**

***25.** (9) $\begin{bmatrix} -2 & 3 & -4 \\ 0 & -3 & -1 \\ 5 & 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 4 & 7 \\ -6 & 1 \\ 3 & 6 \end{bmatrix}$

***26.** (9) $\begin{bmatrix} -2 & -1 \\ 6 & 4 \end{bmatrix} \cdot \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

27. (3) **Astronomy** The speed of light is about 3×10^8 meters/second. If Saturn is about 1.35 $\times\ 10^{12}$ meters from the sun, how many seconds would it take light to reach Saturn from the Sun? Round the answer to the nearest hundredth of a second.

Justify **Determine if these statements are a tautology, a contradiction, or neither. Justify your answer by constructing truth tables.**

28. (Inv. 1) $(p \wedge q) \rightarrow \neg p$ **29.** (Inv. 1) $(p \vee q) \vee q$

30. (5) **Justify** Does the Commutative Property hold true for matrix subtraction? Justify your answer.

LAB 3

Calculating Points on a Graph

Graphing Calculator Lab (Use with Lessons 13, 15, and 30)

Calculating *y*-Values and Roots

A graphing calculator can be used to calculate y-values and zeros of equations. Use the equation $y = 2x - 4$ and find the y-values for $x = 3.7$ and $x = 0$. Then find the zero.

Graphing Calculator Tip

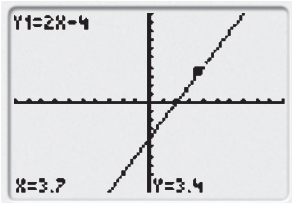
For help with graphing equations, see the graphing calculator keystrokes on page 19.

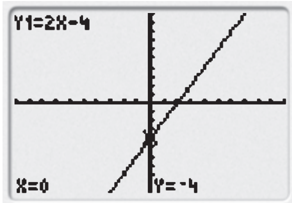
1. Enter the equation into the Y= editor.

2. Graph the equation.

3. Calculate the y-values.

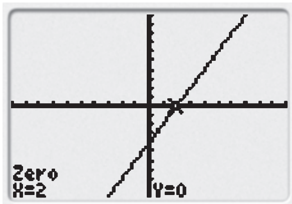
   Press 2nd TRACE to open the CALC menu, and select **1: value.** Enter the x-value and press ENTER. The x- and y-values will be displayed at the bottom of the screen and highlighted on the graph.





4. Calculate the zero.

   Press 2nd TRACE and select **2: zero.** Use the arrow keys to move the cursor to the left of the zero and press ENTER. Move the cursor to the right of the zero and press ENTER. Move the cursor to the approximate location of the zero and press ENTER. The zero will be displayed at the bottom of the screen and highlighted on the graph.



Calculating Minimums and Maximums

A graphing calculator can be used to find minimums and maximums.

Find the minimum of $y = x^2 + 3x - 4$.

1. Enter the equation into the Y= editor.

2. Graph the equation.

3. Calculate the minimum.

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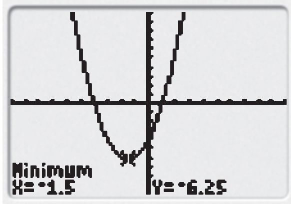
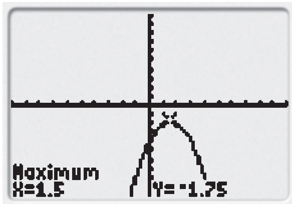
Press **2nd** **TRACE** and select **3: minimum.** Use the arrow keys ◀ ▲ ▼ ▶ to move the cursor to the left of the minimum and press **ENTER**. Move the cursor to the right of the minimum and press **ENTER**. Move the cursor to the approximate location of the minimum and press **ENTER**. The minimum will be displayed at the bottom of the screen and highlighted on the graph.



4. Calculate the maximum.

   The same process can be repeated to find the maximum of a parabola like $y = -x^2 + 3x - 4$. Simply select **4: maximum** from the CALC menu and repeat the same steps.



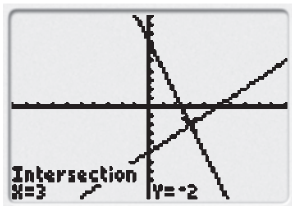
Calculating an Intersection Point

A graphing calculator can be used to find the intersection of two lines. Find the intersection of $y = x - 5$ and $y = -3x + 7$.

1. Enter the equations into the **Y=** editor.

2. Graph the equations.

3. Press **2nd** **TRACE** and select **5: intersect.** Press **ENTER** to select the first line, and then **ENTER** to select the second line. Use the arrow keys ◀ ▲ ▼ ▶ to move the cursor to the approximate location of the intersection and press **ENTER**. The solution is displayed as a decimal at the bottom of the screen.



Graphing Calculator Practice

a. Use a graphing calculator to find the zero of $y = -17x + 51$.

b. Use a graphing calculator to find the maximum of the parabola $y = -x^2 - 4x + 5$.

c. Use a graphing calculator to find the intersection of $y = -x + 2$ and $y = -5x - 1$.

LESSON 13

Graphing Linear Equations I

Warm Up

1. (4) **Vocabulary** The ____________ line test is used to determine if a graph is that of a function.

2. (SB) Subtract $-8 - (-6)$.

3. (2) Evaluate $-5x + 3y$ for $x = -2$ and $y = -3$.

New Concepts

An equation such as $y = 2x - 3$ is a linear equation in two variables. An ordered pair (x, y) is a solution to an equation in two variables if substituting the values of both x and y into the equation produces a true statement. For example, $(4, 5)$ is a solution to $y = 2x - 3$ since $5 = 2(4) - 3$. The graph of an equation in two variables is the set of all points (x, y) that are solutions to the equation.

Graphing Equations in Two Variables
Step 1: Construct a table of values. Choose a reasonable value for x and solve the equation for y. Repeat this step several times.
Step 2: Plot the points represented by the solutions to the equation.
Step 3: Connect the points to form a line. Extend the line and draw arrowheads on the ends to indicate that the line extends to infinity.

Example 1 Graphing a Linear Equation Using a Solution Table

Graph the equation $y = -2x + 1$.

Math Reasoning

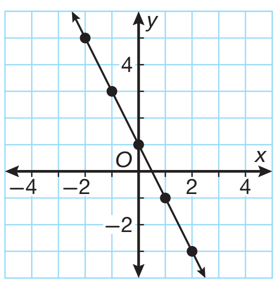
Verify Have students use a graphing calculator to check the y values.

SOLUTION

Construct a table of values.

x	-2	-1	0	1	2
y	5	3	1	-1	-3

Plot and connect the points. The points should form a line.



The points where the graph intersects the axes on the coordinate plane are called the **intercepts.** The point where the graph intersects the x-axis is called the ***x*-intercept,** and the point where the graph intersects the y-axis is called the ***y*-intercept.**

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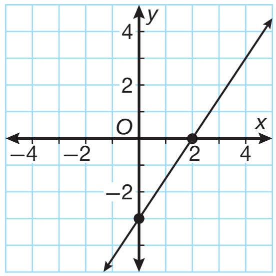
Graphing a Linear Equation Using Intercepts

Find the x-intercept.	Let $y = 0$. Solve the equation for x.
Find the y-intercept.	Let $x = 0$. Solve the equation for y.
Plot and connect the intercepts.	
Extend the line and add arrowheads to the ends.	

Example 2 Graphing a Linear Equation Using Intercepts

Graph $3x - 2y = 6$ using the intercepts.

SOLUTION



Let $y = 0$. Solve for x.

$$3x - 2(0) = 6$$
$$\frac{3x}{3} = \frac{6}{3}$$
$$x = 2$$

Let $x = 0$. Solve for y.

$$3(0) - 2y = 6$$
$$\frac{-2y}{-2} = \frac{6}{-2}$$
$$y = -3$$

Plot and connect points (2, 0) and (0, −3). Extend the line through the points and draw arrows on the ends of the line.

Caution

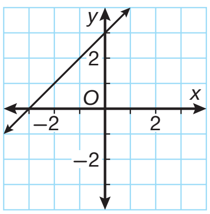
When finding the x intercept, do not make x equal 0. Make $y = 0$ then solve for x.

Slope of a Line

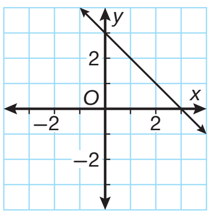
The **slope** of a line is the ratio of the vertical change (rise) to the horizontal change (run).

If (x_1, y_1) and (x_2, y_2) are the coordinates of any two points on the same line, the slope m of that line is calculated using the formula:

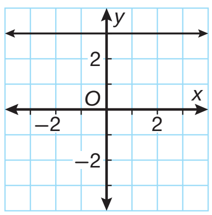
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{rise}{run}$$



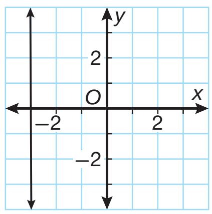
Rises



Falls



Horizontal



Vertical

Example 3 Classifying Lines Using Slope

Calculate the slope of the lines that contain the following pairs of points. Tell whether the line rises, falls, is horizontal, or is vertical.

a. $(-2, 3), (-1, 5)$

SOLUTION

$$m = \frac{5 - 3}{-1 - (-2)} = \frac{2}{1} = 2$$

Because m is positive, the line rises.

b. $(-4, -1), (2, -1)$

SOLUTION

$$m = \frac{-1 - (-1)}{2 - (-4)} = \frac{0}{6} = 0$$

Because $m = 0$, the line is horizontal.

c. $(6, -2), (6, -3)$

SOLUTION

$$m = \frac{-3 - (-2)}{6 - 6} = \frac{-1}{0}$$

Because m is undefined, the line is vertical.

d. $(3, 7), (-1, 10)$

SOLUTION

$$m = \frac{10 - 7}{-1 - 3} = \frac{3}{-4}$$

Because m is negative, the line falls.

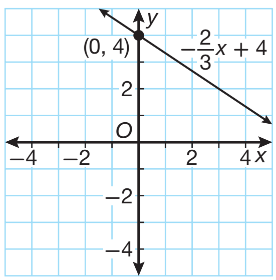
Math Reasoning

Verify If the order of the coordinates in the slope equation were reversed, would the value of the slope change?

The Slope-Intercept Form of a Linear Equation

A linear equation written in the form $y = mx + b$ is written in **slope-intercept form,** where m is the slope of the line and b is the y-intercept.

In the equation $y = -\frac{2}{3}x + 4$, the slope is $-\frac{2}{3}$ and the y-intercept is 4.



A line can be graphed using the slope-intercept form by plotting the y-intercept first, then using the slope to move to and plot another point. The slope is like a set of directions that indicate how many units to move and in which direction.

Moving from Point to Point Using the Slope

Positive slope	$\frac{\textit{up}}{\textit{right}}$	or	$\frac{\textit{down}}{\textit{left}}$
Negative slope	$\frac{\textit{up}}{\textit{left}}$	or	$\frac{\textit{down}}{\textit{right}}$

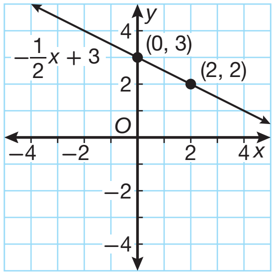
Example 4 Graphing a Line in Slope-Intercept Form

Identify the slope and y-intercept of the line with the given equation. Graph the line.

a. $y = -\frac{1}{2}x + 3$

SOLUTION

The equation is in the form $y = mx + b$. The slope of the line is $-\frac{1}{2}$ and the y-intercept is 3.



Plot the y-intercept (0, 3).

From (0, 3), move down 1 and 2 to the right to plot another point at (2, 2). Draw a line through (0, 3) and (2, 2).

b. $3x - 2y = 6$

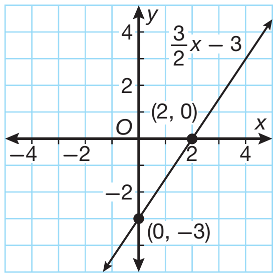
SOLUTION

Rewrite the equation in slope-intercept form by solving for y.

$$\begin{aligned} 3x - 2y &= 6 \\ \underline{-3x \qquad} &\underline{= -3x} \end{aligned}$$ Subtract $3x$ from both sides.

$$\frac{-2y}{-2} = \frac{-3x + 6}{-2}$$ Divide each term by -2.

$$y = \frac{3}{2}x - 3$$ The line has a slope of $\frac{3}{2}$ and a y-intercept of -3.



Plot the y-intercept $(0, -3)$.

From $(0, -3)$, move up 3 and 2 to the right to plot another point at (2, 0). Draw a line through $(0, -3)$ and (2, 0).

Caution

When dividing by a coefficient to isolate y, divide every term in the equation, not just the term with x.

Example 5 Application: Railroads

A section of the Lookout Mountain Incline Railway in Tennessee has a vertical change of 1 mile upward for every 1.37 miles it moves along horizontally. What is the slope of the railroad for this section?

SOLUTION

The slope of the railroad can be calculated using $m = \frac{\text{rise}}{\text{run}}$.

The rise = 1 mi and the run = 1.37 mi.

$$\text{The slope} = \frac{1 \cancel{\text{mi}}}{1.37 \cancel{\text{mi}}}$$

$$= \frac{1}{1.37}$$

$$= 0.73.$$

Hint

Notice that the units must be the same when you are finding the slope so that they cancel out.

Lesson Practice

a. (Ex 1) Graph the equation $y = -\frac{1}{2}x + 3$ by constructing a table of values. (Hint: Choose x-values that are multiples of 2.)

b. (Ex 2) Graph $-x + 4y = 8$ using the intercepts.

Calculate the slope of the lines that contain the following pairs of points. Tell whether the line rises, falls, is horizontal, or is vertical.
(Ex 3)

c. $(-3, 5), (5, -6)$

d. $(0, 4), (-2, 0)$

e. $(-4, 3), (8, 3)$

f. (Ex 4) Identify the slope and y-intercept of the line $y = \frac{1}{3}x + 2$. Graph the line.

g. (Ex 5) **Geography** A section of the Uncanoonuc Mountains, called the South Mountain USC Trail, is located in New Hampshire. The trail has a vertical change of about 24 feet vertically for every 50 feet it moves along horizontally. What is the slope of the trail?

Practice Distributed and Integrated

Determine the constant k, and solve the problem.

***1.** (12) **Multi-Step** The number of elk varied inversely as the number of deer and directly as the number of antelope. When there were 75 elk, there were 85 deer and 15 antelope. How many deer were there when there were 20 elk and 30 antelope? Round the answer down to the nearest whole number.

***2.** (12) **Formulate** The number of daisies varied inversely as the number of sunflowers and directly as the number of roses. When there were 65 daisies, there were 15 roses and 3 sunflowers. How many sunflowers were there when there were 5 daisies and 100 roses?

***3.** (13) Graph the equation $y = -\frac{3}{4}x + 1$ by constructing a table of values.

***4.** (13) Graph $-2x - y = 3$ using the intercepts.

***5.** (8) **Error Analysis** Two farmers solved the following problem:

Strawberries varied jointly as plums and tomatoes. If 500 strawberries went with 4 plums and 25 tomatoes, how many plums would go with 40 strawberries and 2 tomatoes?

Which farmer solved the problem using the correct steps? Explain your reasoning.

Farmer A	Farmer B
$500 = k(4)(25)$ $k = 5$ $40 = 5(p)(2)$ $p = 4$ plums	$500 = k\frac{(25)}{(4)}$ $k = 80$ $40 = 80\frac{(25)}{(p)}$ $p = 50$ plums

Calculate the slope of the lines that contain the following pairs of points. Tell whether the line rises, falls, is horizontal, or is vertical.

***6.** (13) $(-2, 0), (7, -5)$

7. (13) $(0, 4), (-2, 0)$

***8.** (13) Identify the slope and y-intercept of the line $y = -\frac{5}{4}x + 2$.

Analyze **Determine whether each expression is a polynomial. If the expression is a polynomial, rewrite it in standard form. If the expression is not a polynomial, explain why.**

***9.** (11) $25x^3 + x^{-7} + 2xy$

10. (11) $0.25x^4 + 5x^{\frac{4}{2}}$

11. (7) **Geography** The table shows the relationship of the annual estimated population for three cities in the year 2005. The population in Modesto, California, is represented by the variable x.

a. Formulate Write a linear equation to approximate the population in Modesto in 2005.

b. Estimate Find the approximate population of Modesto in 2005. Round to the nearest thousand.

2005 City Population

City	Population
Dallas	$5.879x$
St. Louis	$1.704x$
Modesto	x
Total of 3 cities	1,775,721

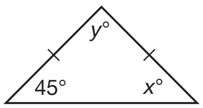
Evaluate.

12. (1) $-2 - 3(-2 - 2) - 5(-5 + 7)$

13. (1) $-[-2(-5 + 2) - (-2 - 3)]$

14. (2) $a^2 - y^3(a - y^2)y^2$ if $a = -2$ and $y = 3$

15. (7) **Geometry** Find x and y.



16. (10) **Justify** Explain why the inequality has no solution. $3x - 5 \geq 3(x + 7)$

17. (10) Solve the inequality, and graph its solution. $7x - 5 < 7(x + 5) + 2$

18. (10) **Civil Engineering** One cubic foot of pennies contains approximately 4.9×10^4 pennies. If approximately 1.8×10^{12} pennies would fit in the Empire State Building in New York City, what is the building's approximate volume in cubic feet? Write your answer in scientific notation. Round to the nearest tenth.

Solve for *p*.

19. (7) $(7p - 6) + 3p - (-2 - p) = 73$

20. (7) $\dfrac{7(p + 3)}{2} = 21$

21. (7) $4p + (p - 4)(2) = 8p - 7$

Use the matrices to find the products. Check the answer on a graphing calculator.

$$A = \begin{bmatrix} -1 & 2 \end{bmatrix},\ B = \begin{bmatrix} 4 & 0 & 2 \\ -3 & 5 & 1 \end{bmatrix},\ C = \begin{bmatrix} 7 & 1 \\ -2 & 0 \\ 0 & -3 \end{bmatrix}$$

22. *(9)* Find AB.

23. *(9)* Find ABC.

24. *(11)* **Citrus Production** The production of oranges and grapefruit (in millions of tons) in the U.S. for the years 2000 through 2006 is modeled by the polynomial functions below. The variable x represents the number of years since 2000.

Oranges $f(x) = -0.044x^3 + 0.258x^2 - 0.657x + 12.883$

Grapefruit $g(x) = 0.004x^3 - 0.065x^2 - 0.040x + 2.695$

What polynomial function is a model for the combined production of oranges and grapefruit in the United States for the years 2000–2006?

Write each fraction or decimal as a percent.

25. *(6)* 0.005

26. *(6)* $\frac{1}{6}$

27. *(6)* $\frac{5}{16}$

***28.** *(12)* **Meteorology** According to the Saffir-Simpson scale, hurricanes fall into five categories based on wind speed. The table below shows the relationship between category, wind speed, and storm surge (the height of waves). Analyze the data to determine the relationship between wind speed and storm surge. Is it a direct, joint, or inverse variation?

Category	Minimum Wind Speed (mph)	Minimum Storm Surge (ft)
1	74	4
2	96	6
3	111	9
4	131	13
5	155	19

29. *(4)* **Multiple Choice** Which of the following sets of ordered pairs are functions?

A $(-7, 6), (-7, 3), (4, 3)$

B $(-2, 5), (5, -2), (-2, 7)$

C $(1, -1), (-1, 1), (2, 1)$

D $(0, 1), (0, 2), (0, 3)$

***30.** *(13)* **Geography** A section of the Uncanoonuc Mountains, called the West Side Trail, is located in New Hampshire. The trail has a vertical change of about 268 feet vertically for every 1000 feet it changes horizontally. What is the slope of the trail?

LESSON
14 Finding Determinants

Warm Up

1. (5) **Vocabulary** A(n) __________ (*element, matrix*) is a rectangular array of numbers.

2. (7) Solve $4x + 6 = 22$.

3. (9) Simplify $\begin{bmatrix} 3 & 2 \\ -1 & 7 \end{bmatrix} \times \begin{bmatrix} 4 & -3 \\ 6 & 8 \end{bmatrix}$.

New Concepts

Every square matrix is associated with one real number called the **determinant** of the matrix. **If a matrix is not a square matrix, it does not have a determinant.** Enclosing the matrix within vertical lines designates the determinant of a matrix.

$$\begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix} \qquad \begin{vmatrix} 4 & 3 \\ 2 & 5 \end{vmatrix}$$

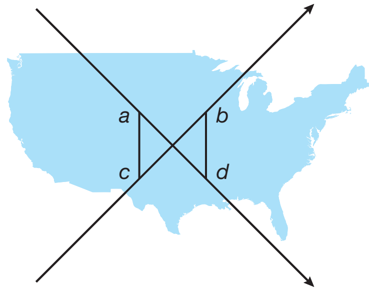
matrix — determinant

The determinant of a 2×2 square matrix is found by subtracting the product of the entries in one diagonal from the product of the entries in the other diagonal. The order in which you subtract is important and is shown by this diagram.

$$\text{Determinant} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - cb$$

$-cb$ $\quad$ $+ad$

This memory device can be used to help remember which product is positive and which is negative. The minus sign points to the cold state of Maine and the plus sign points to the warm state of Florida.



Example 1 Finding the Determinant of a 2 × 2 Matrix

a. Evaluate $\begin{vmatrix} -4 & -3 \\ 2 & 7 \end{vmatrix}$.

SOLUTION Subtract cb from ad.

$$\begin{vmatrix} -4 & -3 \\ 2 & 7 \end{vmatrix} = (-4)(7) - (2)(-3) = -28 - (-6) = -22$$

b. Evaluate $\begin{vmatrix} -7 & 5 \\ 3 & -4 \end{vmatrix}$.

SOLUTION

$$\begin{vmatrix} -7 & 5 \\ 3 & -4 \end{vmatrix} = (-7)(-4) - (3)(5) = 28 - 15 = 13$$

Caution

Subtracting the products in the incorrect order will give you the opposite of the value of the determinant.

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Example 2 Solving Determinant Equations

Find x. $\begin{vmatrix} x+4 & 3 \\ 2 & 4 \end{vmatrix} = 18$

SOLUTION Begin this type of problem the same as you would if you were finding the value of a determinant. Then solve for the missing value.

$(4)(x+4) - (2)(3) = 18$	Write the determinant as an equation.
$4x + 16 - 6 = 18$	Multiply.
$4x = 8$	Simplify.
$x = 2$	Divide.

There are two methods for finding the determinant of a 3×3 matrix with pencil and paper. The first method is known as expansion by minors. The second method is shown in Example 3 part b.

Expansion by Minors

If you pick an element of a matrix and cover its row and column, the determinant of the square matrix that remains is that element's **minor.**

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \quad \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \quad \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$$

To find the determinant, multiply each element in one row or column by its minor. Then subtract the middle product and add the final product.

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1\begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1\begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1\begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$$

Example 3 Finding the Determinant of a 3 × 3 Matrix

Hint

If you choose to expand a row or column that has a zero, it makes your calculations easier.

a. Find the determinant $\begin{vmatrix} 5 & 0 & 1 \\ -4 & 2 & -7 \\ 3 & -6 & 8 \end{vmatrix}$ using expansion by minors.

SOLUTION Find the minors for each element in one row or column.

$$\begin{vmatrix} 5 & 0 & 1 \\ -4 & 2 & -7 \\ 3 & -6 & 8 \end{vmatrix} \quad \begin{vmatrix} 5 & 0 & 1 \\ -4 & 2 & -7 \\ 3 & -6 & 8 \end{vmatrix} \quad \begin{vmatrix} 5 & 0 & 1 \\ -4 & 2 & -7 \\ 3 & -6 & 8 \end{vmatrix}$$

Multiply each minor by its element and combine using the correct signs.

$$5\begin{vmatrix} 2 & -7 \\ -6 & 8 \end{vmatrix} - 0\begin{vmatrix} -4 & -7 \\ 3 & 8 \end{vmatrix} + 1\begin{vmatrix} -4 & 2 \\ 3 & -6 \end{vmatrix}$$

Evaluate the minors and simplify.

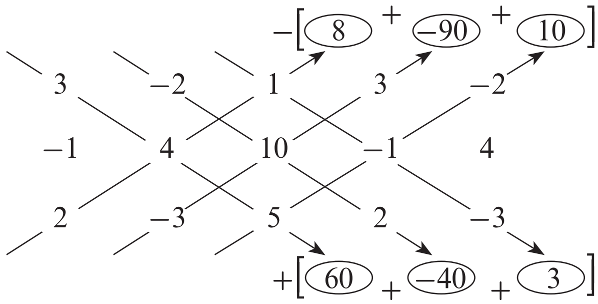
$$5[(2)(8) - (-6)(-7)] - 0[(-4)(8) - (3)(-7)] + 1[(-4)(-6) - (3)(2)]$$
$$= 5[16 - 42] - 0 + 1[24 - 6] = 5(-26) + 18 = -112$$

b. Find the determinant $\begin{vmatrix} 3 & -2 & 1 \\ -1 & 4 & 10 \\ 2 & -3 & 5 \end{vmatrix}$.

SOLUTION To begin the other method, copy the elements of the determinant and then repeat the first two columns to the right of the third column.

$$\begin{matrix} 3 & -2 & 1 & 3 & -2 \\ -1 & 4 & 10 & -1 & 4 \\ 2 & -3 & 5 & 2 & -3 \end{matrix}$$

Multiply on the diagonals as shown and add the products.



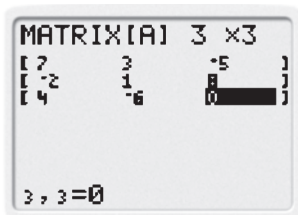
Subtract the sum of the upper products from the sum of the lower products.

$$\begin{vmatrix} 3 & -2 & 1 \\ -1 & 4 & 10 \\ 2 & -3 & 5 \end{vmatrix} = (60 - 40 + 3) - (8 - 90 + 10) = 95$$

Example 4 Using a Calculator to Find the Determinant

Find the determinant $\begin{vmatrix} 7 & 3 & -5 \\ -2 & 1 & 8 \\ 4 & -6 & 0 \end{vmatrix}$ using a calculator.

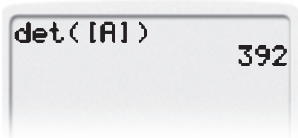
SOLUTION Store the data in a matrix.



Graphing Calculator

For help with storing data in a matrix, refer to page 27.

2nd QUIT MODE; MATRX x^{-1}; Choose MATH; Choose **1:det(**; MATRX x^{-1}; Choose the matrix;); ENTER.



Verify Use expansion by minors to check the calculator answer.

$$7\begin{vmatrix} 1 & 8 \\ -6 & 0 \end{vmatrix} - 3\begin{vmatrix} -2 & 8 \\ 4 & 0 \end{vmatrix} + (-5)\begin{vmatrix} -2 & 1 \\ 4 & -6 \end{vmatrix} =$$

$$7[(1)(0) - (-6)(8)] - 3[(-2)(0) - (4)(8)] + (-5)[(-2)(-6) + (1)(4)] =$$
$$7[0 - (-48)] - 3[0 - 32] + (-5)[12 - 4] = 7(48) - 3(-32) + (-5)(8) = 392$$

Example 5 Application: Geometry

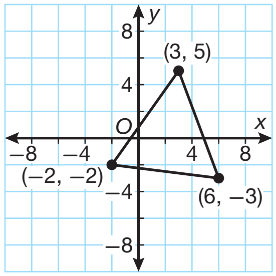
A triangle with vertices (x_1, y_1), (x_2, y_2), and (x_3, y_3) has an area equal to the absolute value of

$$\frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ 1 & 1 & 1 \end{vmatrix}.$$

Math Reasoning

Connect Why do you take the absolute value to find the area?

Find the area of the triangle below.



SOLUTION Substitute the coordinates of the vertices into the given matrix.

$$\frac{1}{2}\begin{vmatrix} 3 & 6 & -2 \\ 5 & -3 & -2 \\ 1 & 1 & 1 \end{vmatrix}$$

Expand by minors.

$$\begin{vmatrix} \cancel{3} & 6 & -2 \\ \cancel{5} & -3 & -2 \\ 1 & \cancel{1} & \cancel{1} \end{vmatrix} \qquad \begin{vmatrix} 3 & \cancel{6} & -2 \\ 5 & \cancel{-3} & -2 \\ \cancel{1} & 1 & \cancel{1} \end{vmatrix} \qquad \begin{vmatrix} 3 & 6 & \cancel{-2} \\ 5 & -3 & \cancel{-2} \\ \cancel{1} & \cancel{1} & 1 \end{vmatrix}$$

$$\frac{1}{2}\begin{vmatrix} 3 & 6 & -2 \\ 5 & -3 & -2 \\ 1 & 1 & 1 \end{vmatrix} = \frac{1}{2}\left[1\begin{vmatrix} 6 & -2 \\ -3 & -2 \end{vmatrix} - 1\begin{vmatrix} 3 & -2 \\ 5 & -2 \end{vmatrix} + 1\begin{vmatrix} 3 & 6 \\ 5 & -3 \end{vmatrix}\right]$$

Simplify.

$$\frac{1}{2}[1[(6)(-2) - (-3)(-2)] - 1[(3)(-2) - (5)(-2)] + 1[(3)(-3) - (5)(6)]] =$$

$$\frac{1}{2}[(-12 - 6) - (-6 - (-10)) + (-9 - 30)] = \frac{1}{2}[-18 - 4 + (-39)] = -\frac{61}{2}$$

The absolute value of $-\frac{61}{2}$ is $\frac{61}{2}$ or $30\frac{1}{2}$.

The area of the triangle is $30\frac{1}{2}$ square units.

Lesson Practice

a. Evaluate $\begin{vmatrix} 1 & -6 \\ 0 & 8 \end{vmatrix}$.
(Ex 1)

b. Find x. $\begin{vmatrix} x-7 & 8 \\ -1 & -5 \end{vmatrix} = 23$
(Ex 2)

c. Find the determinant $\begin{vmatrix} 5 & -9 & -5 \\ -5 & -3 & 8 \\ 2 & -8 & -1 \end{vmatrix}$ using expansion by minors.
(Ex 3)

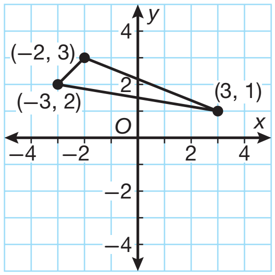
d. Find the determinant $\begin{vmatrix} -4 & 2 & 7 \\ 9 & 3 & -4 \\ 7 & -1 & -2 \end{vmatrix}$.
(Ex 3)

e. Find the determinant $\begin{vmatrix} -8 & -9 & -4 \\ -7 & 4 & 9 \\ 4 & 1 & 7 \end{vmatrix}$ using a calculator.
(Ex 4)

f. A triangle with vertices (x_1, y_1), (x_2, y_2), and (x_3, y_3) has an area equal to the absolute value of
(Ex 5)

$$\frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ 1 & 1 & 1 \end{vmatrix}.$$

Find the area of the triangle.



Practice Distributed and Integrated

***1.** **Multi-Step** The regular price of a cell phone was \$89.99. The price was reduced by 25%.
(6)

a. Find the amount saved. Then find the new price of the cell phone. Round to the nearest hundredth.

b. If the new price is later reduced by \$15, find the percent change from the original price to the lowest price at which the cell phone is sold. Round to the nearest percent.

***2.** **Error Analysis** Two students analyzed the data table shown at right. One student determined that the relationship between the variables is an inverse variation, while the other determined that it is a direct variation. Which student is correct? What error did the other student make?
(12)

Beads

Quantity	Price
1000	\$90
900	\$100
750	\$120
600	\$150
500	\$180

Student A

Quantity	Price	$Q \times P$
1000	\$90	90,000
900	\$100	90,000
750	\$120	90,000
600	\$150	90,000
500	\$180	90,000

Student B

Quantity	Price	$Q \div P$
1000	\$90	90,000
900	\$100	90,000
750	\$120	90,000
600	\$150	90,000
500	\$180	90,000

Identify which equations are direct variations and which are inverse variations.

3. (8, 12) $y = 3x$

4. (8, 12) $y = \frac{1}{2}x$

5. (8, 12) $y = \frac{20}{x}$

Model **Graph the equation by using the stated method.**

6. (13) Graph the equation $y = -\frac{2}{3}x + 4$ using a table of values.

***7.** (13) Graph the equation $y = \frac{5}{4}x + 1$ using its intercepts.

Calculate the slope of the lines that contain the following pairs of points. Tell whether the line rises, falls, is horizontal, or is vertical.

8. (13) $(-8, 2), (3, -1)$

9. (13) $(6, 4), (-5, -2)$

10. (13) $(6, 2), (-5, 2)$

***11.** (14) Evaluate $\begin{vmatrix} -1 & 2 \\ 3 & 5 \end{vmatrix}$.

***12.** (14) Find x. $\begin{vmatrix} 3 & 5 \\ -2x + 1 & -7 \end{vmatrix} = 24$

***13.** (14) **Verify** Evaluate $\begin{vmatrix} 7 & -1 & 2 \\ -3 & 0 & -2 \\ 4 & 3 & 5 \end{vmatrix}$. Show the steps of the method specified.

a. Use expansion by minors.

b. Set up diagonals. Find the difference of products.

***14.** (14) **Graphing Calculator** Use a graphing calculator to find $\begin{vmatrix} 7 & 3 & 8 \\ 0 & -5 & 6 \\ -1 & 2 & -4 \end{vmatrix}$.

***15.** (14) **Geometry** Find the area of the triangle with coordinates $(-2, 5), (7, 3), (-4, -3)$.

16. (5) **Meteorology** The average temperatures were calculated, in degrees Fahrenheit, for the city of Hilo, Hawaii, and for the city of Lihue, Hawaii, from the year 1971 to 2000.

City	Jan	Feb	Mar	Apr	May	Jun
Hilo	71.4	71.5	72.0	72.5	73.7	75.1
Lihue	71.7	71.1	72.7	73.9	75.4	77.7

a. Model Record this information in a matrix and name it matrix A.

b. Let $B = \begin{bmatrix} -32 & -32 & -32 & -32 & -32 & -32 \\ -32 & -32 & -32 & -32 & -32 & -32 \end{bmatrix}$. Convert the temperatures from Fahrenheit to Celsius by applying the operations on matrix A. Round each element to the nearest tenth.

$$C = \frac{5}{9}([A] + [B])$$

17. (8) **Measurement** The number of feet varies directly with the number of yards. If a rope measures 18 feet, it measures 6 yards. Josh is asked to write a proportion to find the number of yards in a rope that measures 33 feet. Josh wrote the proportion $\frac{18}{6} = \frac{x}{33}$. Identify Josh's error and solve the problem correctly.

18. (10) **Formulate** A friend is trying to earn at least $500 this month by delivering newspapers. If she earns $0.25 per delivery, how many papers will she have to deliver? Write an inequality and solve it.

Solve the inequalities.

19. (10) $-1.9 < 2(x - 0.25) \leq 2.5$

***20.** (10) $-2 < \dfrac{2(x-7)}{3} \leq 5$

Basketball **In basketball, points are determined by throwing the ball into the basket, also called making a shot. Use the tables to answer the problems.**

Player	Behind the 3-Point Line	Inside the 3-Point Line	Foul
Lan	2	1	1
Stacey	1	1	1
Robert	2	3	0

Type of Shot	Points
Behind the 3-Point Line	3
Inside the 3-Point Line	2
Foul	1

21. (5) Record Lan's, Stacey's, and Robert's points into a matrix. Name it matrix A. Record the point system into a matrix and name it matrix B.

***22.** (9) Find the total points for each player by finding the product $A \cdot B$.

Determine the kind of variation, if any, for each equation.

23. (8, 12) $y = mx + b$, for $m \neq 0$, $b \neq 0$

24. (8, 12) $y = mx + b$, for $m \neq 0$ and $b = 0$.

Identify the subsets of real numbers of which each number is a member.

25. (1) 0.33

26. (1) 0

27. (1) $\sqrt{5}$

28. (12) **Meteorology** According to the Saffir-Simpson scale, hurricanes fall into five categories based on wind speed. The table at right shows the relationship between category, wind speed, and storm surge (the height of waves). Analyze the data to determine the relationship between wind speed and storm surge. Is it a direct, joint, or inverse variation?

Category	Maximum Wind Speed (mph)	Maximum Storm Surge (ft)
1	95	5
2	100	8
3	130	12
4	155	18
5	155+	19+

29. (11) **Multiple Choice** Which polynomial is in standard form?

A $\frac{1}{2}x^3 - x$

B $\frac{1}{3} - \frac{1}{3}x^3$

C $4x^2 + 3x^3 + 2x^4$

D $x^4 + x^5$

30. (11) **Simple Interest** Al's grandfather deposited the amounts shown in Bank A as gifts on Al's birthday during the years 2002 through 2005. His aunt deposited the amounts shown in Bank B. The value of the accounts on 7/2/06 is represented by these polynomials, where $x = 1 + r$, and r is the interest rate for both accounts:

Bank A $500x^4 + 1000x^3 + 600x^2 + 500x$

Bank B $1000x^4 + 200x^3 + 200x^2 + 200x$

Date	Bank A	Bank B
7/2/02	\$500	\$1000
7/2/03	\$1000	\$200
7/2/04	\$600	\$200
7/2/05	\$500	\$200

a. Write the polynomial that represents the total value of both accounts on 7/2/06.

b. What is the total value of both accounts on 7/2/06 if the interest rate is 5%?

LESSON **15**

Solving Systems of Equations by Graphing

Warm Up

1. **Vocabulary** The __________ of a line is the change in the y-coordinates divided by the change in the x-coordinates. *(13)*
2. Give the slope and the y-intercept of the line with equation $y = -3x + 5$. *(13)*
3. Determine if the point (2, 1) satisfies $y = 3x - 5$ and $y = x + 3$. *(2)*

New Concepts

A **system of equations** is a collection of two or more equations containing two or more of the same variables. A **linear system** contains two linear equations in two like unknowns. Linear systems can be solved by graphing. Both of the equations are graphed on the same coordinate grid. The coordinates of the point where the lines **intersect,** or cross, give the solution.

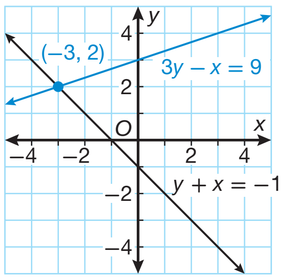
Example 1 Solving Systems Using Tables and Graphs

a. Solve this system by graphing.

$$3y - x = 9$$
$$y + x = -1$$

SOLUTION Solve each equation for y to get the slope-intercept form. Then graph the equations.

$$y = \frac{1}{3}x + 3$$
$$y = -x - 1$$



The solution of the system is (−3, 2).

b. Make a table of values for each equation to solve the system in part **a.**

SOLUTION $3y - x = 9$ $\qquad$ $y = -x - 1$

x	y
−6	1
−3	2
0	3
3	4

x	y
−6	5
−3	2
0	−1
3	−4

When $x = -3$ the y values are the same for both equations. The solution to the system is (−3, 2).

Hint

When choosing values of x to graph an equation, include negative values and zero.

Online Connection
www.SaxonMathResources.com

Not all solutions to a linear system will intersect at integer values of x and y. A graphing calculator enables more exact solutions to be found.

Math Reasoning

Verify Use an algebraic method to verify that the solution (1.6, −0.6) is correct.

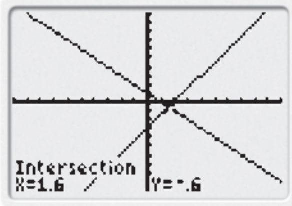
Example 2 Using a Graphing Calculator to Solve Systems

Solve this system by graphing.

$$3x - 2y = 6$$
$$y + x = 1$$

SOLUTION Solve each equation for y to get the slope-intercept form. Then graph the equations and find the point of intersection using a graphing calculator.

$$y = \frac{3}{2}x - 3$$
$$y = -x + 1$$



The lines intersect at (1.6, −0.6), so (1.6, −0.6) is the solution of the system.

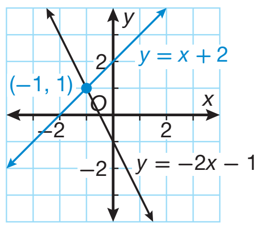
Solutions of systems are ordered pairs. There are three possibilities for the number of solutions to a linear system: one ordered pair, an infinite number of ordered pairs, or no ordered pairs.

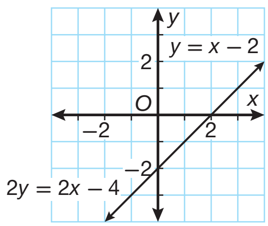
A linear system that has at least one solution is called a **consistent system.**

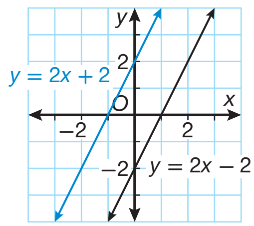
A system with no solution is called an **inconsistent system.**

A linear system is a **dependent system** when one of the equations in the system contains all solutions common to the other equation in the system.

Otherwise the system is an **independent system.**

<table>
<tr><th colspan="3">Classifying Systems of Equations</th></tr>
<tr>
<td>

Lines intersect at one point.
One solution: (−1, 1)
System is consistent and independent.</td>
<td>

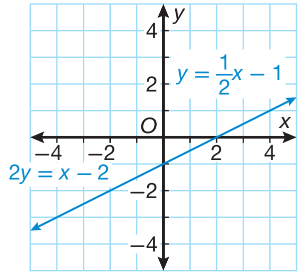
Lines coincide.
Infinite number of solutions.
System is consistent and dependent.</td>
<td>

Lines are parallel.
No points in common.
System is inconsistent.</td>
</tr>
</table>

Example 3 Classifying Linear Systems

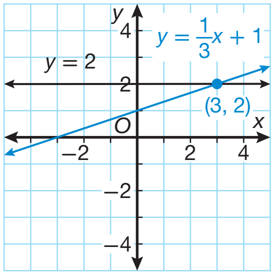
Determine if the following linear systems are consistent and independent, consistent and dependent, or inconsistent. If the system is consistent, give the solution.

a.



SOLUTION consistent and dependent; infinite set of ordered pairs

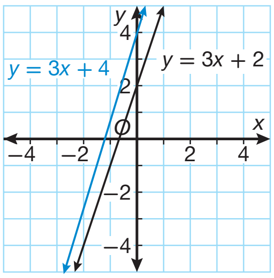
b.



SOLUTION consistent and independent; (3, 2)

c. $3x - y = -2$
$2y = 6x + 8$

SOLUTION Inconsistent



Math Reasoning

Generalize How can it be determined that a system is inconsistent without graphing?

Example 4 Application: Consumer Math

Music Masters sells MP3s for \$0.95 each to nonmembers. Members pay \$5 per year and \$0.90 for each MP3 purchase. After how many MP3 purchases will the total cost be the same for members and nonmembers?

SOLUTION Write two equations in two variables to represent the cost of each type of membership. Solve using a graphing calculator and check by substituting the solution into the equations.

Let x represent the number of MP3s purchased.
Let y represent the cost of the purchase.

Members: $y = 5 + 0.9x$

Nonmembers: $y = 0.95x$

When $x = 100$, the y values are the same for both equations. The cost for members and nonmembers is \$95 when 100 MP3s have been purchased.

Graphing Calculator Tip

The graphs of the lines $y = 0.95x$ and $y = 5 + 0.9x$ appear to be parallel in a standard window. Use the Zoom Out feature to see that the lines do intersect.

Check

$y = 0.95x$ $\qquad$ $y = 5 + 0.9x$

$95 = 0.95(100)$ $\qquad$ $95 = 5 + 0.9(100)$

$95 = 95$ $\qquad$ $95 = 5 + 90$

$95 = 95$

Lesson Practice

a. Solve this system by graphing.
(Ex 1)

$$2x - y = 6$$
$$y + 2x = -10$$

b. Make a table of values for each equation to solve the system in part **a.**
(Ex 1)

$2x - y = 6$

x	y

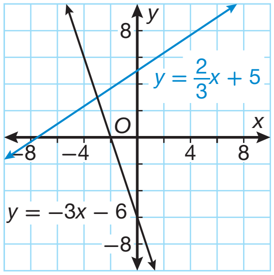
$y + 2x = -10$

x	y

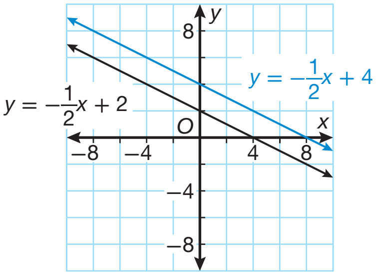
c. Solve this system by graphing.
(Ex 2)

$$5x - 6y = 36$$
$$2y - 3x = -4$$

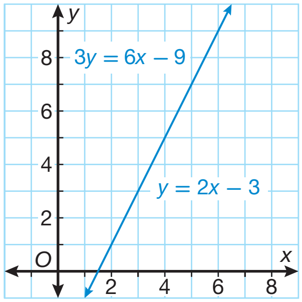
d. Determine if the linear system at the right is consistent and independent, consistent and dependent, or inconsistent. If the system is consistent, give the solution.
(Ex 3)



e. Determine if the linear system at the right is consistent and independent, consistent and dependent, or inconsistent. If the system is consistent, give the solution.
(Ex 3)



f. Determine if the linear system at the right is consistent and independent, consistent and dependent, or inconsistent. If the system is consistent, give the solution.
(Ex 3)



g. (Ex 4) Superfit Gym charges nonmembers \$20 for each fitness class. Members pay \$75 per year and \$5 for each fitness class. After how many fitness classes will the total cost be the same for members and nonmembers?

Practice Distributed and Integrated

Given $h(x) = 6x + 3$ and $p(x) = 2x^2 - x$, find each of the following.

***1.** (4) $p(-5)$

2. (4) $h\left(\frac{1}{2}\right)$

Identify whether the equation is a direct, inverse, or joint variation.

3. (12) $\frac{1}{3}y = x$

***4.** (12) $y = \frac{1}{2}xz$

5. (12) $xy = 10$

Geometry **For each equation identify the kind of variation it represents.**

6. (12) Circumference $= 2\pi \times$ Radius

7. (12) Area of a circle $= \pi(\text{Radius})^2$

***8.** (6) **Multi-Step** A jacket is on sale for 75% off the original price of \$39.99. Find the amount of the price reduction. Then find the new price, which includes a sales tax of 5%.

9. (14) **Multiple Choice** Evaluate $\begin{vmatrix} -1 & 3 \\ 2 & 3 \\ 7 & -4 \end{vmatrix}$.

A 0

B -9

C -29

D This matrix does not have a determinant.

10. (13) Given the equation $8x - 2y = -10$,

a. identify the slope and the y-intercept.

b. graph the line.

Use the slope formula to find the missing coordinate.

***11.** (13) The slope of a line is $\frac{4}{9}$. The points $(3, 8)$ and $(-6, y)$ lie on the line. Find y.

12. (13) The slope of a line is $-\frac{7}{3}$. The points $(4, -4)$ and $(x, 3)$ lie on the line. Find x.

***13.** (6) **Error Analysis** The price of a product decreased from \$9 to \$6. A student incorrectly computed the percent of decrease to be 50%. What was the student's error?

14. (14) Evaluate $\begin{vmatrix} -5 & 2 \\ 1 & -3 \end{vmatrix}$.

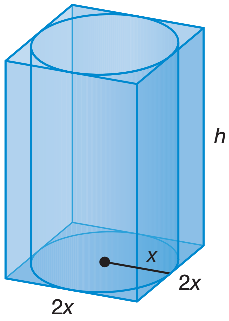
15. (14) Find x. $\begin{vmatrix} 2x & 3 \\ -x - 4 & -1 \end{vmatrix} = 10$

***16.** (15) **Graphing Calculator** Graph to solve the system. Then classify the system.

$2x - y = -2$

$2y = 4x - 4.$

*17. (11) **Woodworking** A lathe operator has a piece of wood in the shape of a square prism with dimensions $2x$, $2x$, and h. He wants to make the largest possible cylinder by turning the wood on the lathe and cutting it down. The cylinder will have radius x and height h. Write a polynomial that represents the volume of wood he will cut off to make the cylinder. (The formula for the volume of a cylinder is $V = \pi r^2 h$.)



18. (12) **Physics** According to Ohm's Law, a direct current flowing in a wire is inversely proportional to the resistance of the wire. If the current in a wire is 0.01 amperes (A) and the resistance is 1200 ohms (Ω), what resistance will result in a current of 0.1 A?

19. (7) **Travel** Aaron is traveling 955 miles from Phoenix, Arizona, to Jackson, Wyoming. He leaves Phoenix, driving 410 miles before stopping for gas in Cortez, Colorado. He makes one last stop in Rock Springs, Wyoming, before traveling the final 180 miles to his destination. Write and solve an equation to find the number of miles Aaron drove between Cortez and Rock Springs.

*20. (4) **Formulate** Write a function that represents the data set $(-4, 16)$, $(1, 1)$, $(3, 9)$, $(4, 16)$, $(7, 49)$.

21. (10) **Business** A snow cone vendor wants to make at least \$2,500 the first month. If he charges \$1.25 per snow cone, how many snow cones will he have to sell? Write and solve an inequality to find how many snow cones he will have to sell.

22. (10) **Verify** Solve $-2(7x + 1) \leq -8x - 9$. Check the answer by substituting an integer that makes the inequality true.

23. (9) **Scoring** Andrew, Max, and Gracie are all taking a test where each type of question is given a certain point value. Use matrix multiplication to find out who scored the greatest number of points on the test.

Questions answered correctly

	Type 1	Type 2	Type 3
Andrew	7	5	4
Max	6	7	3
Gracie	7	4	5

Point Value	
Type 1	3
Type 2	5
Type 3	10

Use the matrices to find the products, if possible.

$$A = \begin{bmatrix} -1 \\ 5 \\ 3 \end{bmatrix} \qquad B = \begin{bmatrix} -2 & 0 & 2 \end{bmatrix}$$

24. (9) AB

25. (9) BA

26. (15) **Multiple Choice** Which is a solution to the system below?

$$-x + 2y = 6$$
$$-3x + 4y = 4$$

A (7, 5) **B** (8, 7) **C** (0, 0) **D** (−16, −11)

***27.** (15) **Graphing Calculator** Use a graphing calculator to determine the solution of the system.

$-2x - y = 2$

$3x + 2y = 1$

***28.** (15) **Write** The solution of a system is a single point. Classify the system and describe the graph.

29. (8) **Meaurement** In a certain recipe, the amount of sugar is directly proportional to the amount of flour. If 3 cups of sugar are used with 8 cups of flour, how many cups of sugar are used with 12 cups of flour?

30. (3) **Population** The U.S. population is approximately 2.99×10^8. In one year, the United States produced 4.78×10^{11} pounds of garbage. Approximately how much garbage did the average American produce that year?

LESSON

16 Using Cramer's Rule

Warm Up

1. **Vocabulary** A __________ (***denominator, determinant***) is a real number associated with a square matrix. (14)

2. Find the determinant of $\begin{vmatrix} 4 & -3 \\ 5 & 2 \end{vmatrix}$. (14)

3. Solve the system. $\begin{aligned} 2x + y &= 2 \\ 4x - y &= 10 \end{aligned}$ (15)

New Concepts

Cramer's rule is a method for solving systems of linear equations using determinants. Elimination can be used to solve a general system of linear equations for x and y, as shown here.

$$\begin{array}{lcccl} ax + by = e & \rightarrow & (d) & \rightarrow & dax + dby = de \\ cx + dy = f & \rightarrow & (-b) & \rightarrow & \underline{-bcx - bdy = -bf} \\ & & & & (ad - bc)x = de - bf \rightarrow x = \dfrac{de - bf}{ad - bc} \end{array}$$

$$\begin{array}{lcccl} ax + by = e & \rightarrow & (-c) & \rightarrow & -cax - cby = -ce \\ cx + dy = f & \rightarrow & (a) & \rightarrow & \underline{acx + ady = af} \\ & & & & (ad - bc)y = af - ce \rightarrow y = \dfrac{af - ce}{ad - bc} \end{array}$$

Observe that the denominators of x and y are the same and that their numerators are different. The same results are found by simplifying the following expressions.

$$x = \frac{\begin{vmatrix} e & b \\ f & d \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}} = \frac{ed - fb}{ad - cb} \qquad y = \frac{\begin{vmatrix} a & e \\ c & f \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}} = \frac{af - ce}{ad - cb}$$

Math Language

When a constant is multiplied by a variable, it is called a **coefficient.**

Note that the elements of the determinants in the denominators are the coefficients of x and y in the given equations. For this reason, this matrix is called the **coefficient matrix.** The matrices in the numerators are different. In the matrices in the numerators, the constants replace the coefficients of x when we solve for x, and the constants replace the coefficients of y when we solve for y.

Cramer's Rule
The solutions of the linear system $\begin{aligned} ax + by &= e \\ cx + dy &= f \end{aligned}$ are
$x = \dfrac{\begin{vmatrix} e & b \\ f & d \end{vmatrix}}{D}$ and $y = \dfrac{\begin{vmatrix} a & e \\ c & f \end{vmatrix}}{D}$, where D is the coefficient matrix.

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Example 1 Using Cramer's Rule to Solve Systems of Equations

Use Cramer's rule to solve $\begin{aligned} 3x + 2y &= -1 \\ 4x - 3y &= 10 \end{aligned}$.

SOLUTION The denominator of both x and y is the determinant of the coefficient matrix.

$$x = \frac{\begin{vmatrix} & \\ & \end{vmatrix}}{\begin{vmatrix} 3 & 2 \\ 4 & -3 \end{vmatrix}} \qquad y = \frac{\begin{vmatrix} & \\ & \end{vmatrix}}{\begin{vmatrix} 3 & 2 \\ 4 & -3 \end{vmatrix}}$$

The numerator determinants are the same, except that the constants -1 and 10 replace the coefficients of x when we solve for x and replace the coefficients of y when we solve for y.

$$x = \frac{\begin{vmatrix} -1 & 2 \\ 10 & -3 \end{vmatrix}}{\begin{vmatrix} 3 & 2 \\ 4 & -3 \end{vmatrix}} = \frac{3 - 20}{-9 - 8} = 1 \qquad y = \frac{\begin{vmatrix} 3 & -1 \\ 4 & 10 \end{vmatrix}}{\begin{vmatrix} 3 & 2 \\ 4 & -3 \end{vmatrix}} = \frac{30 + 4}{-9 - 8} = -2$$

Caution

The order of the elements in the coefficient matrix is always the same.

Sometimes when using Cramer's rule, the determinant of the coefficient matrix is equal to zero. This makes the denominator of the solutions equal to zero, which makes the expression undefined.

Classifying Systems by Their Solutions		
If $D \neq 0$, the system has exactly **one unique solution.** A system that has any number of solutions is considered consistent.	If $D = 0$ but neither of the numerators is zero, the system has **no solution** and is considered inconsistent.	If $D = 0$ and at least one of the numerators is zero, the system has an **infinite number of solutions,** which means it is dependent and consistent.

Example 2 Interpreting a Denominator of Zero

a. Use Cramer's rule to solve $\begin{aligned} 3x + 2y &= 5 \\ 3x + 2y &= 8 \end{aligned}$.

SOLUTION Set up and simplify the equations for x and y using the coefficient matrix and the constants.

$$x = \frac{\begin{vmatrix} 5 & 2 \\ 8 & 2 \end{vmatrix}}{\begin{vmatrix} 3 & 2 \\ 3 & 2 \end{vmatrix}} = \frac{10 - 16}{6 - 6} = \frac{-6}{0} \qquad y = \frac{\begin{vmatrix} 3 & 5 \\ 3 & 8 \end{vmatrix}}{\begin{vmatrix} 3 & 2 \\ 3 & 2 \end{vmatrix}} = \frac{24 - 15}{6 - 6} = \frac{9}{0}$$

Division by zero is undefined, so using Cramer's rule did not provide a solution. Neither of the numerators is zero, so there is no solution. This is because the graphs of the equations are parallel lines and parallel lines never intersect.

Math Reasoning

Generalize Multiplying $3x + 2y = 5$ by 2 results in $6x + 4y = 10$, which is the same line. Give one more equation of this line.

b. Use Cramer's rule to solve the system.
$$3x + 2y = 5$$
$$6x + 4y = 10$$

SOLUTION Set up and simplify the equations for x and y.

$$x = \frac{\begin{vmatrix} 5 & 2 \\ 10 & 4 \end{vmatrix}}{\begin{vmatrix} 3 & 2 \\ 6 & 4 \end{vmatrix}} = \frac{20 - 20}{12 - 12} = \frac{0}{0} \qquad y = \frac{\begin{vmatrix} 3 & 5 \\ 6 & 10 \end{vmatrix}}{\begin{vmatrix} 3 & 2 \\ 6 & 4 \end{vmatrix}} = \frac{30 - 30}{12 - 12} = \frac{0}{0}$$

In addition to the denominators being zero, both of the numerators are equal to zero, so there is an infinite number of solutions to this system. This is because the graphs of the equations are the same line.

Example 3 Application: Science

The molecular weight of H_2O is 18 atomic mass units, which means that 2 hydrogen (H) atoms and 1 oxygen (O) atom have a combined weight of 18 amu. Write a system of equations that describes the data in the table and use Cramer's rule to find the atomic weights of hydrogen and carbon (C).

Name	Formula	Molecular weight
Benzene	C_6H_6	78
Naphthalene	$C_{10}H_8$	128

Reading Math

The subscript number after each letter in a formula stands for the number of atoms of each element in that compound.

SOLUTION Using C and H as the variables, the system that represents the data is

$$6C + 6H = 78$$
$$10C + 8H = 128$$

Solving this system with Cramer's rule will result in the weights of each element.

Remember that the coefficient matrix will be in the denominator for each variable. Use the constants in the numerator.

$$C = \frac{\begin{vmatrix} 78 & 6 \\ 128 & 8 \end{vmatrix}}{\begin{vmatrix} 6 & 6 \\ 10 & 8 \end{vmatrix}} = \frac{624 - 768}{48 - 60} = \frac{-144}{-12} = 12$$

$$H = \frac{\begin{vmatrix} 6 & 78 \\ 10 & 128 \end{vmatrix}}{\begin{vmatrix} 6 & 6 \\ 10 & 8 \end{vmatrix}} = \frac{768 - 780}{48 - 60} = \frac{-12}{-12} = 1$$

Check The atomic weight of carbon is 12 amu and that of hydrogen is 1 amu.

Substitute the values back into each equation to make sure they are correct.

$$6(12) + 6(1) = 72 + 6 = 78$$
$$10(12) + 8(1) = 120 + 8 = 128$$

Lesson Practice

a. (Ex 1) Use Cramer's rule to solve the system. $\begin{aligned} 2x + 2y &= 3 \\ 3x + 8y &= 7 \end{aligned}$

b. (Ex 2) Use Cramer's rule to solve the system. $\begin{aligned} 2y - x &= 6 \\ 2y - x &= -2 \end{aligned}$

c. (Ex 2) Use Cramer's rule to solve the system. $\begin{aligned} x + 2y &= -4 \\ 3x + 6y &= -12 \end{aligned}$

d. (Ex 3) The molecular weight of H_2O is 18 atomic mass units, which means that 2 hydrogen (H) atoms and 1 oxygen (O) atom have a combined weight of 18 amu. Write a system of equations that describes the data in the table and use Cramer's rule to find the atomic weights of nitrogen (N) and oxygen (O).

Name	Formula	Molecular weight
Nitrogen oxide	N_2O_4	92
Nitrous oxide	N_2O	44

Practice Distributed and Integrated

1. (4) **Multi-Step** One T-shirt costs $15.50. This can be expressed as the ordered pair (1, 15.5). The store is having a buy-one-get-one-free sale on T-shirts. Find the total cost if 2, 3, 4, 5, and 6 T-shirts are purchased. Express the answers as ordered pairs in set notation. Identify the domain and range. Determine if the set of ordered pairs represents a function. Explain.

***2.** (15) **Error Analysis** After examining the two function tables below, a student concluded that the two equations were inconsistent.

$$y = -\frac{7}{3}x + 3$$

x	-2	-1	0	1
y	$-\frac{5}{2}$	-1	2	5

$$y = \frac{1}{3}x - 1$$

x	-2	-1	0	1
y	$-\frac{5}{3}$	$-\frac{4}{3}$	-1	$-\frac{2}{3}$

a. Describe the student's error.

b. How can this error be avoided?

c. Solve the system.

3. (4) **Model** One day the value of one U.S. dollar was equivalent to 0.77 euro. On the same day, one U.S. dollar was equivalent to 1.24 Canadian dollars. Write a function to represent the value of Canadian dollars in euros.

***4.** (5) **Graphing Calculator** Solve the system: $\begin{aligned} x - 2y &= 6 \\ -6x + 3y &= 1 \end{aligned}$. Round to the nearest tenth.

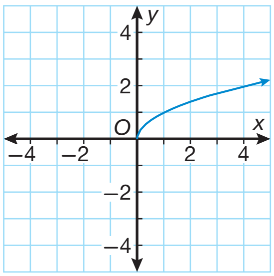
5. (6) The number of students attending a certain sports event decreased from 2080 to 1575 from one school year to the next.

a. Estimate Estimate the percent of change.

b. Find the actual percent of change. Round to the nearest percent.

Which of the following depict functions? Identify the domain and the range.

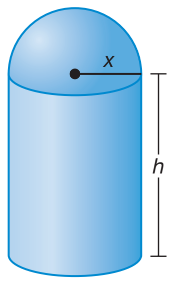
6. (4)



7. (4)

x	y
−2	−1
−2	3
−2	7

8. (11) **Measurement** This figure consists of a cylinder with a hemisphere on top. Write a polynomial in x that represents the total surface area of the figure. For reference, formulas for surface area are given below.



Cylinder: $SA = 2\pi r^2 + 2\pi rh$

Sphere: $SA = 4\pi r^2$

(Hint: The figure has only one circular base.)

Given matrices A, B, and C, multiply, if possible.

$$A = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} 3 & 2 \\ -1 & 1 \end{bmatrix} \qquad C = \begin{bmatrix} 0 & 2 & 0 \\ 3 & 1 & 4 \end{bmatrix}$$

9. (9) CA

10. (9) AB

11. (9) BC

Solve.

12. (7) $\frac{4}{7} + \frac{x+2}{3} = \frac{5}{3}$

13. (7) $\frac{5}{3} - \frac{x-4}{2} = \frac{1}{2}$

14. (7) $x - 7 + \frac{x}{4} = -\frac{1}{3}$

***15.** (15) **Demography** The population of Sacramento, California, in 2005 was 451,743 and increased by 2038 people by 2006. The population of Miami, Florida, in 2005 was 386,619 and increased by 17,429 people by 2006. If the population of these cities continued to grow at these rates, then the following equations would represent the population of the cities, where x is the number of years after 2005 and y is the population.

$y = 2038x + 451{,}743$ Sacramento's Population

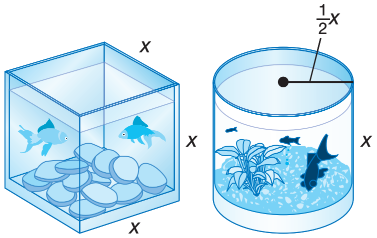
$y = 17{,}429x + 386{,}619$ Miami's Population.

a. In what year would the populations of Sacramento and Miami be equal?

b. About how many people are in each city when the populations are equal?

16. Find x. $\begin{vmatrix} x & 2 & 1 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 5$
(14)

17. **Aquarium Design** The aquariums shown each require the same-sized shelf, but they hold different amounts of water. Write a polynomial that represents how much more water the cube-shaped aquarium holds than the cylindrical aquarium. (Hint: The formula for the volume of a cylinder is $V = \pi r^2 h$.)
(11)



18. **Explain** Why is there no solution for the compound inequality $-10 > n > -1$?
(10)

Solve the inequalities.

19. $3x + 1 > -2$ or $6 < 2x - 4$
(10)

20. $\frac{1}{3}x + 5 \leq 6$ or $-8 \leq \frac{1}{2}x - 7$
(10)

*21. **Construction** A contractor is replacing the tile in two bathrooms. For one bathroom, she purchased 416 large tiles and 256 small tiles at a cost of \$233.60. For the other bathroom, she purchased 400 large tiles and 512 small tiles at a cost of \$251.20. How much does each size tile cost?
(16)

Use Cramer's rule to solve each system.

*22. $\begin{aligned} -3x + 4y &= 8 \\ x + 4y &= 4 \end{aligned}$
(16)

*23. $\begin{aligned} -3x + 5y &= 8 \\ -3x + 5y &= 4 \end{aligned}$
(16)

*24. $\begin{aligned} 2x + 4y &= 10 \\ -5x - 10y &= -25 \end{aligned}$
(16)

*25. **Multiple Choice** Match the solution with the system of equations.
(16)

$$x = \frac{\begin{vmatrix} 8 & 2 \\ 1 & -1 \end{vmatrix}}{\begin{vmatrix} 5 & 2 \\ 3 & -1 \end{vmatrix}} \qquad y = \frac{\begin{vmatrix} 5 & 8 \\ 3 & 1 \end{vmatrix}}{\begin{vmatrix} 5 & 2 \\ 3 & -1 \end{vmatrix}}$$

A $\begin{aligned} 8x + 2y &= 5 \\ x - y &= 3 \end{aligned}$

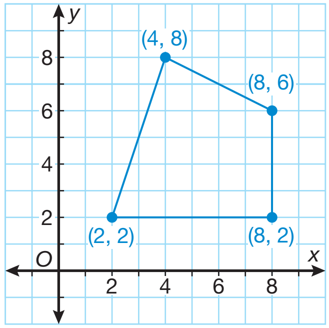
B $\begin{aligned} 5x + 8y &= 2 \\ 3x + y &= -1 \end{aligned}$

C $\begin{aligned} 5x + 2y &= 8 \\ 3x - y &= 1 \end{aligned}$

D $\begin{aligned} 8x + y &= 5 \\ 2x - y &= 2 \end{aligned}$

*26. **Write** When using Cramer's rule to solve systems of equations, what types of systems could exist if the determinant of the coefficient matrix equals zero? Describe the graphs of the lines.
(16)

*27. **Geometry** A quadrilateral with vertices (x_1, y_1), (x_2, y_2), (x_3, y_3), and (x_4, y_4) has an area equal to the absolute value of
(14)



$$\frac{1}{4}\begin{vmatrix} x_1 & x_2 & x_3 & x_4 \\ y_1 & y_2 & y_3 & y_4 \\ 1 & 1 & 1 & 1 \\ -1 & 1 & -1 & 1 \end{vmatrix}.$$ Find the area of the quadrilateral.

28. Find the x- and y-intercepts of the line with equation $3x - \frac{1}{2}y = 6$.
(13)

29. **Meteorology** The average temperatures were calculated, in degrees Fahrenheit, for the city of Kodiak, Alaska, and for the city of Valdez, Alaska, from the year 1971 to 2000.
(5)

City	Jan	Feb	Mar	Apr	May	Jun
Kodiak	29.7	29.9	32.6	37.3	43.5	49.2
Valdez	21.9	24.8	29.8	37.7	45.8	52.2

a. Model Record this information in a matrix and name it Matrix A.

b. Let $B = \begin{bmatrix} -32 & -32 & -32 & -32 & -32 & -32 \\ -32 & -32 & -32 & -32 & -32 & -32 \end{bmatrix}$. Convert the temperatures from Fahrenheit to Celsius by applying the operations on Matrix A. Round each element to the nearest tenth.

$$C = \frac{5}{9}([A] + [B])$$

30. **Geography** Christine is hiking a trail to the summit of a mountain. The mountain has a vertical change of 15 feet upward for every 25 feet of horizontal distance.
(13)

a. What is the slope of the mountain?

b. If the trail has changed vertically by 450 feet, how much has it changed horizontally?

LAB

4

Changing the Line and Window of a Graph

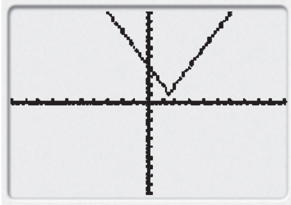
Graphing Calculator Lab (Use with Lessons 17 and 39)

Graphing Calculator Tip

For help with graphing equations, see the graphing calculator keystrokes on page 19.

Adjusting the Viewing Window

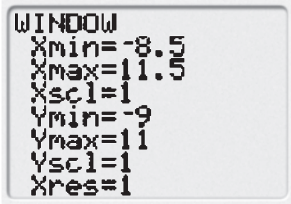
The viewing window of the graphing calculator can be adjusted to best display a graph.

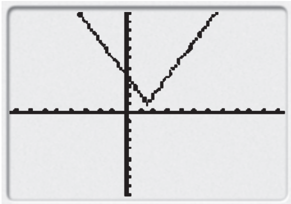


1. Graph the function $y = |2x - 3| + 1$.

2. Center the window on the vertex.

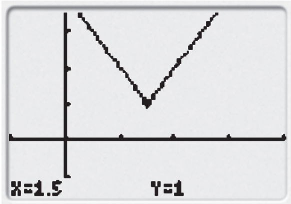
   The vertex of this function is located at (1.5, 1). To center the graph on this vertex, the x- and y-values need to be translated: $(x, y) \rightarrow (x + 1.5, y + 1)$. Press WINDOW, use the arrow keys ◄ ▲ ▼ ► to highlight Xmin, enter the value -8.5, and press ENTER. Repeat for Xmax, Ymin, and Ymax. Press GRAPH to display the graph centered on the vertex.





3. Use the zoom features to better see the graph.

   To zoom in on a certain area of a graph, press ZOOM and press 2 to select **2: Zoom In**. The graph will then be displayed with a cursor. Use the arrow keys ◄ ▲ ▼ ► to move the cursor to the area you would like to zoom in on, and then press ENTER. Follow the same method but select **3: Zoom Out** to zoom the graph out.

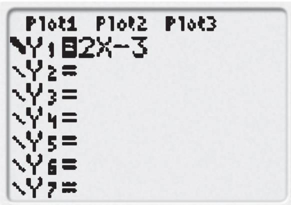


   To return a graph to the standard settings where $-10 \leq x \leq 10$ and $-10 \leq y \leq 10$, press ZOOM, use the arrow keys ◄ ▲ ▼ ► to select **6: ZStandard** and press ENTER .

Using Different Drawing Tools

A graphing calculator can be used to change the style of a line.

1. Graph the equation $y = 2x - 3$.



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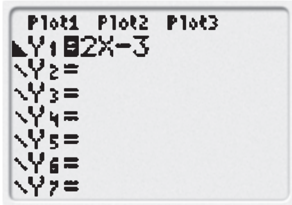
2. Change the graph to thick-line style.

   Press Y= and ◀ ◀ to highlight the line symbol to the left of the equation $y = 2x - 3$.

   Press ENTER to change to the thick-line symbol. Press GRAPH to see the thick-line graph displayed.



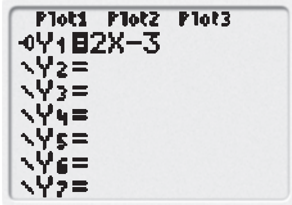
3. Change the graph to a less-than-line style.

   Press Y= and highlight the line symbol again.

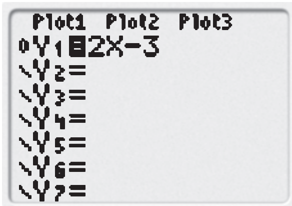
   Press ENTER twice this time until you see the less-than-line symbol. Press GRAPH to see the less-than-line graph of $y \leq 2x - 3$ displayed. (The same method can be used to graph $y \geq 2x - 3$ by selecting the greater-than-line tool.)

4. Investigate the other drawing tools.

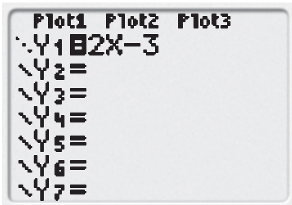
   After the less-than-line symbol is the path symbol. This tool traces the graph and draws the path. This is especially helpful in drawing parametric equations.



   After the path symbol is the animate symbol. This tool traces the graph without drawing the path. This tool is primarily used in physics applications.



   The last tool is the dot tool, which will represent each plotted point as a dot. This tool is helpful when drawing step functions or other noncontinuous functions.



Graphing Calculator Practice

a. Use the graphing calculator to graph $y \geq 5 - x$.

b. Use the graphing calculator to graph $y = x^2 - 4x + 9$ and center the graph on (2, 5).

c. Use the graphing calculator to graph $y = |6x + 3| - 5$ and zoom in on the vertex.

LESSON 17 Solving Equations and Inequalities with Absolute Value

Warm Up

1. **Vocabulary** A __________ (***conjunction, disjunction***) is a compound inequality containing the word *and*. *(10)*

2. Solve $x \geq -1$ and $x \leq 4$. *(10)*

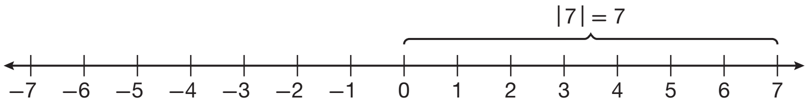
3. Solve $x \geq -1$ or $x \leq 4$. *(10)*

New Concepts

The **absolute value** of a number is the distance along the x-axis from the origin to the graph of the number. It is important to note that the absolute value of every number except zero is greater than zero.

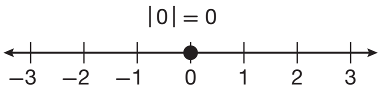
$$\text{If } x > 0, |x| = x.$$

$$|7| = 7$$



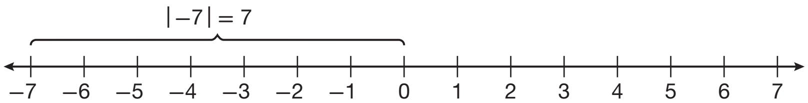
$$\text{If } x = 0, |x| = x.$$

$$|0| = 0$$



$$\text{If } x < 0, |x| = -x.$$

$$|-7| = -(-7) = 7$$



Reading Math

The equation $|x - a| = k$ can be read as "the distance from x to a is k units."

To solve absolute value equations, use inverse operations to isolate the absolute value expression on one side of the equation. Then consider the two cases below.

If $|x - a| = k$

$$x - a = -k \quad \text{or} \quad x - a = k$$

$$\text{then } x = a - k \quad \text{or} \quad x = a + k$$

The two equations above are derived, or obtained, from the original absolute value equation. Derived equations may result in **extraneous solutions.** These solutions do not satisfy the original absolute value equation. Therefore, you need to check all possible solutions.

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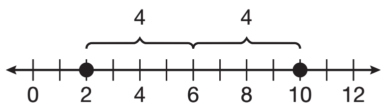
Example 1 Solving Absolute Value Equations

a. Solve $|x - 6| = 4$. Graph the solution.

SOLUTION

$$x - 6 = -4 \quad \text{or} \quad x - 6 = 4$$
$$x = -4 + 6 \qquad\qquad x = 4 + 6$$
$$x = 2 \qquad\qquad x = 10$$



Check $|2 - 6| = |-4| = 4 \qquad |10 - 6| = |4| = 4$

The solutions are 2 and 10.

b. Solve $|2x + 4| - 5 = 1$.

SOLUTION

$$|2x + 4| - 5 = 1$$
$$|2x + 4| = 6$$
$$2x + 4 = 6 \quad \text{or} \quad 2x + 4 = -6$$
$$2x = 2 \qquad\qquad 2x = -10$$
$$x = 1 \qquad\qquad x = -5$$

Check $|2(1) + 4| - 5 \stackrel{?}{=} 1 \qquad |2(-5) + 4| - 5 \stackrel{?}{=} 1$

$$|2 + 4| \stackrel{?}{=} 6 \qquad |-10 + 4| \stackrel{?}{=} 6$$
$$6 = 6 \ \checkmark \qquad |-6| \stackrel{?}{=} 6$$
$$6 = 6 \ \checkmark$$

The solutions are 1 and -5.

Hint

An extraneous solution may occur anytime there is a variable outside the absolute value symbol.

c. Solve $|3x + 5| = -10$.

SOLUTION No solution. The absolute value of a number is never negative, as it represents distance.

d. Solve $|3x + 18| = 6x$. Check for extraneous solutions.

SOLUTION $3x + 18 = 6x \quad \text{or} \quad 3x + 18 = -6x$

$$18 = 3x \qquad\qquad 18 = -9x$$
$$6 = x \qquad\qquad -2 = x$$

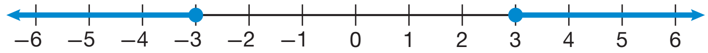
Check Evaluate $|3x + 18| = 6x$ for $x = 6$ and $x = -2$.

$$|3(6) + 18| \stackrel{?}{=} 6(6) \qquad |3(-2) + 18| \stackrel{?}{=} 6(-2)$$
$$|18 + 18| \stackrel{?}{=} 36 \qquad |-6 + 18| \stackrel{?}{=} -12$$
$$|36| \stackrel{?}{=} 36 \qquad |12| \stackrel{?}{=} -12$$
$$36 = 36 \ \checkmark \qquad 12 \neq -12$$

The solution is 6. -2 is an extraneous solution.

Absolute value inequalities can be solved by rewriting and solving them as compound inequalities. Absolute value inequalities are either conjunctions or disjunctions.

$$x \leq 3 \text{ and } x \geq -3$$
$$|x| \leq 3$$



$$x \geq 3 \text{ or } x \leq -3$$
$$|x| \geq 3$$

Conjunctions can be replaced with absolute value statements of less than. Disjunctions can be replaced with absolute value statements of greater than.

If a is a positive number, then

$\lvert x \rvert < a$	is the same as the conjunction	$x < a$ and $x > -a$ or $-a < x < a$
$\lvert x \rvert > a$	is the same as the disjunction	$x > a$ or $x < -a$

Note that statements like $|x| < -4$ and $|x| > -4$ are special cases. The first inequality has no solution, while the second is all real numbers.

Example 2 Solving Absolute Value Inequalities with Conjunctions

Reading Math

"And" means to include in the solution all numbers common to both inequalities.

a. Solve $|x - 5| \leq 2$. Graph the solution.

SOLUTION Rewrite the inequality as a conjunction, then solve for x.

$$x - 5 \leq 2 \quad \text{and} \quad x - 5 \geq -2$$
$$x \leq 7 \quad \text{and} \quad x \geq 3$$

$$3 \leq x \leq 7$$

b. Solve $-2|3x| + 4 \geq 6$.

SOLUTION Always isolate the absolute value term before rewriting the inequality as a conjunction or disjunction.

$$-2|3x| + 4 \geq 6 \quad \longrightarrow \quad -2|3x| \geq 2 \quad \longrightarrow \quad |3x| \leq -1$$

Since the absolute value of a number is always greater than or equal to zero, there is no solution.

Example 3 Solving Absolute Value Inequalities with Disjunctions

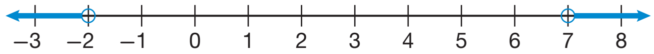
Reading Math

"Or" means to include in the solution all numbers described by both inequalities.

a. Solve $|2x - 5| > 9$. Graph the solution.

SOLUTION Rewrite the inequality as a disjunction and then solve for x.

$$2x - 5 > 9 \quad \text{or} \quad 2x - 5 < -9$$
$$2x > 14 \qquad 2x < -4$$
$$x > 7 \quad \text{or} \quad x < -2$$



$$x < -2 \text{ or } x > 7$$

b. Solve $-|4x - 2| < 1$.

SOLUTION Isolate the absolute value term.

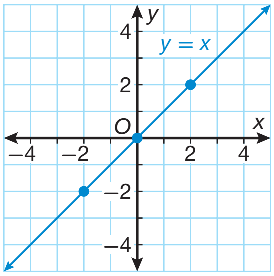
$$-|4x - 2| < 1 \quad \longrightarrow \quad |4x - 2| > -1$$

Since the absolute value of a number is always greater than or equal to zero, the inequality will be true for any number substituted for x.

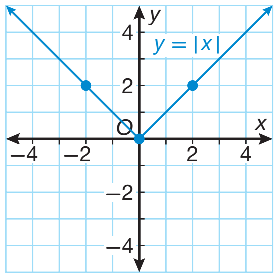
The solution is all real numbers.

The graphs in the previous examples are one-dimensional because the only variable is x. An **absolute value function** is a function that contains an absolute value expression. The graph of the equation $y = |x|$, or $f(x) = |x|$, will be two-dimensional because there are two variables, x and y.

x	y
2	2
0	0
–2	–2



x	y
2	2
0	0
–2	2



The graph on the left is the graph of $y = x$. The graph on the right is the graph of $f(x) = |x|$, the parent function of absolute value functions. It does not go below the x-axis because y equals the absolute value of x and the absolute value of a number is never negative.

Math Reasoning

Generalize Give the parent function for a linear function.

A **parent function** is the simplest function of a particular type. New functions can be graphed by **transformations,** or changes, to the graph of the parent function. These changes may involve a change in the size, shape, orientation, or position of the parent function.

Hint

Since $f(x) = |x + 4| = |x - (-4)|$, the h value is negative, not positive.

Exploration Transforming $f(x) = |x|$

Use a table of values to graph each of the following equations.

a. $f(x) = -|x|$ **b.** $f(x) = \frac{1}{2}|x|$ **c.** $f(x) = |x - 4|$

d. $f(x) = |x + 4|$ **e.** $f(x) = |x| - 4$ **f.** $f(x) = |x| + 4$

The graph of the absolute value function $y = a|x - h| + k$ can be used to model the transformations of the graph of the parent function $f(x) = |x|$.

g. Use the graphs of the functions to describe the effect of h and k on the graph of the parent function.

h. How is the shape and orientation of the graph determined?

i. Can the equation $y = a|x - h| + k$ be used to determine the **vertex,** or turning point, of the graph?

Hint

The number inside the absolute value symbol will be the "starting point"; the number outside the absolute value symbol will be the variance.

Example 4 Application: Chemistry

For hydrogen to be a liquid, it must be within 2° of −257°C. Write an absolute value equation to determine the least and greatest temperatures at which hydrogen will remain a liquid. Solve the equation and graph the solution.

SOLUTION Let t = temperature.

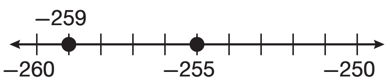
$$|t - (-257)| = 2$$

$$|t + 257| = 2$$

$$t + 257 = 2 \quad \text{and} \quad t + 257 = -2$$

$$t = -255 \qquad\qquad t = -259$$

The least temperature is −259°C and the greatest temperature is −255°C.



Lesson Practice

a. (Ex 1) Solve $|x + 3| = 7$. Graph the solution.

b. (Ex 1) Solve $|3x - 3| + 5 = 8$.

c. (Ex 1) Solve $|7x - 2| = -8$.

d. (Ex 1) Solve $|4x - 1| = 2x$. Check for extraneous solutions.

e. (Ex 2) Solve $|x + 2| < 8$. Graph the solution.

f. (Ex 2) Solve $3|5x| - 6 \leq 6$.

g. (Ex 3) Solve $|-2x + 9| \geq 7$. Graph the solution.

h. (Ex 3) Solve $-|2x + 3| \leq 5$.

i. (Ex 4) For nitrogen to be a liquid, it must be within 13° of −333°F. Write an absolute value equation to determine the least and greatest temperatures at which nitrogen will remain a liquid. Solve the equation and graph the solution.

Practice Distributed and Integrated

*1. (17) **Construction Engineering** A company's safety regulations require the wire rope used to suspend a 100-ton load to have a thickness that is within $\frac{3}{8}$ inch of $1\frac{3}{4}$ inches. Write and solve an absolute value equation to find the minimum safe thickness of rope.

2. (13) **Explain** Two points, $(3, -7)$ and $(3, 5)$, lie on a line. Using the slope formula, explain why the slope does not exist. What kind of line do these points lie on?

*3. (17) **Generalize** Will the inequality $3|x - 7| > -12$ have a solution?

Use Cramer's rule to solve each system.

*4. (16) $\begin{aligned} 4x - 2y &= 6 \\ 10x - 5y &= 15 \end{aligned}$

5. (16) $\begin{aligned} -5x + 2y &= 7 \\ 10x - 4y &= 3 \end{aligned}$

6. (16) $\begin{aligned} -3x + 2y &= 4 \\ x + 2y &= 10 \end{aligned}$

*7. (15) **Multiple Choice** Which is *not* a solution to the system below?

$$\begin{aligned} 2x - 5y &= -3 \\ -10x + 25y &= 15 \end{aligned}$$

A (1, 1) **B** (0, 0)

C (6, 3) **D** All of these are solutions.

Find the product.

8. (9) $\begin{bmatrix} 1 & 0 & -2 \\ 0 & 3 & -1 \\ 2 & 0 & 4 \end{bmatrix} \cdot \begin{bmatrix} 4 & -3 \\ 1 & 2 \\ 0 & -1 \end{bmatrix}$

9. (9) $\begin{bmatrix} 2 & -3 & 1 & -2 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 \\ -3 & 2 \\ 1 & 0 \\ 4 & 6 \end{bmatrix}$

10. (8) **Temperature** Juanita is conducting an experiment where the temperature of a liquid varies directly with time. Her observations are recorded in the table below.

Temperature	80 degrees	120 degrees	? degrees
Time	120 seconds	180 seconds	240 seconds

a. Using the values in the table, identify the constant of variation.

b. Find the missing temperature in the table.

c. Determine an accurate prediction for the temperature of the liquid after 360 seconds.

11. (13) **Multiple Choice** Identify the slope of the line with the equation $2x + 9y = 18$.

A 2 **B** $\frac{2}{9}$ **C** $-\frac{9}{2}$ **D** $-\frac{2}{9}$

12. *(8)* **Baseball** When a baseball pitcher throws a baseball, the ball is slowed down by wind resistance. The relationship between the force (F) of wind resistance and velocity (v) of the baseball is a direct variation of the form $F = kv^2$. The data tables below represent sample pitches from two ball games.

Pitcher A

Ball Speed (mph)	Force
90	1620
91	1656
92	1693
93	1730
94	1767
95	1805

Pitcher B

Ball Speed (mph)	Force
90	2025
91	2070
92	2116
93	2162
94	2209
95	2256

a. Find the constant of variation for each pitcher.

b. Based on your results, which pitcher was throwing the ball harder?

c. What could account for the difference in the force of wind resistance?

13. *(16)* **Multi-Step** Use Cramer's Rule to solve the system. Check your solution by graphing.

$$\begin{aligned} -2x + 3y &= 0 \\ x + 3y &= 6 \end{aligned}$$

14. *(11)* **Error Analysis** A student says that the polynomial $ab^4 + a^2b^2 - 3$ is of higher degree than the polynomial $a^2b^3 + ab + 2$. What is the error? Explain.

Solve the equation or inequality.

15. *(7)* $\frac{y-2}{4} = 7y - 2 - 6(y + 1)$

16. *(10)* $2 + \frac{1}{3}x \leq 1\frac{2}{3}$ and $-2x - 5 \leq 7$

17. *(10)* $-\frac{1}{3} - \frac{2x}{3} > \frac{1}{3}$ and $-7x - \frac{1}{2} < 6\frac{1}{2}$

***18.** *(17)* $|-2x + 7| - 3 = 10$.

***19.** *(17)* **Verify** Solve $|8x - 3| = 16x$. Check for extraneous solutions by substituting the solution into the equation.

***20.** *(17)* Solve $|3x + 2| \leq 14$. Graph the solution.

21. *(15)* **Geometry**

a. Graph the system below. Describe the shape that the lines form.

$$\begin{aligned} 3x - 2y &= 2 \\ -3x - 2y &= -34 \\ x - 2y &= 6 \end{aligned}$$

b. Find the vertices of the shape.

22. *(14)* A triangle has vertices (1, 2), (1, −4), and (4, −4). Find the area of the triangle by writing and solving a determinant.

Use a table to graph the equation. (Hint: Choose x-values that are multiples of the slope's denominator.)

23. (13) $y = -\frac{5}{3}x - 2$

24. (13) $y = -\frac{1}{2}x + 5$

***25.** (14) **Graphing Calculator** Use a graphing calculator to evaluate

$$2\begin{vmatrix} -2 & 1 & 4 \\ -5 & 7 & 0 \\ 8 & -3 & 1 \end{vmatrix} - 3\begin{vmatrix} 0 & 1 & 4 \\ 1 & 7 & 0 \\ -6 & -3 & 1 \end{vmatrix} - 1\begin{vmatrix} 0 & -2 & 4 \\ 1 & -5 & 0 \\ -6 & 8 & 1 \end{vmatrix} - 5\begin{vmatrix} 0 & -2 & 1 \\ 1 & -5 & 7 \\ -6 & 8 & -3 \end{vmatrix}.$$

26. (3) **Solar System** The average distance between the earth and sun is approximately 1.495×10^{11} m. Earth's orbit around the sun is nearly circular. For estimation purposes, assume that Earth's orbit is circular.

a. Estimate the distance in kilometers that the earth travels around the sun in one year. Write your answer in scientific notation. Round to the nearest hundredth.

b. Estimate the average speed at which the earth travels around the sun in km/s. Use 365.25 days per year.

***27.** (13) Find the slope of the line between $(-2, -7)$ and $(2, 1)$. Determine if the line rises, falls, or is horizontal or vertical.

***28.** (8) **Landscaping** When mixing concentrated fertilizer for a garden, the amount of fertilizer varies directly with the amount of water. If 2 cups of fertilizer are used in 16 gallons of water, how much fertilizer is used in 48 gallons of water?

Determine the kind of variation, if any, for each equation.

29. (12) $y = ax^2 + bx + c$, for $a, b, c \neq 0$

30. (12) $y = ax^2 + bx + c$, for any values of a, b, and c.

LESSON 18

Calculating with Units of Measure

Warm Up

1. **Vocabulary** 1 foot and 12 inches are ________. (*equivalent, proportional*)

2. (3) Simplify $\frac{x^4 \cdot x^2}{x^3}$.

3. (SB) Simplify $\frac{6}{7} \times \frac{5}{6} \times \frac{3}{5} \times \frac{2}{3}$.

New Concepts

You can use unit **conversion factors** to change one unit of measure to another. Dimensional analysis involves choosing the appropriate conversion factor to produce the appropriate unit.

Example 1 Choosing the Appropriate Conversion Factor

Reading Math

The slash (/) is used to symbolize the canceling of factors or, as in this lesson, units.

a. Change 600 inches to feet.

SOLUTION Put 600 in. over 1.

$$\frac{600 \text{ in.}}{1}$$

There are 12 inches in a foot, so the conversion factor is either $\frac{1 \text{ ft}}{12 \text{ in.}}$ or $\frac{12 \text{ in.}}{1 \text{ ft}}$. If you choose $\frac{1 \text{ ft}}{12 \text{ in.}}$ as the conversion factor, the inches will cancel.

$$\frac{600 \cancel{\text{in.}}}{1} \times \frac{1 \text{ ft}}{12 \cancel{\text{in.}}} = 50 \text{ ft}$$

b. Change 8 hours to minutes.

SOLUTION Put 8 hours over 1.

$$\frac{8 \text{ hr}}{1}$$

There are 60 minutes in an hour, so the conversion factor is either $\frac{1 \text{ hr}}{60 \text{ min}}$ or $\frac{60 \text{ min}}{1 \text{ hr}}$. You want the hours to cancel, so choose $\frac{60 \text{ min}}{1 \text{ hr}}$.

$$\frac{8 \cancel{\text{hr}}}{1} \times \frac{60 \text{ min}}{1 \cancel{\text{hr}}} = 480 \text{ min}$$

Sometimes you may need to use more than one conversion factor.

Example 2 Using Multiple Conversion Factors

a. Convert 42 yd^2 to in^2.

SOLUTION Put 42 yd^2 over 1.

$$\frac{42 \text{ yd}^2}{1}$$

There are 3 feet in a yard and 12 inches in a foot, so

$$\frac{42 \cancel{\text{yd}^2}}{1} \times \frac{3 \cancel{\text{ft}}}{1 \cancel{\text{yd}}} \times \frac{3 \cancel{\text{ft}}}{1 \cancel{\text{yd}}} \times \frac{12 \text{ in.}}{1 \cancel{\text{ft}}}$$
$$\times \frac{12 \text{ in.}}{1 \cancel{\text{ft}}} = 54{,}432 \text{ in}^2$$

b. Convert 3000 seconds to hours.

SOLUTION Put 3000 s over 1.

$$\frac{3000 \text{ s}}{1}$$

There are 60 seconds in a minute and 60 minutes in an hour, so

$$\frac{3000 \cancel{\text{s}}}{1} \times \frac{1 \cancel{\text{min}}}{60 \cancel{\text{s}}} \times \frac{1 \text{ hr}}{60 \cancel{\text{min}}} = \frac{5}{6} \text{ hr}$$

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> **Math Language**
>
> **Per** means "for each" and implies division.

Example 3 Converting Rates

Convert 50 feet per second to miles per hour.

SOLUTION Write 50 feet per second as a fraction. $\frac{50\text{ ft}}{1\text{ s}}$

Convert feet to miles and seconds to hours. There are 5280 feet in a mile, so the conversion factor is $\frac{1\text{ mi}}{5280\text{ ft}}$. There are 60 seconds in a minute and 60 minutes in an hour, so the other conversion factor is $\frac{3600\text{ s}}{1\text{ hr}}$.

$$\frac{50\cancel{\text{ ft}}}{1\cancel{\text{ s}}} \times \frac{1\text{ mi}}{5280\cancel{\text{ ft}}} \times \frac{3600\cancel{\text{ s}}}{1\text{ hr}} = \frac{(50)(3600)}{5280}\ \frac{\text{mi}}{\text{hr}}$$

Using a calculator, this simplifies to about 34 mph.

The **accuracy** of a measurement refers to how closely it corresponds to the actual value being measured. The **precision** of a measurement refers to the number of **significant digits** it contains. The following table illustrates how to find the number of significant digits in a measured value.

Significant Digits

Rule	Example	Number of Significant Digits
Nonzero digits are significant.	325.6	4
Zeros between significant digits are significant.	5003.09	6
Leading zeros are not significant.	0.00047	2
Zeros after the last nonzero digit that are also to the right of a decimal point are significant.	0.0001800	4

The following rules are used to determine the number of significant digits that should be in the answer to a calculation with measurements:

Rules for Significant Digits in Measurements
When adding or subtracting, the answer should have the same number of decimal places as the measurement with the fewest decimal places. When multiplying or dividing, the answer should have the same number of significant digits as the measurement with the fewest significant digits.

Example 4 Using Significant Digits

Simplify each expression.

a. 5.38 mi + 6.495 mi + 0.5 mi

SOLUTION Add the three distances.

5.38 mi + 6.495 mi + 0.5 mi = 12.375 mi

Since 0.5 mi has only one decimal place, the answer must be rounded to one decimal place, resulting in 12.4 mi.

b. 2.09 ft × 0.050 ft × 303 ft

SOLUTION Multiply the three lengths.

2.09 ft × 0.050 ft × 303 ft = 31.6635 ft^3

Since 0.050 ft has only two significant digits, the answer can only have two significant digits, resulting in 32 ft^3.

Example 5 Application: Sports

Natcar is an annual robotic car race. In 2007, the winning car completed the course with an average speed of 7.57 feet per second. What is this speed in miles per hour?

SOLUTION Write the given speed as a fraction.

$$\frac{7.57 \text{ ft}}{1 \text{ s}}$$

One of the conversion factors needs to have *ft* in its denominator and the other needs *s* in its numerator.

$$\frac{7.57 \cancel{\text{ft}}}{1 \cancel{\text{s}}} \times \frac{1 \text{ mi}}{5280 \cancel{\text{ft}}} \times \frac{3600 \cancel{\text{s}}}{1 \text{ hr}} = \frac{(7.57)(3600)}{5280} \frac{\text{mi}}{\text{hr}}$$

This answer simplifies to about 5.16 mph.

Math Reasoning

Predict If the builder of a car wanted it to reach 10 mph, about how fast would it need to go in feet per second?

Lesson Practice

a. (Ex 1) Change 720 inches to feet.

b. (Ex 1) Change 6 hours to minutes.

c. (Ex 2) Convert 52 yd^2 to in^2.

d. (Ex 2) Convert 2700 seconds to hours.

e. (Ex 3) Convert 80 feet per second to miles per hour.

f. (Ex 4) Simplify the expression 6.21 mi + 3.672 mi + 0.8 mi.

g. (Ex 4) Simplify the expression 8.08 ft $\times$ 0.020 ft $\times$ 407 ft.

h. (Ex 5) RaceUSA is an annual boxcar race. In 2007, the winning car completed the course with an average speed of 24 feet per second. What is this speed in miles per hour?

Practice Distributed and Integrated

Use Cramer's rule to solve each system.

***1.** (16) $\begin{aligned} x &= y + 1 \\ 3x + 2y &= 8 \end{aligned}$

2. (16) $\begin{aligned} 3x - y &= 22 \\ 2x + 3y &= -11 \end{aligned}$

3. (16) $\begin{aligned} x + y &= 20 \\ 5x + 10y &= 200 \end{aligned}$

Solve.

***4.** (17) $-5|2x - 7| - 4 = -34$

5. (17) $\left|\frac{1}{2}x + 1\right| \le -\frac{1}{2}$

6. (17) $|8 - 3x| > 9$

Identify whether the equation is a direct, inverse, or joint variation. Find the constant of variation.

7. (12) $\frac{y}{x} = \sqrt{2}$

8. (12) $y = \frac{15}{x}$

***9.** (15) **Chemical Mixture** A chemist has one solution that is 10% iodine and another that is 50% iodine. How many milliliters of each should the chemist use to make 100 mL of a mixture that is 20% iodine?

a. Formulate Set up a system of equations that models the problem.

b. Graphing Calculator Use a graphing calculator to solve the system.

10. (12) **Multiple Choice** In the data set at right, what is the relationship between x and y?

A joint variation
B direct variation
C inverse variation
D none of the above

x	1	2	3	4
y	4	2	$1\frac{1}{3}$	1

11. (15) **Sports** Suppose you and a friend are practicing basketball. To make practice more interesting, you invent a game. Every time you make a basket, you will gain 2 points. Every time your friend makes a basket, you will lose 1 point. Likewise, every time your friend makes a basket, he will gain 2 points, and every time you make a basket, he will lose 1 point. Both of you start with 0 points. At the end of the game, you have 16 points and your friend has 10.

a. Formulate Write a system of equations to represent this situation.

b. How many baskets did each of you make?

12. (12) **Geometry** As the diameter (d) of a circle increases in size, the circumference (C) increases. Likewise, as the diameter decreases in size, so does the circumference. The constant of variation between C and d is π. Describe the kind of variation between circumference and diameter. Write the equation.

13. (14) **Error Analysis** Explain the error in the work below. Then find the correct determinant.

$$\begin{vmatrix} -2 & 6 \\ 3 & -1 \end{vmatrix} = (6)(3) - (-2)(-1) = 18 - 2 = 16$$

Convert the units and unit rates.

***14.** (18) Change 0.075 hours to seconds.

***15.** (18) Convert 1224 in^2 to yd^2.

***16.** (18) Convert 0.26 miles per hour to feet per minute.

***17.** (18) **Space Travel** The Endeavor, a space shuttle, has speeds of about 27,404 feet per second. Convert the speed into kilometers per hour and miles per hour.

18. (17) **Verify** Show that $|2x + 3| = 16x$ has two solutions, one of which is extraneous.

***19.** (17) **Analyze** Write the four resulting inequalities that are used to solve $|-3x + 5| \geq |x|$.

20. (17) **Chemical Elements** For oxygen to be a liquid, it must be within 35.845° of $-332.925°$F.

a. Write an absolute value equation to determine the least and greatest temperatures at which oxygen will remain a liquid.

b. Solve the equation.

21. (13) **Explain** Give a written explanation on how to graph an equation using intercepts.

22. (11) **Analyze** Give an example of two trinomials whose sum has 6 terms.

Solve for *x*.

***23.** (7, 14) $\begin{vmatrix} \frac{x}{2} & 4 \\ 3 & -10 \end{vmatrix} = -(-x + 7)$

24. (7, 14) $\begin{vmatrix} -3 & 2 \\ 4x - 1 & 2x - \frac{1}{3} \end{vmatrix} = -16x + 7$

Multi-Step **Given the slope and a point of the line, graph the line. Determine the *y*-intercept of the line.**

25. (13) $m = \frac{5}{3}\ (-3, 1)$

26. (13) $m = -\frac{4}{5}\ (5, -5)$

***27.** (3, 18) **Solar System** The earth revolves about the sun at about 29.8 kilometers per second. Convert the rate into meters per hour. Write the answer in scientific notation rounded to the thousandths.

$A = \begin{bmatrix} 6 & -3 \\ 1 & 4 \end{bmatrix}$ **and** $B = \begin{bmatrix} 0 & 4 \\ -2 & 6 \end{bmatrix}$**. Find each product.**

28. (9) Find AB.

29. (9) Find BA.

30. (9) Find A^2.

LESSON 19

Multiplying Polynomials

Warm Up

1. (11) **Vocabulary** The 5 in the term $5x$ is called the __________.
2. (1) Simplify using the Distributive Property: $5(3 + 4)$.
3. (2) Combine like terms: $3x^2 + 5x - 8x^2 - x$.
4. (3) Simplify $2a^3 \cdot 3a^3$.

New Concepts

To multiply two polynomials, each term of the first polynomial is multiplied by each term of the other polynomial. After each of these products is found, the like terms are combined. To multiply two binomials a method called FOIL is often used. The letters in FOIL stand for where the terms are found in the binomials.

The FOIL Method
Multiply $(x + 3)(x - 5)$.
The **F**irst terms are x and x. Their product is x^2.
The **O**utside terms are x and -5. Their product is $-5x$.
The **I**nside terms are 3 and x. Their product is $3x$.
The **L**ast terms are 3 and -5. Their product is -15.
Combine the like terms in $x^2 - 5x + 3x - 15$.
$(x + 3)(x - 5) = x^2 - 2x - 15$

Example 1 Multiplying Using the FOIL Method

Multiply using the FOIL method.

a. $(a + 8)(a - 5)$

SOLUTION

F: $a \cdot a = a^2$ **O:** $a \cdot (-5) = -5a$ **I:** $8 \cdot a = 8a$ **L:** $8 \cdot (-5) = -40$

$= a^2 - 5a + 8a - 40$ Multiply.

$= a^2 + 3a - 40$ Combine like terms.

b. $(2x - 3)(3x + 2)$

SOLUTION

F: $2x \cdot 3x = 6x^2$ **O:** $2x \cdot 2 = 4x$ **I:** $-3 \cdot 3x = -9x$ **L:** $-3 \cdot 2 = -6$

$= 6x^2 + 4x - 9x - 6$ Multiply.

$= 6x^2 - 5x - 6$ Combine like terms.

Math Reasoning

Write Explain in writing why $(-5 + a)(8 + a)$ would produce the same result as $(a + 8)(a - 5)$.

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Example 2 Multiplying a Binomial by a Trinomial

Multiply $(a + 5)(3a^2 - 2a + 7)$.

SOLUTION Multiply each term of the binomial by each term of the trinomial.

$(a + 5)(3a^2 - 2a + 7)$

$= a(3a^2) + a(-2a) + a(7) + 5(3a^2) + 5(-2a) + 5(7)$

$= 3a^3 - 2a^2 + 7a + 15a^2 - 10a + 35$ Multiply.

$= 3a^3 + 13a^2 - 3a + 35$ Combine like terms.

More than two polynomials can be multiplied together. The Associative Property of Multiplication holds for polynomials. Multiply two of the polynomials and then multiply that product by the remaining polynomial.

Example 3 Multiplying Three Binomials

Multiply $(r + 3)(r - 2)(r + 5)$ in two different ways using the Associative Property of Multiplication.

SOLUTION

Multiply $(r + 3)(r - 2)$.

$(r + 3)(r - 2)$

$= r(r) + r(-2) + 3(r) + 3(-2)$ Use FOIL.

$= r^2 - 2r + 3r - 6$ Multiply.

$= r^2 + r - 6$ Combine like terms.

$(r + 5)(r^2 + r - 6)$ Multiply by $(r + 5)$.

$= r(r^2) + r(r) + r(-6) + 5(r^2) + 5(r) + 5(-6)$ Distribute.

$= r^3 + r^2 - 6r + 5r^2 + 5r - 30$ Multiply.

$= r^3 + 6r^2 - r - 30$ Combine like terms.

SOLUTION

Multiply $(r - 2)(r + 5)$.

$(r - 2)(r + 5)$

$= r(r) + r(5) + (-2)r + (-2)5$ Use FOIL.

$= r^2 + 5r - 2r - 10$ Multiply.

$= r^2 + 3r - 10$ Combine like terms.

$(r + 3)(r^2 + 3r - 10)$ Multiply by $(r + 3)$.

$= r(r^2) + r(3r) + r(-10) + 3(r^2) + 3(3r) + 3(-10)$ Multiply.

$= r^3 + 3r^2 - 10r + 3r^2 + 9r - 30$ Multiply.

$= r^3 + 6r^2 - r - 30$ Combine like terms.

Math Reasoning

Verify Does multiplying $(r + 3)(r + 5)$ and then multiplying that result by $(r - 2)$ lead to the same result as the two methods shown in Example 3?

Some binomial products appear so frequently that memorizing the patterns they form results in convenience and efficiency while solving problems.

Caution

$(a+b)^2 \neq a^2 + b^2$

$(a-b)^2 \neq a^2 - b^2$

Special Product Patterns	
Sum and difference	$(a+b)(a-b) = a^2 - b^2$
Square of a sum	$(a+b)^2 = (a+b)(a+b) = a^2 + 2ab + b^2$
Square of a difference	$(a-b)^2 = (a-b)(a-b) = a^2 - 2ab + b^2$

Example 4 Multiplying Using Special Product Patterns

Multiply.

a. $(2y-5)^2$

SOLUTION

$(2y-5)^2$

$= (2y-5)(2y-5)$

$= (2y)^2 - 2(2y)(5) + (5)^2$

$= 4y^2 - 20y + 25$

b. $(5x+2y)(5x-2y)$

SOLUTION

$(5x+2y)(5x-2y)$

$= (5x)^2 - (2y)^2$

$= 25x^2 - 4y^2$

c. $(3q+2r)^2$

SOLUTION

$(3q+2r)^2$

$= (3q+2r)(3q+2r)$

$= (3q)^2 + 2(3q)(2r) + (2r)^2$

$= 9q^2 + 12qr + 4r^2$

Many problems involving unknown quantities in geometric figures and designs can be solved using polynomials to represent dimensions.

Example 5 Application: Geography

The National Mall in Washington, D.C., forms a rectangle and includes many important sites. The length of the mall from the steps of the U.S. Capitol to the Lincoln Memorial can be expressed as $(x - 0.1)$ miles. The width of the mall is measured across the Washington Monument and can be expressed as $(x - 0.9)$ miles. Write a polynomial to express the area of the National Mall.

SOLUTION

The area of a rectangle is equal to the length times the width.

Area of the mall $= (x-0.1)(x-0.9)$

$= x(x) + x(-0.9) + (-0.1)x + (-0.1)(-0.9)$ Use FOIL.

$= x^2 - 0.9x - 0.1x + 0.09$ Multiply.

$= x^2 - x + 0.09$ Combine like terms.

The area of the National Mall is $x^2 - x + 0.09$ square miles.

Lesson Practice

a. (Ex 1) Multiply using the FOIL method. $(a + 7)(a - 4)$

b. (Ex 1) Multiply using the FOIL method. $(4x + 5)(3x - 2)$

c. (Ex 2) Multiply $(a + 9)(2a^2 - 6a + 5)$.

d. (Ex 3) Multiply $(r + 3)(r - 6)(r + 2)$ in two different ways using the Associative Property of Equality.

e. (Ex 4) Multiply $(4y + 3)^2$.

f. (Ex 4) Multiply $(7a + 3b)(7a - 3b)$.

g. (Ex 4) Multiply $(5q + 4r)^2$.

h. (Ex 5) The food court at the Galleria Mall forms a rectangle with eateries on all sides. The length of the food court can be expressed as $(x - 15)$ yards. The width of the food court can be expressed as $(x - 12)$ yards. Write a polynomial to express the area of the food court at the Galleria Mall.

Practice Distributed and Integrated

Simplify each expression.

1. (18) 2.25 yd + 7.6 yd + 0.58 yd

2. (18) 2.08 ft $\times$ 0.033 ft $\times$ 15.5 ft

3. (8, 12) **Geometry** Which type of variation is represented by the equation Area = length $\times$ width, if the Area = 25 ft^2? Find the constant of variation.

4. (8, 12) **Analyze** Which type of variation is represented by the equation $d = r \times t$ if d is held constant? r is held constant? t is held constant?

5. (18) Kellie's last three walks were 2.25 mi, 3.2 mi, and 2.31 mi. What is the sum of the distances Kellie walked?

6. (17) **Multiple Choice** Solve $|2x + 1| < 4$.

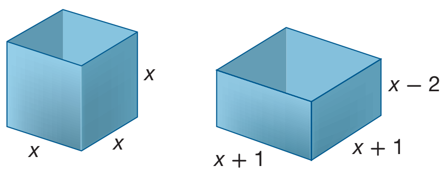
A $-\frac{5}{2} < x < \frac{3}{2}$

B $\frac{3}{2} < x < -\frac{5}{2}$

C $x < -\frac{5}{2}$ or $x > \frac{3}{2}$

D $x < \frac{3}{2}$

***7.** (11, 19) **Packaging Technology** An engineer has a box in the shape of a cube. She wants to know how the volume is affected if she adds 1 unit to each base edge and subtracts 2 units from the height. Does the volume increase, decrease, or remain the same? Explain your answer.



8. (13) Graph to determine the solution of $\begin{aligned} x + 2y &= 2 \\ y &= -\frac{1}{2}x - 3 \end{aligned}$.

9. (18) **Explain** How can feet per second be converted to miles per hour?

*10. (15) **Chemical Mixture** Ocean water is about 3.5% salt. A chemist has a solution that is 25% salt and 75% water. If the chemist needs to make 1600 mL of a solution that is 15% salt, about how much ocean water should the chemist use? Round the answer to the nearest whole number.

a. Set up a system of equations that models the problem.

b. Graphing Calculator Use a graphing calculator to solve the system.

11. (12) **Data Analysis** Each of the data sets shown is either a direct or an inverse variation. For each data set, find the constant of variation.

a.

x	y
1	1.5
2	3
3	4.5
4	6
5	7.5

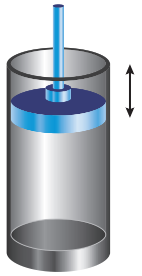
b.

x	y
1	4
2	2
3	$\frac{4}{3}$
4	1
5	$\frac{4}{5}$

c.

x	y
2	$\frac{4}{3}$
3	2
4	$\frac{8}{3}$
5	$\frac{10}{3}$

*12. (12) **Generalize** In the diagram, there is a volume (V) of air in a cylinder with a movable pressure device that applies pressure (P) on the column of air. When downward pressure is applied, the volume of air decreases. When pressure is released by moving the pressure device up, the volume of air increases. What would need to be true for the relationship between P and V to be an inverse variation? What would be the equation to model this situation?



Multiply.

*13. (19) $(8x + 6)(7x - 9)$

*14. (19) $(y - 3)(y^2 + 2y - 5)$

15. (18) **Error Analysis** Two students converted 200 feet per second to miles per hour. Which student is correct? Explain the error in the other student's work.

Student A

$$\frac{200\text{ ft}}{1\text{ s}} \times \frac{1\text{ mi}}{5280\text{ ft}} \times \frac{360\text{ s}}{1\text{ hr}} = \frac{(200)(360)}{5280}\,\frac{\text{mi}}{\text{hr}}$$

Using a calculator, this simplifies to about 13.6 miles per hour.

Student B

$$\frac{200\text{ ft}}{1\text{ s}} \times \frac{1\text{ mi}}{5280\text{ ft}} \times \frac{3600\text{ s}}{1\text{ hr}} = \frac{(200)(3600)}{5280}\,\frac{\text{mi}}{\text{hr}}$$

Using a calculator, this simplifies to about 136 miles per hour.

Convert each of the following measurements.

16. (18) 8 yd^2 to in^2

17. (18) 72 hours to seconds

18. (15) **Multiple Representations** The ordered pairs $(-1, -\frac{5}{2})$, $(0, -1)$, $(2, 2)$, and $(4, 5)$ are values in one function of a system that is consistent and dependent. Complete the table to represent ordered pairs of the other function in the same system.

x	0	2	4	6
y				

19. (18) **Multi-Step** While bicycling one weekend, Jess recorded the total distance and time he traveled each time that he stopped. The table shows his records.

Time (hours)	Distance Traveled (miles)
0.5	6
1	12
1.5	18
3	36

a. What was Jess's average speed, in miles per hour, for the total trip?

b. Convert Jess's speed in miles per hour to feet per second.

c. How long, in seconds, would it take Jess to travel 500 feet?

Simplify.

20. (3) $\frac{5x^2y}{7y^5} \cdot \frac{28z}{115x^{-2}y^{-3}}$

21. (3) $\frac{3x^2yz^{-4}}{18xy^{-2}} \div \frac{9y^4}{54x^6z^8}$

***22.** (17) **Chemical Elements** For helium to be a liquid, it must be within 3.0935° of −454.9065°F.

a. **Formulate** Write an absolute value equation to determine the least and greatest temperatures at which helium will remain a liquid.

b. Solve the equation.

***23.** (17) **Analyze** Write the distance from each point to the origin as an absolute value expression. Then simplify the expression.

a. $(5, 0)$ **b.** $(-7, 0)$ **c.** $(0, 9)$ **d.** $(0, -8)$

Simplify the expressions.

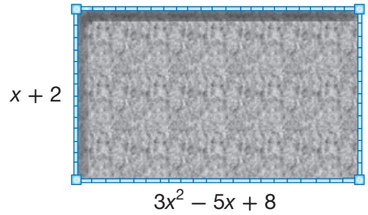
***24.** (19) $(-3y + 7)^2$

***25.** (19) $(4a + 6b)(4a - 6b)$

26. (11) **Generalize** Consider the following polynomials: $P_1 = 2x^5 + x^3$, $P_2 = -2x^5 - x^3 + x$, $P_3 = x^2$, and $P_4 = x^5$. Find the number of terms in each of these sums: $P_1 + P_2$, $P_1 + P_3$, and $P_1 + P_4$. Use the results to make a generalization about the number of terms in the sum of two polynomials compared to the number of terms in the polynomials that make up that sum.

27. (3) **Solar System** The mass of the moon is 7.3477×10^{22} kg. If the mass of Earth is 5.9736×10^{24} kg, how many times larger is Earth than the moon?

***28.** (19) **Exterior Design** The diagram at the right shows the area in Lucy's backyard that she is preparing for planting.



a. Write an expression for the area of the garden.

b. Simplify the expression using multiplication.

c. Find the area of the garden if $x = 3$ feet.

Calculate the slope of the lines that contain the following pairs of points. Tell whether the line rises, falls, is horizontal, or is vertical.

29. (13) $\left(-\frac{1}{2}, \frac{1}{2}\right)$ and $\left(-\frac{1}{2}, \frac{3}{4}\right)$

30. (13) $\left(-1, \frac{1}{2}\right)$ and $\left(3, \frac{1}{2}\right)$

LESSON 20

Performing Operations with Functions

Warm Up

1. **Vocabulary** A __________ is a mapping between two sets that associates with each element in the domain a unique element in the range. (7)

2. Evaluate $f(x) = 3x + 5$, for $x = -2$. (4)

3. True/False: The points (0, 4), (1, 7), and (1, –2) represent a function. (4)

New Concepts

You can add, subtract, multiply, and divide functions in several ways.

Notation for Operations with Functions

Operation	Notation
Addition	$(f+g)(x) = f(x) + g(x)$
Subtraction	$(f-g)(x) = f(x) - g(x)$
Multiplication	$(fg)x = f(x) \cdot g(x)$
Division	$(f/g)(x) = \frac{f(x)}{g(x)}$, when $g(x) \neq 0$

The domain of the sum of two functions is all the numbers that are common to both of the original domains.

Example 1 Finding the Sum and Difference of Functions Numerically and Algebraically

Given $g(x) = x + 4$; $D = \{\text{Integers}\}$, $h(x) = x - 2$; $D = \{\text{Reals}\}$

a. Find $(h + g)(2)$ numerically.

SOLUTION Note that $D = \{\text{Integers}\}$ is common to both functions. Find $h(2)$ and $g(2)$. Then add.

$$h(2) = 2 - 2 \qquad g(2) = (2) + 4$$
$$h(2) = 0 \qquad g(2) = 6$$
$$(h + g)(2) = (0) + (6) = 6$$

The numerical sum of $(h + g)(2) = 6$.

b. Find the algebraic sum $(h + g)(x)$.

$$h(x) = x - 2$$
$$\underline{g(x) = x + 4}$$
$$h(x) + g(x) = 2x + 2$$

The algebraic sum of $(h + g)(x) = 2x + 2$.

Math Reasoning

Verify Check that $(h + g)(2) = 6$ using the algebraic sum. Show your work.

c. Find $(g - h)(-2)$ numerically.

SOLUTION Find $g(-2)$ and $h(-2)$.

$$g(-2) = (-2) + 4 \qquad h(-2) = (-2) - 2$$
$$g(-2) = 2 \qquad h(-2) = -4$$
$$(g - h)(-2) = (2) - (-4) = 6$$

The numerical difference of $(g - h)(-2) = 6$.

d. Find the algebraic difference $(g - h)(x)$.

$$g(x) - h(x) = (x + 4) - (x - 2) = x + 4 - x + 2 = 6$$

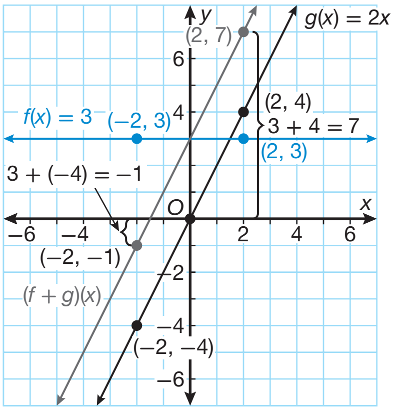
The algebraic difference of $(g - h)(x) = 6$.

You can also find the sum and difference of functions using a coordinate plane.

Example 2 Finding the Sum and Difference of Functions Geometrically

Given $f(x) = 3$; $D = \{\text{Reals}\}$, $g(x) = 2x$; $D = \{\text{Reals}\}$

a. Find $(f + g)(x)$ geometrically.



SOLUTION

Step 1: Write $(f + g)(x)$ as $f(x) + g(x)$ and replace both notations $f(x)$ and $g(x)$ with y.

Step 2: Graph the equations on the coordinate plane.

Step 3: Given D for both f and g is $\{\text{Reals}\}$, let $x = 2$ and $x = -2$.

Step 4: Find the vertical distance from the x-axis to the y-coordinates $f(2)$ and $g(2)$. Find $(2, 0)$ on the x-axis and count the units vertically up or down to the y-coordinate of each function, f and g. Add the vertical distances from the x-axis to $f(2)$ and $g(2)$: $3 + 4 = 7$. Plot the point $(2, 7)$.

Step 5: Find the y-coordinate of each function at $x = -2$. Add the vertical distances from the x-axis to $f(-2)$ and $g(-2)$: $3 + (-4) = -1$. Plot the point $(-2, -1)$. Draw a line through $(2, 7)$ and $(-2, -1)$.

Math Reasoning

Verify How can the algebraic sum $(f + g)(x)$ be used to verify that the line graphed as the sum of f and g is correct?

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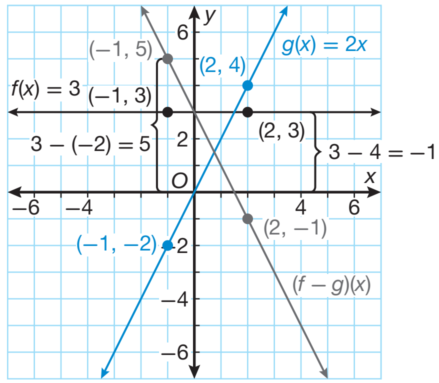
b. Find $(f - g)(x)$ geometrically.



SOLUTION

Step 1: Write $(f - g)(x)$ as $f(x) - g(x)$ and replace both notations $f(x)$ and $g(x)$ with y.

Step 2: Graph the equations.

Step 3: Given D for both f and g is {Reals}, let $x = 2$ and $x = -1$.

Step 4: Find the vertical distance from the x-axis to the y-coordinates $f(2)$ and $g(2)$. Find $(2, 0)$ on the x-axis and count the units vertically up or down to the y-coordinate of each function, f and g. Subtract the vertical distances from the x-axis to $f(2)$ and $g(2)$: $3 - 4 = -1$. Plot the point $(2, -1)$.

Step 5: Repeat the process in **Step 4** for $x = -1$. Plot the point $(-1, 5)$. Draw a line through $(2, -1)$ and $(-1, 5)$.

Example 3 Multiplying Functions Numerically and Algebraically

a. Find $(hg)(-4)$ where $h(x) = x + 3$; $D = \{\text{Reals}\}$, and $g(x) = x - 6$; $D = \{\text{Negative integers}\}$.

SOLUTION

Method 1: Note that the common domain is the negative integers. Find $h(-4)$ and $g(-4)$, then multiply these answers.

$$h(-4) = -4 + 3 \qquad g(-4) = (-4) - 6$$
$$h(-4) = -1 \qquad g(-4) = -10$$
$$(hg)(-4) = (-1)(-10) = 10$$

Method 2: Find $hg(x)$ algebraically. Then find $hg(-4)$.

$$(hg)(x) = (x + 3)(x - 6)$$
$$(hg)(x) = x^2 - 3x - 18$$
$$(hg)(-4) = (-4)^2 - 3(-4) - 18$$
$$(hg)(-4) = 16 + 12 - 18$$
$$(hg)(-4) = 10$$

b. Find $(fg)(-4)$ where $f(x) = x + 3$; $D = \{\text{Reals}\}$, and $g(x) = x - 5$; $D = \{\text{Positive integers}\}$.

SOLUTION

The common domain is the positive integers. Since –4 is a negative integer, $(fg)(-4)$ cannot be found. The problem has no answer.

Math Reasoning

Generalize Find $(f/g)(0)$ for $f(x) = 2x + 1$ and $g(x) = x$. Explain your answer.

Example 4 Dividing Functions Algebraically and Numerically

Find $(f/g)(6)$ if $f(x) = x + 2$; $D = \{\text{Integers}\}$, and $g(x) = x + 3$; $D = \{\text{Positive whole numbers}\}$.

SOLUTION Note that $D = \{\text{Positive whole numbers}\}$ is the common domain.

Method 1: Find $f(6)$ and $g(6)$ and then divide these answers.

$$f(6) = 6 + 2 \qquad g(6) = 6 + 3$$
$$f(6) = 8 \qquad g(6) = 9$$
$$(f/g)(6) = \frac{8}{9}$$

Method 2: Find $(f/g)(x)$ algebraically. Then find $(f/g)(6)$.

$$(f/g)(x) = \frac{f(x)}{g(x)} = \frac{x + 2}{x + 3}$$

The algebraic solution for $(f/g)(x)$ is $\frac{x+2}{x+3}$. Now find $(f/g)(6)$.

$$\frac{6 + 2}{6 + 3} = \frac{8}{9}$$

Example 5 Application: Business

Betsy makes memory wire bracelets to earn extra cash. The cost of making a bracelet is given by the function $f(x) = \$2 + \$5.75x$. Her income from the sale of bracelets is given by the function $g(x) = \$10x$. The number of bracelets is represented by x. Find the function representing her profit, $p(x)$. Then find the profit resulting from the sale of 5 bracelets.

SOLUTION Her profit is represented by the function $p(x) = g(x) - f(x)$ since the income from each bracelet, $g(x)$, minus the cost of making each bracelet, $f(x)$, gives the profit, $p(x)$, for each bracelet, x.

$$p(x) = g(x) - f(x) = 10(x) - (2 + 5.75(x))$$
$$10x - 2 - 5.75(x) = 4.25x - 2$$

So $p(x) = 4.25x - 2$. The profit resulting from the sale of 5 bracelets is found by evaluating $p(5)$.

$$p(5) = 4.25(5) - 2 = 21.25 - 2 = 19.25$$

Betsy's profit on 5 bracelets is \$19.25.

Lesson Practice

a. *(Ex 1)* Given $f(x) = 3x - 5$; $D = \{\text{Reals}\}$, $g(x) = x - 3$; $D = \{\text{Integers}\}$ find $(f + g)(6)$ numerically.

b. *(Ex 1)* Given $f(x) = 3x - 5$; $D = \{\text{Reals}\}$, $g(x) = x - 3$; $D = \{\text{Integers}\}$ find the algebraic sum $(f + g)(x)$.

c. *(Ex 1)* Given $f(x) = 7x - 8$; $D = \{\text{Reals}\}$, $g(x) = x + 5$; $D = \{\text{Integers}\}$ find $(f - g)(3)$ numerically.

d. Given $f(x) = 7x - 8$; $D = \{\text{Reals}\}$, $g(x) = x + 5$; $D = \{\text{Integers}\}$ find the algebraic difference $(f - g)(x)$.
(Ex 1)

e. Given $f(x) = 8$; $D = \{\text{Reals}\}$, $g(x) = 3x$; $D = \{\text{Reals}\}$ find $(f + g)(x)$ geometrically.
(Ex 2)

f. Given $f(x) = 8$; $D = \{\text{Reals}\}$, $g(x) = 3x$; $D = \{\text{Reals}\}$ find $(f - g)(x)$ geometrically.
(Ex 2)

g. Find $(hg)(-4)$ where $h(x) = x + 6$; $D = \{\text{Reals}\}$, and $g(x) = x - 5$; $D = \{\text{Negative integers}\}$.
(Ex 3)

h. Find $(fg)(-4)$ where $f(x) = x + 9$; $D = \{\text{Reals}\}$, and $g(x) = x - 4$; $D = \{\text{Positive integers}\}$.
(Ex 3)

i. Find $(f/g)(7)$ where $f(x) = x + 3$; $D = \{\text{Integers}\}$, and $g(x) = x + 5$; $D = \{\text{Whole numbers}\}$.
(Ex 4)

j. Patty sells pastries to bakeries to earn extra cash. The cost of making one pastry is given by the function $f(x) = \$3 + \$6.25x$. Her income from the sale of pastries is given by the function $g(x) = \$12x$. The number of pastries is represented by x. Find the function representing Patty's profit $p(x)$. Then find her profit from the sale of 6 pastries.
(Ex 5)

Practice Distributed and Integrated

1. *(18)* **Write** Explain how to simplify this problem: 12.65 in. + 30.1 in. + 15.843 in.

2. *(16)* **Multi-Step** The solution to the system of equations $\begin{array}{l} x + y = 5 \\ px + qy = -4 \end{array}$ is (3, 2).

a. Use Cramer's rule to set up equations for x and y.

b. Set $x = 3$ and $y = 2$. Simplify to write an equation with variables p and q.

c. Graph the line from part **b.**

d. Choose a point (p, q) along the line in part **c.** Substitute those values for p and q into the original system to write a system of equations with variables x and y.

e. Graph your system from part **d** to check your answer. How can you tell if your answer is correct?

Solve the equations.

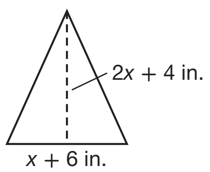
3. *(7)* $3(5r - 10) = 8(2r - 4) + 5r$

4. *(7)* $\frac{3}{4}\left(\frac{1}{2}x - 4\right) = (x - 7)$

***5.** *(16)* **Graphing Calculator** Solve the system using Cramer's Rule. Verify the solution by using a graphing calculator.

$$\begin{cases} 2x + 3y = 54 \\ -x + 6y = 48 \end{cases}$$

6. *(19)* **Geometry** Write an expression for the area of the triangle and simplify.



*7. (18) **Transportation** The rate of one knot equals one nautical mile per hour. One nautical mile is 1852 meters. What is the speed in meters per second of a ship traveling at 20 knots?

*8. (18) **Bird Flight** The peregrine falcon is perhaps the fastest animal on earth. In a stoop, or dive, the peregrine has been clocked at speeds of over 180 miles per hour. Convert this to feet per seconds.

9. (15) A scientist has one solution that is 10% iodine and another that is 40% iodine. How much of each should the scientist use to produce 100 mL of a mixture that is 25% iodine?

Multiply the polynomials.

10. (19) $(x + 7)(x - 9)$

11. (19) $(x + 3)^2 (x - 2)$

12. (19) **Write** Explain how to multiply $(2x + 8)(3x - 6)$ using the FOIL method.

13. (12) **Justify** Is the equation $xyz = 0$ a joint variation? Why or why not?

*14. (16) **Graphing Calculator** The population of Tulsa, Oklahoma, in 2005 was 381,479 and increased by 1393 people by 2006. The population of Tampa, Florida, in 2005 was 325,800 and increased by 7088 people by 2006. If the population of these cities continued to grow at these rates, then the following equations would represent the population of the cities, where x is the number of years after 2005 and y is the population.

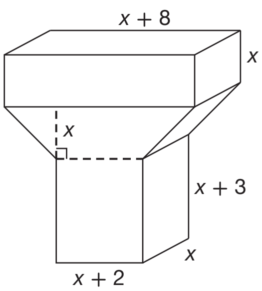
$y = 1393x + 381{,}479$ Tulsa's Population

$y = 7088x + 325{,}800$ Tampa's Population

a. In what year would the populations of Tulsa and Tampa be equal?

b. About how many people would be in each city when the populations are equal?

*15. (11) **Architecture** A concrete support for a highway bridge consists of (from the bottom up) a rectangular prism, a trapezoidal prism, and another rectangular prism, with the dimensions shown. Write a polynomial in standard form to represent the total volume. (Hint**:** The formula for the area of a trapezoid is $A = \frac{1}{2}(b_1 + b_2)h$.)



Find the percent of change in each situation. Round to the nearest tenth.

16. (6) The population changes from 135,000 people to 150,000 people.

17. (6) A price changes from $2.49 for a cup of tea to $2.59.

18. (6) The energy usage changes from 25,500 trillion Btu to 22,500 trillion Btu.

Determine the kind of variation, if any, for each equation.

19. (12) $y = \frac{1}{x}$, for $x \neq 0$

20. (12) $z = \frac{1}{xy + a}$, for $x, y \neq 0$ and any value of a

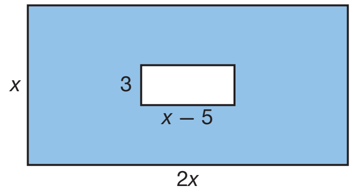
21. (14) Write the determinant as an equation, and then solve the equation.

$$\begin{vmatrix} -x & 2 & -2 \\ 0 & -3 & 1 \\ 2x & 8 & -3 \end{vmatrix} = -4x + 10$$

***22.** (20) **Error Analysis** Two students were evaluating $(f - g)(3)$, given $f(x) = 5x - 2$ and $g(x) = 9x + 20$. Who is correct? Explain the error.

Student A	Student B
$(f-g)(3) = 5(3) - 2 - (9(3) + 20)$ $= 15 - 2 - 27 - 20$ $= -34$	$(f-g)(3) = 5(3) - 2 - (9(3) + 20)$ $= 15 - 2 - 27 + 20$ $= 6$

23. (11, 19) **Geometry** Write a polynomial in standard form that represents the shaded area formed by the two rectangles.



24. (20) **Spending and Saving** The amount of money Hannah has saved is given by the function $f(x) = 20x + 100$. Jess, Hannah's sister, has started spending her money and this is given by the function $g(x) = -10x + 400$. The number of weeks the girls are spending and saving is represented by x.

a. **Formulate** Find the function representing the amount of money that the girls have combined, $h(x)$.

b. Find the amount of money that the girls would have together in 3 weeks.

c. Who will have more money in 15 weeks?

***25.** (20) Given $f(x) = 6x + 13$ and $g(x) = 7x - 15$, find $(f + g)(x)$.

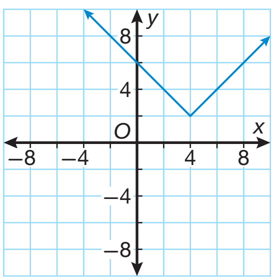
***26.** (20) Explain how to find $(f \cdot g)(x)$, given $f(x) = 5x - 9$ and $g(x) = 9x + 8$.

***27.** (20) **Multiple Choice** Given $f(x) = -2x + 11$ and $g(x) = 6x - 1$, find $(f + g)(8)$.

A 42 **B** 52 **C** 74 **D** 76

28. (17) Solve $|-3x - 4| \geq 2$, then graph the solution. Is $x = 4$ a solution to the inequality?

29. (17) **Multiple Representations** Write the absolute value function shown in the graph.



***30.** (17) **Chemical Elements** For neon to be a liquid, it must be within 1.2715° of –247.1755°C.

a. **Formulate** Write an absolute value inequality to determine the least and greatest temperatures at which neon will remain a liquid.

b. Solve the equation.

INVESTIGATION

2

Solving Parametric Equations

House Painting A house painter charges \$30 per hour. On average, she paints 200 square feet of a wall per hour.

In t hours, the painter paints x square feet and earns y dollars. The following equations describe the area painted and amount earned in terms of t.

$$x = 200t$$
$$y = 30t$$

1. Suppose the painter works for 7 hours.

a. How many square feet does she paint?

b. How much does she earn?

2. Complete the table.

t (hours)	0	1	2	3	4	5
x (square feet)	0	200				
y (dollars)	0	30				

3. **Model** Use the table to write six (x, y) coordinate pairs. (Do not use the t-values.) Graph and connect the points.

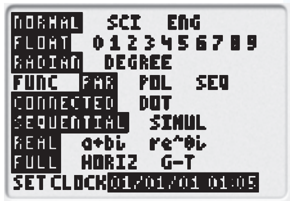
4. **Write** Describe the graph. What is the real-world meaning of the point (200, 30)?

Hint

Time is often the parameter when a real-world situation is modeled by parametric equations.

When two variables are expressed in terms of a third variable, the equations used are called **parametric equations.** In the activity above, the parametric equations are $x = 200t$ and $y = 30t$. The variable in both equations, t, is the **parameter.**

5. **Graphing Calculator** A graphing calculator can graph parametric equations.



a. To put the calculator in parametric mode, press MODE. Use the arrow keys to highlight "PAR," and then press ENTER.

b. Press the Y= button to enter the parametric equations. In $X1_T$, enter 200, and then press the X,T,θ,n key to insert the variable t. In $Y1_T$, enter 30, and then press X,T,θ,n.

c. Press the WINDOW key. Enter 0 for Tmin, 10 for Tmax, and 1 for Tstep. Enter 0 for Xmin, 2000 for Xmax, and 500 for Xscl. Enter 0 for Ymin, 300 for Ymax , and 100 for Yscl.

d. Press GRAPH to graph the equation. Compare the graph to the one from question 3 above.

Online Connection
www.SaxonMathResources.com

A projectile is an object that falls to earth due to the force of gravity. Parametric equations can describe the path of a projectile.

Golf A golfer hits his first shot on a hole. When the golf ball leaves the club, it is traveling 120 feet per second horizontally and 96 feet per second vertically.

The parametric equations that describe the golf ball's motion are

$$x = 120t$$
$$y = -16t^2 + 96t$$

where x is the horizontal distance the golf ball has traveled and y is the height of the ball.

Math Reasoning

Formulate Using the parametric equations describing the path of the golf ball, how can an equation be written for y in terms of x?

Notice where the initial speeds appear in the equations. The initial horizontal speed, 120 ft/s, is the coefficient of t in the equation for x. The initial vertical speed, 96 ft/s, is the coefficient of t in the equation for y.

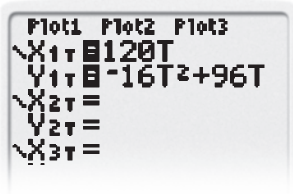
6. Complete the table.

***t* (seconds)**	0	1	2	3	4	5	6
***x* (feet)**	0						
***y* (feet)**	0						

7. **Write** Use the table to write (x, y) coordinate pairs. Graph and connect the points. Describe the graph.

8. **Graphing Calculator** Use a graphing calculator to graph the parametric equations.

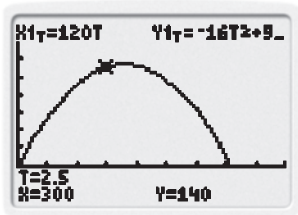
a. Press the Y= button. Enter the parametric equations, using the x^2 button to enter the exponent.



b. Press the WINDOW key. Enter 0 for Tmin, 6 for Tmax, and 0.5 for Tstep. Enter 0 for Xmin, 900 for Xmax, and 100 for Xscl. Enter –50 for Ymin, 200 for Ymax, and 25 for Yscl. Then press GRAPH to see the graph.

c. Press the TRACE key. Notice the blinking cursor at the origin, and the x-, y-, and t-values for that point at the bottom of the screen.

d. Use the right and left arrows to move along the graph. Watch the values of the variables change. When does the golf ball reach its highest point, and what is its height at that time? How long does it take the golf ball to hit the ground, and how many feet does it travel horizontally?



Math Reasoning

Generalize Gravity on the moon is about one-sixth as strong as it is on the earth. Would a golf ball hit in the same way on the moon go higher, farther, or both, than the one in this Activity did on the earth?

Investigation Practice

Suppose t represents a Great Dane puppy's age in months, x is its shoulder height in inches, and y is its weight in pounds. Then these parametric equations approximate the typical growth of a puppy from age 2 months to age 12 months.

$$x = 1.8t + 11.4$$
$$y = 10t$$

a. Complete the table.

t (months)	2	4	6	9	12
x (inches)	15	18.6			
y (pounds)	20	40			

b. Graph the (x, y) coordinate pairs. Describe the graph.

c. **Predict** According to the graph, how heavy will a 24-inch-tall puppy be?

A bus traveling at 40 miles per hour uses 5 gallons of fuel per hour. The fuel tank has a capacity of 60 gallons.

d. Let t be the time in hours, x be the distance traveled, and y be the number of gallons remaining in the fuel tank. Write parametric equations for x and y in terms of t.

e. Make a table of values and graph the (x, y) coordinate pairs. Describe the graph.

f. Use the graph to determine how far the bus can travel on one tank of fuel. How many hours can the bus be driven before it needs refueling?

Graphing Calculator **Suppose the golfer in the activity on page 144 hits a second shot. This time, the horizontal speed of the ball is only 60 feet per second, but the vertical speed is 128 feet per second.**

g. Write a pair of parametric equations for the horizontal distance traveled by the golf ball (x) and the height of the golf ball (y).

h. Graph the equations on a graphing calculator. Adjust Tmax, Xmax, and/or Ymax to see the complete flight of the ball.

i. Use TRACE to move along the graph. What do the coordinates of each point mean?

j. For this shot, when does the golf ball reach its highest point, and what is its height at that time? How long does it take the golf ball to hit the ground, and how many feet does it travel?

k. **Write** Compare the maximum heights and the distances traveled for the two shots. Explain why the differences make sense.