LESSON 31

Multiplying and Dividing Rational Expressions

Warm Up

1. (28) **Vocabulary** A rational expression is __________ if its denominator equals zero.

2. (SB) Multiply. $\frac{3}{10} \cdot \frac{5}{9}$

3. (SB) Divide. $\frac{2}{5} \div \frac{1}{20}$

4. (2) Evaluate $4xy^3$ for $x = 3$, $y = -2$.

New Concepts

To multiply rational expressions, multiply the numerators and multiply the denominators. To simplify a rational expression, factor and then divide out all common factors. To evaluate an algebraic expression for given values of the variables, substitute the given values, and then simplify the numerical expression.

Recall that a rational expression is undefined if its denominator equals zero. In this lesson, assume that all rational expressions are defined unless otherwise specified.

Example 1 Multiplying Rational Expressions

a. Multiply, and then evaluate for $a = 4$, $b = 5$: $\frac{5ab^2}{6b^2} \cdot \frac{12a^2b^2}{3ab^3}$.

SOLUTION Multiply:

$$\frac{5ab^2}{6b^2} \cdot \frac{12a^2b^2}{3ab^3} = \frac{60a^3b^4}{18ab^5}$$ Multiply numerators and denominators.

$$= \frac{\cancel{6} \cdot 10 \cdot \cancel{a} \cdot a^2 \cdot \cancel{b^4}}{3 \cdot \cancel{6} \cdot \cancel{a} \cdot b \cdot \cancel{b^4}}$$ Factor. Then divide out common factors.

$$= \frac{10a^2}{3b}$$ Write the algebraic expression in simplified form.

Evaluate the expression for $a = 4$, $b = 5$:

$$\frac{10a^2}{3b} = \frac{10 \cdot 4^2}{3 \cdot 5}$$ Substitute 4 for *a* and 5 for *b*.

$$= \frac{2 \cdot \cancel{5} \cdot 4^2}{3 \cdot \cancel{5}}$$ Factor as needed to divide out all common factors.

$$= \frac{32}{3}$$ Write the numerical expression in simplified form.

The product is the expression $\frac{10a^2}{3b}$. Its value is $\frac{32}{3}$ for the given values of a and b.

Hint

Another way to simplify $\frac{10 \cdot 4^2}{3 \cdot 5}$ is to multiply, divide, and then simplify.

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b. Multiply, and then identify all values of x that make the expression undefined: $\frac{15x - 3x^2}{x^2 - 9} \cdot \frac{x^2 - 8x + 15}{x^2 - 10x + 25}$.

SOLUTION

$$\frac{15x - 3x^2}{x^2 - 9} \cdot \frac{x^2 - 8x + 15}{x^2 - 10x + 25}$$

$$= \frac{3x(5 - x)}{(x - 3)(x + 3)} \cdot \frac{(x - 3)(x - 5)}{(x - 5)(x - 5)}$$ Factor.

$$= \frac{3x(-1)\cancel{(x - 5)}\cancel{(x - 3)}\cancel{(x - 5)}}{\cancel{(x - 3)}(x + 3)\cancel{(x - 5)}\cancel{(x - 5)}}$$ Rewrite $(5 - x)$ as $(-1)(x - 5)$ so that the common factor $(x - 5)$ appears. Then multiply. Then divide out common factors.

$$= \frac{-3x}{(x + 3)}$$

Identify values of x that make the *given* expression undefined:

The given expression is $\frac{15x - 3x^2}{x^2 - 9} \cdot \frac{x^2 - 8x + 15}{x^2 - 10x + 25}$, which is equivalent to $\frac{3x(5 - x)}{(x - 3)(x + 3)} \cdot \frac{(x - 3)(x - 5)}{(x - 5)(x - 5)}$. The factors in the denominator are $(x - 3)$, $(x + 3)$, and $(x - 5)$. These factors are zero when x is 3, −3, and 5, respectively. So, the values of x that make the given expression undefined are 3, −3, and 5.

Caution

To identify values that make an expression undefined, use the given expression, not the simplified result.

c. Multiply $\frac{5x + 10}{3x^2 + 2x - 1} \cdot \frac{9x^2 - 1}{x + 2} \cdot \frac{x + 1}{15x + 5}$.

SOLUTION

$$\frac{5x + 10}{3x^2 + 2x - 1} \cdot \frac{9x^2 - 1}{x + 2} \cdot \frac{x + 1}{15x + 5}$$

$$= \frac{\cancel{5(x + 2)}}{\cancel{(3x - 1)(x + 1)}} \cdot \frac{\cancel{(3x + 1)(3x - 1)}}{\cancel{x + 2}} \cdot \frac{\cancel{x + 1}}{\cancel{5(3x + 1)}}$$ Factor. Divide by the common factors and then multiply.

$$= 1$$

d. Multiply: $\frac{x + 4}{x^2 - 4x} \cdot (2x^2 - 9x + 4)$

SOLUTION

$$\frac{x + 4}{x^2 - 4x} \cdot \frac{2x^2 - 9x + 4}{1}$$ Rewrite the polynomial as a rational expression with a denominator of 1. Then factor, multiply, and divide out common factors.

$$= \frac{x + 4}{x\cancel{(x - 4)}} \cdot \frac{(2x - 1)\cancel{(x - 4)}}{1}$$

$$= \frac{(x + 4)(2x - 1)}{x}$$

Dividing by a Rational Expression

The **reciprocal** of a real number $\frac{c}{d}$ is $\frac{d}{c}$, provided that $c \neq 0$ and $d \neq 0$.

To divide by a rational expression, multiply by its reciprocal: $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$, where b, c, and d are all nonzero.

Example 2 Dividing Rational Expressions

Hint

One quick way to check simplification is to substitute values for the variables in the original expression and evaluate. They should be the same.

a. Divide, and then evaluate for $x = -2$, $y = 3$: $\frac{12x^3}{5y} \div \frac{4x^5}{5x^3y^2}$.

SOLUTION

$\frac{12x^3}{5y} \div \frac{4x^5}{5x^3y^2} = \frac{12x^3}{5y} \cdot \frac{5x^3y^2}{4x^5}$ Multiply by the reciprocal.

$= \frac{60x^6y^2}{20x^5y}$ Multiply numerators and denominators.

$= \frac{3 \cdot \cancel{20} \cdot x \cdot \cancel{x^5} \cdot \cancel{y} \cdot y}{\cancel{20} \cdot \cancel{x^5} \cdot \cancel{y}}$ Factor. Then divide out common factors.

$= \frac{3xy}{1}$ Write the algebraic expression in simplified form.

$= 3xy$

Evaluate for $x = -2$, $y = 3$:

$3xy = 3(-2)(3)$ Substitute -2 for x and 3 for y, and then simplify.

$= -18$

The quotient is the expression $3xy$. Its value is -18 for the given values of x and y.

Caution

Do not divide out terms of polynomials.

$\frac{x+1}{x-3} \neq \frac{1}{-3}$

(unless $x = 0$).

b. Divide, and then identify all values of x that make the expression undefined: $\frac{x+1}{x-2} \div \frac{x^2-3x}{x^2-2x}$.

SOLUTION Divide:

$\frac{x+1}{x-2} \div \frac{x^2-3x}{x^2-2x} = \frac{x+1}{x-2} \cdot \frac{x^2-2x}{x^2-3x}$ Multiply by the reciprocal.

$= \frac{x+1}{x-2} \cdot \frac{x(x-2)}{x(x-3)}$ Factor.

$= \frac{(x+1)\cancel{x}\cancel{(x-2)}}{\cancel{(x-2)}\cancel{x}(x-3)}$ Divide out common factors.

$= \frac{x+1}{x-3}$

Identify values of x that make the *given* expression undefined:

The given expression in factored form is $\frac{x+1}{x-2} \div \frac{x(x-3)}{x(x-2)}$. In a division expression of the form $\frac{a}{b} \div \frac{c}{d}$, b, c, and d must all be nonzero. So, in this case, $x - 2$, $x(x-3)$, and $x(x-2)$ must all be nonzero. Therefore, the factors $(x-2)$, x, and $(x-3)$ must all be nonzero. So the values of x that make the given division expression undefined are 2, 0, and 3.

Example 3 Simplifying a Rational Expression Containing Multiplication and Division

Simplify: $\dfrac{45}{x^2 + 2x - 15} \cdot \dfrac{2x^2 + 10x}{5x^2y} \div \dfrac{1}{x - 3}$

SOLUTION

$$\frac{45}{x^2 + 2x - 15} \cdot \frac{2x^2 + 10x}{5x^2y} \div \frac{1}{x - 3}$$

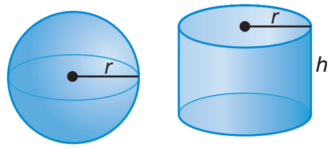
$$= \frac{45}{x^2 + 2x - 15} \cdot \frac{2x^2 + 10x}{5x^2y} \cdot \frac{x - 3}{1}$$ Multiply by the reciprocal.

$$= \frac{\cancel{5} \cdot 9}{\cancel{(x + 5)}\cancel{(x - 3)}} \cdot \frac{2 \cdot \cancel{x} \cdot \cancel{(x + 5)}}{\cancel{5} \cdot \cancel{x} \cdot x \cdot y} \cdot \frac{\cancel{(x - 3)}}{1}$$ Factor and divide out common factors.

$$= \frac{18}{xy}$$

Example 4 Application: Determining a Dimension

An engineer is designing two storage tanks. One tank is to be a sphere and the other a cylinder. He wants the tanks to have the same volume and the same radius. What must be the height of the cylinder, in terms of the radius?



SOLUTION

$$V_{sphere} = V_{cylinder}$$

$$\frac{4\pi}{3}r^3 = \pi r^2 h$$

$$\frac{1}{\pi r^2} \cdot \frac{4\pi}{3}r^3 = \frac{1}{\pi r^2} \cdot \pi r^2 h$$ Multiply both sides by the reciprocal of πr^2.

$$\frac{1}{\pi r^2} \cdot \frac{4\pi}{3}r^3 = h$$

$$\frac{1}{\cancel{\pi} \cdot \cancel{r^2}} \cdot \frac{4 \cdot \cancel{\pi} \cdot r \cdot \cancel{r^2}}{3} = h$$ Factor and divide out common factors.

$$\frac{4r}{3} = h$$

Check Substitute $\frac{4r}{3}$ for h in the expression for the volume of a cylinder. The result is the expression for the volume of a sphere, so the answer is correct:

$$V_{cylinder} = \pi r^2 h = \pi r^2 \cdot \frac{4r}{3} = \frac{4\pi r^3}{3} = V_{sphere}$$

Math Reasoning

Analyze Suppose the engineer wanted the tanks to have the same surface area. What must be the height of the cylinder in terms of the radius?

Lesson Practice

a. (Ex 1) Multiply, and then evaluate for $x = 5$: $\frac{4x^2 - 9}{x + 1} \cdot \frac{x^2 + x}{4x^2 + 12x + 9}$.

b. (Ex 1) Multiply, and then identify all values of x that make the expression undefined: $\frac{x^2 + 5x - 14}{x^2 + x - 6} \cdot \frac{x^2 - x - 12}{5x^2 - 20x}$.

c. (Ex 1) Multiply: $\frac{x^2 - 4}{7x^2 + 14x - 56} \cdot \frac{7x - 21}{x + 4} \cdot \frac{x + 4}{x^2 - x - 6}$.

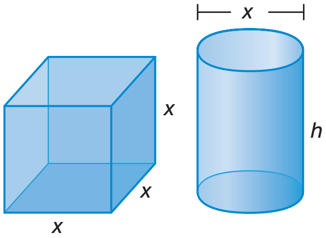
d. (Ex 1) Multiply: $\frac{3x - 9}{6x^2 + 12x} \cdot (x^2 - 4x - 12)$.

e. (Ex 2) Divide, and then evaluate for $x = -1$, $y = -6$: $\frac{3x^2y^5}{7xy^8} \div \frac{9x^8y^2}{14xy^6}$

f. (Ex 2) Divide, and then identify all values of x that make the expression undefined: $\frac{1}{x^2 + 2x} \div \frac{2x - 1}{x^3 + 7x^2 + 10x}$

g. (Ex 3) Simplify: $\frac{16 - x^2}{4xy} \cdot \frac{x^2y}{x - 4} \div \frac{x + 4}{4}$

h. (Ex 4) A cube and a cylinder are to have the same volume. The diameter x of the cylinder is equal to the length of the edge of the cube. What must be the height of the cylinder, in terms of x?



Practice Distributed and Integrated

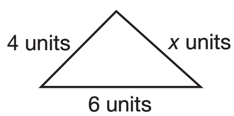
1. (9) **Generalize** Given the dimensions of four matrices, A: 2 × 4, B: 2 × 2, C: 4 × 2, D: 4 × 4, determine all of the possible multiplication pairs and the dimensions of each product matrix.

***2.** (31) If $f(x) = \frac{x^2 - 100}{x^2 + 20x + 100}$ and $g(x) = \frac{10x + 100}{3x - 30}$, what is $f(x) \cdot g(x)$ in simplest form?

***3.** (28) **Engineering** The Channel Tunnel is a tunnel that runs under the English Channel. It is cylindrical in shape. Write the volume to lateral area ratio as a simplified rational expression. Then find the ratio given that the tunnel has an inner diameter of 7.6 meters. (Hint: The lateral area of a cylinder is the same as the surface area minus the area of the bases.)

4. (8) When mixing concentrated fertilizer for a garden, the amount of fertilizer varies directly with the amount of water. If 2 cups of fertilizer are used in 16 gallons of water, how much fertilizer is used in 48 gallons of water?

5. (10) **Geometry** Write a compound inequality using the fact that the sum of the lengths of any two sides of a triangle is greater than the length of the third side (Triangle Inequality) using the diagram. (Hint: Remember that length cannot be negative.)

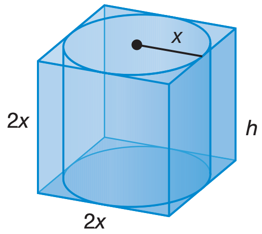


Use the description to write the vertex form of a quadratic equation.

***6.** (30) The parent function is shifted 4 units right and 6 units down.

***7.** (30) The parent function is reflected across the x-axis, and then shifted 3 units up.

8. (11) **Woodworking** A lathe operator has a piece of wood in the shape of a square prism with dimensions $2x$, $2x$, and h. He wants to make the largest possible cylinder by turning the wood on the lathe and cutting it down. The cylinder will have radius x and length h. Write a polynomial that represents the volume of wood he will cut off to make the cylinder.



Multiply.

***9.** (31) $\frac{3a^3b^2}{5a^5} \cdot \frac{10a^3b}{7a^4b^2}$

***10.** (31) $\frac{2xy}{3y^{11}} \cdot \frac{6x^2y}{3xy}$

11. (23) **Verify** Use FOIL to verify that $a = c + d$ and $b = cd$ in the formula:

$x^2 + ax + b = (x + c)(x + d)$.

12. (13) **Mountains** Mt. Elbert in Colorado has an elevation of 14,433 ft and from the center, it runs horizontally approximately 22,105 ft. Find the approximate slope of Mt. Elbert, round your answer to the thousandths place.

13. (11) **Multiple Choice** Which polynomial is in standard form?

A $\frac{1}{2}x^3 - x$ **B** $\frac{1}{3} - \frac{1}{3}x^3$ **C** $4x^2 + 3x^3 + 2x^4$ **D** $x^4 + x^5$

14. (18) Convert 87 ft^2 to square inches.

15. (10) **Analyze** Find the largest two consecutive even integers with a total sum less than or equal to 130.

16. (14) **Error Analysis** Explain the error in the student's work. Then find the correct determinant. $\begin{vmatrix} -2 & 6 \\ 3 & -1 \end{vmatrix} = (6)(3) - (-2)(-1) = 18 - 2 = 16$

***17.** (15) **Graphing Calculator** Solve the system of equations using a graphing calculator.

$$2x - 3y = 4$$
$$4x + y = 1$$

18. (17) **Multi-Step** $|ax + b| \leq c$, where $a > 0$ and $c > 0$.

a. Solve the inequality for x in terms of a, b, and c.

b. Apply this general solution to solve $|2x + 3| \leq 5$.

19. (13) **Coordinate Geometry** Graph the pentagon formed by these five equations on a coordinate plane.

$$x = 3 \qquad x = -3 \qquad y = -4 \qquad y = \frac{2}{3}x + 3 \qquad y = -\frac{2}{3}x + 3$$

a. Is the pentagon regular?

b. What are the coordinates of the vertices of the pentagon?

***20.** (30) **Error Analysis** A student said the graph of the function $y = (x - 6)^2$ is a shift of the parent function 6 units to the left because h is negative. Explain and correct the error in the student's thinking.

21. (19) **Multiple Choice** Multiply $(x + 11)(x - 8)$.
A $x^2 - 19x + 88$ **B** $x^2 + 3x - 88$ **C** $x^2 - 3x - 88$ **D** $x^2 + 19x - 88$

Identify the following equations as direct or inverse variations.

22. (12) $y = 3x$

23. (12) $y = \frac{20}{x}$

24. (21) **Analyze** Describe how a graph of 2 linear equations looks when the system has:
a. exactly one solution.

b. an infinite number of solutions.

c. no solution.

***25.** (31) **Write** Describe how to find the following product in simplest form: $\frac{a^2 - 49}{a - 7} \cdot \frac{a}{2a^2 + 14a}$.

***26.** (28) **Multi-Step** **a.** Simplify $\frac{4x^2 - 13x + 3}{9 - 6x + x^2}$.

b. Evaluate the simplified expression for $x = 9$.

27. (24) **Money** A parking meter contains 157 coins worth $26.95. If the meter only accepts quarters and dimes, how many of each type of coin are in the meter?

Given $f(x) = 5x - 9$ and $g(x) = 9x + 8$.

28. (20) Find $(f \cdot g)(3)$.

29. (20) Find $\left(\frac{f}{g}\right)(2)$.

30. (Inv 3) Find the intercepts of the equation $10x + 3y - 8z = 12$.

LESSON 32

Solving Linear Systems with Matrix Inverses

Warm Up

1. **Vocabulary** A ________ is a rectangular array of elements.
(5)

2. Multiply the matrix by the scalar. $-2[1 \ \ 3 \ \ -4]$
(9)

3. Multiply the matrices. $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} 0 & 4 \\ 1 & 5 \end{bmatrix}$
(9)

New Concepts

Exploration **Exploring Matrix Inverses**

1. Find products AB and BA for $A = \begin{bmatrix} -2 & 0 \\ 5 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -0.5 & 0 \\ 2.5 & 1 \end{bmatrix}$.

2. Find products PQ and QP for $P = \begin{bmatrix} 8 & 5 \\ 6 & 4 \end{bmatrix}$ and $Q = \begin{bmatrix} 2 & -2.5 \\ -3 & 4 \end{bmatrix}$.

3. Find products ST and TS for $S = \begin{bmatrix} 3 & 1 \\ 11 & 4 \end{bmatrix}$ and $T = \begin{bmatrix} 4 & -1 \\ -11 & 3 \end{bmatrix}$.

4. What do you notice about the products of the matrices in Problems 1–3?

5. Explain what is special about the matrix you found to be the product in Problems 1–3.

Hint

Recall that an identity matrix has ones on the diagonal and zeros everywhere else.

If A is any $n \times n$ matrix and I is the $n \times n$ identity matrix, then $AI = IA = A$. The **multiplicative inverse of a square matrix,** if it exists, is a matrix such that the product of it and another matrix forms an identity matrix. The inverse of matrix A is notated A^{-1}. So, two matrices A and B are inverses of each other if $A \cdot B = B \cdot A = I$. Likewise, $A \cdot A^{-1} = A^{-1} \cdot A = I$.

Example 1 Verifying Inverses

Hint

Matrix multiplication is not commutative. Check the product in both orders to verify that the matrices are inverses.

Determine whether the matrices are inverses.

a. $P = \begin{bmatrix} 2 & 6 \\ 1 & 4 \end{bmatrix}$ and $Q = \begin{bmatrix} 2 & -3 \\ -0.5 & 1 \end{bmatrix}$

SOLUTION

Determine whether $P \cdot Q = I$ and $Q \cdot P = I$.

$$P \cdot Q = \begin{bmatrix} 2 & 6 \\ 1 & 4 \end{bmatrix} \cdot \begin{bmatrix} 2 & -3 \\ -0.5 & 1 \end{bmatrix} = \begin{bmatrix} 4-3 & -6+6 \\ 2-2 & -3+4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$$Q \cdot P = \begin{bmatrix} 2 & -3 \\ -0.5 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 6 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 4-3 & 12-12 \\ -1+1 & -3+4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$P \cdot Q = I$ and $Q \cdot P = I$, so P and Q are inverses.

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b. $R = \begin{bmatrix} 3 & 2 \\ 5 & 1 \end{bmatrix}$ and $S = \begin{bmatrix} 4 & 1 \\ -2 & 3 \end{bmatrix}$

SOLUTION Determine whether $R \cdot S = I$ and $S \cdot R = I$.

$$R \cdot S = \begin{bmatrix} 3 & 2 \\ 5 & 1 \end{bmatrix} \cdot \begin{bmatrix} 4 & 1 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 12 - 4 & 3 + 6 \\ 20 - 2 & 5 + 3 \end{bmatrix} = \begin{bmatrix} 8 & 9 \\ 18 & 8 \end{bmatrix} \neq I$$

$R \cdot S \neq I$, so R and S are not inverses.

> **Reading Math**
>
> If A is the matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then these all represent the determinant of A:
>
> - det A
> - $\det\begin{bmatrix} a & b \\ c & d \end{bmatrix}$
> - $|A|$
> - $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$

Recall, that to find the determinant of a 2×2 matrix, use the following method.

$$\det\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - cb$$

An $n \times n$ matrix A has an inverse if and only if $\det A \neq 0$. The inverse of matrix A is denoted A^{-1}. If A does not have an inverse, then A is called a **singular matrix.**

Inverse of a 2 × 2 Matrix
If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $ad - cb \neq 0$, then the inverse of A is:
$A^{-1} = \frac{1}{\lvert A \rvert}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{ad - cb}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

Example 2 Finding Inverses

Find the inverse of each matrix, if it exists.

a. $T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \end{bmatrix}$

SOLUTION First find the determinant:
$|T| = (3)(2) - (-1)(4) = 6 + 4 = 10.$

$$T^{-1} = \frac{1}{10}\begin{bmatrix} 2 & -4 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 0.2 & -0.4 \\ 0.1 & 0.3 \end{bmatrix}$$

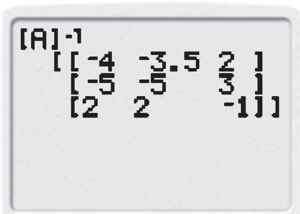
b. $Y = \begin{bmatrix} 2 & 4 \\ 5 & 10 \end{bmatrix}$

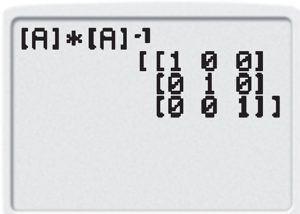
SOLUTION Find the determinant: $|Y| = (2)(10) - (5)(4) = 20 - 20 = 0.$
The inverse of Y does not exist because $|Y| = 0$.

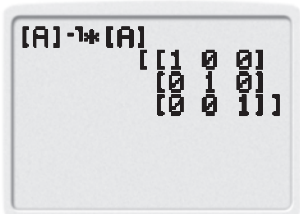
c. $A = \begin{bmatrix} -2 & 1 & -1 \\ 2 & 0 & 4 \\ 0 & 2 & 5 \end{bmatrix}$

SOLUTION Use a graphing calculator to find the inverse matrix. Then use the calculator to verify that $AA^{-1} = I$ and $A^{-1}A = I$.

> **Graphing Calculator Tip**
>
> For help in inputting matrices and finding inverses with the graphing calculator, see the lab on page 27.







A **matrix equation** represents a system of equations in matrix form where the variable is a matrix.

Using an Inverse to Solve a Matrix Equation		
If matrix A has an inverse and the matrix equation $AX = B$ has a solution, then the solution is $X = A^{-1}B$.	$AX = B$	
	$A^{-1}AX = A^{-1}B$	Multiply both sides by A^{-1}.
	$IX = A^{-1}B$	$A^{-1}A = I$ by definition of inverse.
	$X = A^{-1}B$	$IX = X$ by definition of identity.

Example 3 Solving a Matrix Equation

Solve for matrix X. $\underbrace{\begin{bmatrix} 12 & 7 \\ 5 & 3 \end{bmatrix}}_{A} X = \underbrace{\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}}_{B}$

SOLUTION

$$X = A^{-1}B = \begin{bmatrix} 3 & -7 \\ -5 & 12 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} -15 & -17 \\ 26 & 29 \end{bmatrix}$$

Check $AX = \begin{bmatrix} 12 & 7 \\ 5 & 3 \end{bmatrix}\begin{bmatrix} -15 & -17 \\ 26 & 29 \end{bmatrix} = \begin{bmatrix} -180 + 182 & -204 + 203 \\ -75 + 78 & -85 + 87 \end{bmatrix}$

$$= \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix} = B$$

Caution

To solve $AX = B$, you must multiply both sides of the equation by A^{-1} from the left.

$(A^{-1})\,AX = (A^{-1})B$

When the matrix equation $AX = B$ is used to represent a system of equations, A is called the **matrix of coefficients,** X is called the **matrix of variables,** and B is called the **matrix of constants.**

Example 4 Using an Inverse Matrix to Solve a Linear System

Solve the system.

$$\begin{aligned} 3x + 2y + z &= 1 \\ x - y - 3z &= 12 \\ -x + y + 4z &= -15 \end{aligned}$$

SOLUTION

Write the system as a matrix equation. $\underbrace{\begin{bmatrix} 3 & 2 & 1 \\ 1 & -1 & -3 \\ -1 & 1 & 4 \end{bmatrix}}_{A}\underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 1 \\ 12 \\ -15 \end{bmatrix}}_{B}$

Solve the equation. $X = A^{-1}B = \begin{bmatrix} 0.2 & 1.4 & 1 \\ 0.2 & -2.6 & -2 \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 12 \\ -15 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ -3 \end{bmatrix}$

The solution to the system is $x = 2$, $y = -1$, $z = -3$.

Hint

Remember
1 kilogram = 1000 grams

Example 5 Application: Analyzing Nutrition and Cost

A 460-gram package of trail mix costs $4.42 and contains 59 grams of protein. The mix contains peanuts ($12 per kilogram, 25% protein), raisins ($5.50 per kilogram, 3% protein), and almonds ($14 per kilogram, 13% protein). How much of each ingredient is in the trail mix?

SOLUTION Write a system.

$p + r + a = 460$	masses of ingredients, using grams
$0.25p + 0.03r + 0.13a = 59$	masses of protein in the ingredients
$0.012p + 0.0055r + 0.014a = 4.42$	costs of ingredients, using grams

Write the matrix equation. $\underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 0.25 & 0.03 & 0.13 \\ 0.012 & 0.0055 & 0.014 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} p \\ r \\ a \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 460 \\ 59 \\ 4.42 \end{bmatrix}}_{B}$

Solve the equation. $X = A^{-1}B = \begin{bmatrix} 160 \\ 200 \\ 100 \end{bmatrix}$

The solution to the system is $p = 160$, $r = 200$, $a = 100$. The trail mix contains 160 grams of peanuts, 200 grams of raisins, and 100 grams of almonds.

Check The total mass of ingredients is $160 + 200 + 100 = 460$ grams. The total mass of protein is $(0.25)(160) + (0.03)(200) + (0.13)(100) = 59$ grams.

The total cost is $(0.012)(160) + (0.0055)(200) + (0.014)(100) = \$4.42$.

Lesson Practice

a. (Ex 1) Determine whether $Q = \begin{bmatrix} -1 & 2 \\ 1.5 & -2.5 \end{bmatrix}$ and $W = \begin{bmatrix} 5 & 4 \\ 3 & 2 \end{bmatrix}$ are inverses.

b. (Ex 1) Determine whether $A = \begin{bmatrix} 2 & -1 \\ -4 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} \frac{1}{2} & -1 \\ -\frac{1}{4} & \frac{1}{3} \end{bmatrix}$ are inverses.

c. (Ex 2) Find the inverse of $A = \begin{bmatrix} 5 & -2 \\ -3 & 1 \end{bmatrix}$, if it exists.

d. (Ex 2) Find the inverse of $B = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$, if it exists.

e. (Ex 2) Use a calculator to find and verify the inverse of $A = \begin{bmatrix} 2 & 1 & -2 \\ 5 & 3 & 0 \\ 4 & 3 & 8 \end{bmatrix}$.

f. (Ex 3) Solve for matrix X. $\underbrace{\begin{bmatrix} 3 & 7 \\ 1 & 4 \end{bmatrix}}_{A} X = \underbrace{\begin{bmatrix} -1 & -8 \\ -3 & -24 \end{bmatrix}}_{B}$

g. (Ex 4) Solve the system.

$$\begin{aligned} -5x - y + z &= -16 \\ 3x + 2y - z &= 12 \\ x + y + 2z &= -10 \end{aligned}$$

h. (Ex 5) A 350-gram entrée contains black beans (3% fat and 23% protein), chicken breast (20% fat and 80% protein), and rice (2% fat and 8% protein). The entrée has 24.7 grams of fat and 115.3 grams of protein. How much of each ingredient is in the entrée?

Practice Distributed and Integrated

***1.** (29) **Verify** Is the ordered triple (5, −3, −1) a solution of the system:

$$\begin{aligned} -2x - y - 2z &= 5 \\ x - 4y + 3z &= 14 \quad ? \\ -x + 4y - 3z &= -6 \end{aligned}$$

2. (25) **Multi-Step** A data set consists of the following numbers: −4, −7, 12, −2, 8, 3, −1, 2, 4, −52

a. Find the mean of the data. **b.** Find the standard deviation.

c. Identify any outliers.

d. Describe how any outlier affects the mean and the standard deviation of this data set.

3. (10) Solve the inequality: $3t + 4 < 2(t + 3) + t$. Show your work.

***4.** (31) **Multiple Choice** Which shows all the values of x that make the division expression $\frac{x^2 - 1}{x} \div \frac{3 - x}{2x + 2}$ undefined?

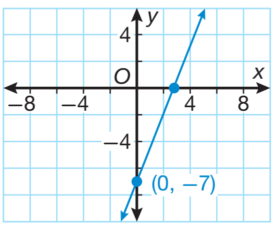
A 0, 3, −1 **B** 0, −3, 1 **C** 2, 3, −1 **D** 2, −3, 1

5. (11) **Analyze** If the degree of every term of a polynomial is even and all the coefficients are positive (including the constant term), can the polynomial have a negative value? Explain.

***6.** (32) Write the matrix equation for the system and solve.

$$3x - y = 5 \text{ and } y = 2x - 4$$

7. (13) **Multiple Choice** Choose the letter that best represents the equation of the graph:



A $y = \frac{2}{5}x - 7$ **B** $y = -\frac{5}{2}x - 7$

C $y = \frac{5}{2}x - 7$ **D** $y = -\frac{2}{5}x - 7$

8. (15) **Verify** Verify that (1.3, −2.7) is a solution of $\begin{aligned} 5.3x - 0.8y &= 9.05 \\ -3.0x + 1.4y &= -7.68 \end{aligned}$.

*9. Divide. $\frac{36x^5y^{10}}{7y^4} \div \frac{12y^4}{21x^2}$
(31)

10. **Recreational Planning** A playground that is to be rectangular in shape has an area modeled by the function $f(x) = -x^2 + 12x$ where x represent the length of one side of the playground in meters.
(27)

a. What value of x gives the playground the greatest possible area?

b. What is the greatest possible area?

*11. **Graphing Calculator** An object dropped from a height 40 feet above the ground after x seconds is represented by $y = -16x^2 + 40$. When the object is dropped from 10 feet above the ground, the function becomes $y = -16x^2 + 10$.
(30)

a. Graph the functions on a graphing calculator and describe the transformation that changes the graph of the first function to the second.

b. For which quadrant or quadrants does the graph make sense? Why?

Write matrix equations for the systems of equations

*12. (32)
$$\begin{aligned} 2x + 4y + z &= 16 \\ x - 5y + z &= -5 \\ x + y + z &= 7 \end{aligned}$$

*13. (32)
$$\begin{aligned} 3x + 4z &= 13 \\ 15y - 2z &= 20 \\ 6z &= 6 \end{aligned}$$

14. Written in vertex form, what are the values of a, h, and k in $f(x) = -2\left(x - \frac{1}{2}\right)^2 - 5$?
(27)

15. **Write** Describe how to rewrite the polynomial $x + 4x + 12 - 5x^2 - x$ as an equivalent polynomial in standard form.
(11)

*16. **Nutrition** A sandwich consists of Swiss cheese, ham, and rye bread. The sandwich contains 37.4 grams of carbohydrates, 46 grams of fat, and 36.6 grams of protein. How much of each ingredient is in the sandwich?
(32)

	Swiss Cheese	Ham	Rye Bread
Carbohydrates	6%	9%	77%
Fat	64%	48%	11%
Protein	30%	43%	12%

17. **Error Analysis** Explain and correct the error a student made below.
(28)

$$\frac{10 - 2r}{r^2 - 10r + 25} = \frac{2(5 - r)}{(r - 5)(r - 5)} = \frac{2\cancel{(r - 5)}}{\cancel{(r - 5)}(r - 5)} = \frac{2}{r - 5}$$

18. **Geometry** For each equation identify the kind of variation it represents.
(12)

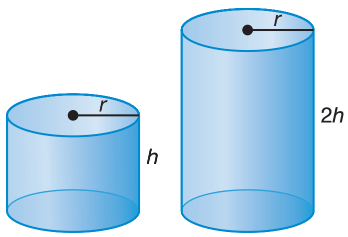
a. Area of a circle = $\pi(\text{radius})^2$ **b.** Area of a rectangle = length × width

*19. **Multi-Step** A container manufacturer is considering doubling the height of a cylindrical container.
(31)

a. Find the ratio of surface area to volume for the small container.

b. Find the ratio of surface area to volume for the large container.

c. A smaller ratio of surface area to volume results in a more economical container. Which container is more economical? Explain.



20. (17) **Exercise** For a 20-year old to workout within his cardio range, his heart rate should be within 25 beats per minute of 155 bpm.

a. Write an absolute value inequality to determine the lowest and highest heart rate at which he will be in his cardio range.

b. Solve the inequality.

21. (3) Simplify $\dfrac{x^2x^0x^{-1}(x^{-2})^2yx^{-3}}{(x^2y)^{-3}xyx^{-2}x^2}$.

22. (19) **Basketball** The length of a regulation high school basketball court is represented by the expression $(x + 24)$ feet. The width of a regulation high school basketball court is represented by the expression $(x - 10)$ feet.

a. Write an expression that represents the area of a regulation high school basketball court.

b. Evaluate the area of the court if $x = 60$.

***23.** (32) **Analyze** Suppose matrix A has an inverse and the matrix equation $AX = B$ has a solution. Explain why $X = A^{-1}B$ is the solution, but $X = BA^{-1}$ is not the solution.

24. (20) Given $f(x) = -7x + 4$ and $g(x) = 9x + 2$, represent $(f + g)(x)$ geometrically.

25. (28) **Probability** A mathematician has a number of red, white, and blue marbles. The number of red marbles is the same as the number of blue marbles and the number of white marbles is the square of the number of red marbles. If the mathematician puts all the marbles in a bag and randomly chooses one marble, what simplified rational expression represents the probability of pulling out one red marble?

Solve.

26. (24) $\frac{3}{8}x - \frac{1}{3}y = -9$
$2x + 4y = 4$

27. (7) $\dfrac{-3 - x}{2} - \dfrac{x}{2} = 7$

***28.** (29) Use
$$\begin{aligned} 9x &= y - 3z - 21 \\ 4y - 6z &= 8x + 10 \\ 56x - 28y &= 2 - 42z \end{aligned}$$

a. Write the equations in standard form.

b. Estimate the number of solutions. Explain your answer.

c. Solve the system of equations to verify your estimate.

29. (23) Solve $x^2 - 5x - 36 = 0$

30. (14) **Error Analysis** Explain the error in the work below. Then find the correct determinant.

$$\begin{vmatrix} 1 & 2 & -2 \\ -1 & 4 & 3 \\ 0 & 1 & 2 \end{vmatrix} = 1\begin{vmatrix} 4 & 3 \\ 1 & 2 \end{vmatrix} - 1\begin{vmatrix} 2 & -2 \\ 1 & 2 \end{vmatrix} + 0\begin{vmatrix} 2 & -2 \\ 4 & 3 \end{vmatrix} = 5 - 6 + 0 = -1$$

LESSON 33

Applying Counting Principles

Warm Up

1. **Vocabulary** The middle number in an ordered list of numbers is called the __________.
(25)

2. Find the mean of this data set: 12, 20, 20, 30.
(25)

3. Find the median of this data set: 10, 20, 40, 30, 32, 38.
(25)

New Concepts

In counting theory, an **experiment** is any process that results in one or more **outcomes.** If a process is repeated one or more times, each time it is performed is sometimes called a **trial,** and the experiment consists of all the trials.

Math Language

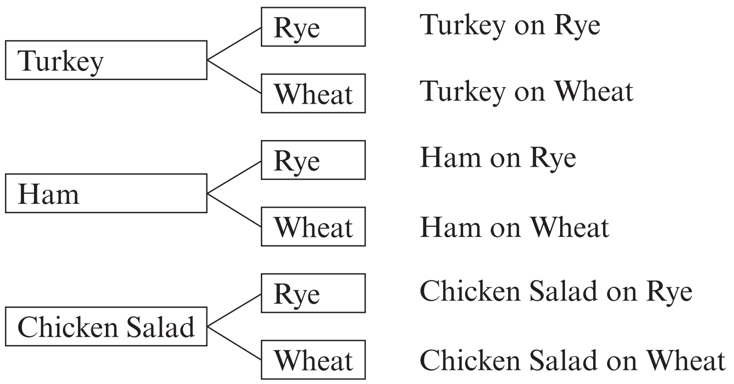
A **tree diagram** is a branching diagram that shows all possible combinations or outcomes of an experiment.

Example 1 Using a Tree Diagram

A cafeteria offers turkey, ham, and chicken salad sandwiches. The bread choices are rye and wheat. How many different sandwiches can be ordered?

SOLUTION

Ordering a sandwich can be considered a trial of an experiment. Each different sandwich is an outcome. Make a tree diagram to show all possible outcomes.



There are 6 different sandwiches that can be ordered.

The **sample space** for an experiment is the set of all possible outcomes. A tree diagram was used in Example 1 to show a sample space. An **event** is any subset of a sample space, so an event is any outcome or set of outcomes. If you need only to count outcomes rather than actually show them, one method you can sometimes use is to apply the **Addition Counting Principle.**

Addition Counting Principle
Suppose a trial can result in any of n_1 outcomes from one category, any of n_2 outcomes from another category, and so on. If there are k different categories of outcomes, then the total number of outcomes that can result is $n_1 + n_2 + \ldots + n_k$.

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Two events in a sample space are **mutually exclusive** if they have no outcomes in common. To find the number of outcomes in an event that consists of mutually exclusive events, use the Addition Counting Principle.

Example 2 Using the Addition Counting Principle

Find the number of outcomes in each event.

a. Draw an ace or a face card by drawing a card at random from a standard deck.

SOLUTION

The events *draw an ace* and *draw a face card* are mutually exclusive. There are 4 aces. There are 12 face cards: 4 jacks, 4 queens, and 4 kings.

$4 + 12 = 16$

There are 16 outcomes in the event *draw an ace or a face card.*

b. Choose a prime number or a multiple of 6 or a multiple of 10 by choosing a number at random from the whole numbers 1 through 20.

SOLUTION

The three events *choose a prime number, choose a multiple of 6*, and *choose a multiple of 10* are mutually exclusive. There are 8 prime numbers: 2, 3, 5, 7, 11, 13, 17, and 19. There are 3 multiples of 6: 6, 12, and 18. There are 2 multiples of 10: 10 and 20.

$8 + 3 + 2 = 13$

There are 13 outcomes in the event *choose a prime number or a multiple of 6 or a multiple of 10.*

Math Language

An event that is the union or intersection of two events is a **compound event.** The event *draw an ace or a face card* is a compound event.

The **Fundamental Counting Principle** uses multiplication instead of addition.

Fundamental Counting Principle
Suppose k items are to be chosen. If there are n_1 ways to choose the first item, n_2 ways to choose the second item, and so on, then there are $n_1 \cdot n_2 \cdot \ldots \cdot n_k$ ways to choose all k items.

Example 3 Using the Fundamental Counting Principle

A student is choosing a three-letter password to use for email.

a. How many passwords are possible if letters may be repeated?

SOLUTION

Because letters may be repeated, there are 26 choices for each letter.

$26 \cdot 26 \cdot 26 = 17{,}576$

There are 17,576 possible passwords.

b. How many passwords are possible if letters may not be repeated?

SOLUTION

Because letters may not be repeated, there are 26 choices for the first letter, 25 choices for the second letter, and 24 choices for the third letter.

$26 \cdot 25 \cdot 24 = 15{,}600$

There are 15,600 possible passwords.

Two events are **independent** if the probability of one event is not affected by whether or not the other event occurs. Two events are **dependent** if the probability of one event is affected by whether or not the other event occurs.

Example 4 Comparing Independent and Dependent Events

The letters A through E are written on 5 index cards. A card is chosen at random and then another card is chosen at random. Determine whether the events in each case are independent or dependent.

a. Choose A and then choose B, if the first card is replaced before the second card is chosen.

SOLUTION

The events are independent because there is a 1 in 5 probability of choosing B second, regardless of what is chosen first.

b. Choose A and then choose B, if the first card is not replaced before the second card is chosen.

SOLUTION

The events are dependent. If A is chosen first, there is a 1 in 4 probability of choosing B second. But if A is not chosen first, then it is possible that B is chosen first, in which case there is a zero probability of choosing B second.

> **Math Reasoning**
>
> **Analyze** How is finding the probability of independent events different than finding the probability of dependent events?

Example 5 Application: Sports

There are 6 positions in volleyball: left front, center front, right front, left back, center back, and right back. How many ways can 6 players be placed in the 6 positions?

	Net	
LF	CF	RF
LB	CB	RB

SOLUTION

Use the Fundamental Counting Principle.

$6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$

There are 720 ways to place 6 players in the 6 positions.

Check The method makes sense because any of the 6 players can be LF. Then any of the 5 remaining players can be CF, and so on: any of 4 can be RF, any of 3 can be LB, either of 2 can be CB, and the last player would be RB.

Lesson Practice

a. *(Ex 1)* A student has four choices for a T-shirt: red, white, blue, and black. He has 2 choices for pants: jeans or khakis. How many different outfits can he make, using one T-shirt and one pair of pants? Make a tree diagram to show all possible outcomes.

b. *(Ex 2)* A card is drawn at random from a standard 52-card deck. How many outcomes are in the event *draw a heart or a club*?

c. *(Ex 2)* A number is chosen at random from the whole numbers 21 through 40. How many outcomes are in the event *choose a perfect square or a multiple of 8 or a prime number*? List all the outcomes.

d. *(Ex 3)* A student is choosing a four-digit password to use for her bank account. How many passwords are possible if digits may be repeated?

e. *(Ex 3)* A student is choosing a four-digit password to use for her bank account. How many passwords are possible if digits may not be repeated?

There are five tiles in a bag, labeled W, O, R, D, and S. A tile is chosen at random and then another tile is chosen at random. Determine whether the following events are independent or dependent.
(Ex 4)

f. Choose W and then choose S, if the first tile is replaced before the second tile is chosen.

g. Choose W and then choose S, if the first tile is not replaced before the second tile is chosen.

h. *(Ex 5)* There are 9 players in a starting baseball lineup. How many ways can batting positions 1, 2, and 3 be filled, choosing from 9 players?

Practice Distributed and Integrated

***1.** *(30)* **Geometry** The function representing the area of a rectangle whose length is 12 units longer than its width is $f(x) = (x + 6)^2 - 36$ where x is the width of the rectangle. Graph this function. For which quadrant or quadrants does the graph make sense? Why?

2. *(10)* Solve and graph the compound inequality $4 < 2(x - 2) \leq 16$.

Use the letters A-B-C-D-E-F-G.

***3.** *(33)* How many seven-letter passwords are possible if letters can be repeated?

***4.** *(33)* How many seven-letter passwords are possible if letters cannot be repeated?

5. *(26)* Find the equation of the line through $(-3, -5)$ and $(4, -5)$.

6. *(Inv 2)* **Airplanes** As an airplane ascends after takeoff, its altitude increases at a rate of 45 ft/s while its distance on the ground from the airport increases at 210 ft/s. Write parametric equations to model the location of the plane.

***7.** Write the matrix equation that represents the system of equations.
(32)

$$p + q + r = 6$$
$$0.4p + 0.5q + 2r = 5.7$$
$$2.2p + 3.4q + r = 12$$

8. **Error Analysis** Two students evaluated $(f - g)(7)$, given $f(x) = 5x + 8$ and $g(x) = 9x + 2$. Who is correct? Explain the error.
(20)

Student A
$(f - g)(7)$ $= 5(7) + 8 - (9(7) + 2)$ $= 35 + 8 - 63 - 2$ $= -22$

Student B
$(f - g)(7)$ $= 5(7) + 8 - (9(7) + 2)$ $= 35 + 8 - 63 + 2$ $= -2$

9. Given the polynomial functions $f(x) = \frac{1}{2}x^2 - 3$, $g(x) = 2x^2$, and $h(x) = 4x$,
(11, 19)

a. write the sum $(f + g + h)(x)$ in standard form.

b. write the product $(f \cdot g \cdot h)(x)$ in standard form.

***10.** **Justify** Is multiplication of rational expressions commutative? Is division of rational expressions commutative? Justify your answers.
(31)

11. **Multiply** Assume all expressions are defined. $\frac{2x^4y^5}{3x^2} \cdot \frac{15x^2}{8x^3y^2}$
(31)

***12.** Using the numbers 2-4-6 and the letters P-A-R-K, how many one-digit, two-letter passwords can be made, if repeats are allowed and the password starts with a digit?
(33)

13. **Multi-Step** $\begin{vmatrix} x & 2 \\ -6 & 4 \end{vmatrix} = 8$
(14)

a. Write the determinant as an equation.

b. Solve for x.

c. Check your answer by substituting for x and finding the determinant with a calculator.

***14.** Find the inverse of $M = \begin{bmatrix} 4 & 8 \\ 2 & 6 \end{bmatrix}$, if it exists.
(32)

Write each equation in standard form.

15. $y = 1.5x + 3$
(26)

16. $y = -0.05x + 20$
(26)

17. **Ticket Prices** A zoo charges \$10.95 for each child's admission and \$14.95 for each adult admission. If a busload of 45 people attend, and together they pay \$516.75, how many adults and how many children are admitted?
(16)

a. Set up equations relating the number of children and adults to the total admissions. Let c represent the number of children and a the number of adults.

b. Use Cramer's Rule to solve the system of equations.

18. **Write** Without solving the system, can you tell how many solutions this system of equations has? Explain. $\begin{array}{l} y + x = 2 \\ y + 1 = -x \end{array}$
(21)

19. *(19)* Simplify: $\dfrac{(xm^{-2})^0x^0m^0}{xx^2m^0(2x)^{-2}}$

***20.** *(33)* **Write** Describe how to determine the number of different arrangements of the letters M, A, T, and H.

21. *(23)* **Multiple Choice** Factor the expression $-12x^4 + 6x^2 + 18x$.

A $-6x(2x^3 - x - 3)$ **B** $-6(2x^4 - x^2 - 3)$

C $-6x(2x^3 + x + 3)$ **D** $-6(2x^4 + x^2 + 3)$

***22.** *(27)* **Graphing Calculator** Use a graphing calculator to determine the y-intercept of the quadratic function $y = x^2 + x - 2$.

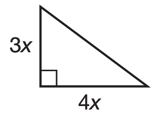
***23.** *(33)* **Sports** There were 12 players on the United States team that competed in the 2006 Women's Basketball (under 19) World Championship tournament. How many ways can 5 starters be chosen from 12 players?

24. *(16)* **Motion** A speed boat leaves a nearby shore of a lake and moves at 25 knots toward the opposite shore, 2 nautical miles away. At the same time, a pleasure boat leaves the opposite shore and moves at 10 knots toward the speed boat. When will the boats meet?

a. Set up a system of equations representing the position of each boat relative to the nearby shore.

b. Use Cramer's Rule to solve the system.

***25.** *(31)* **Measurement** Write a rational expression in simplest form for the ratio of area to perimeter for the right triangle.



***26.** *(Inv 3)* Find the intercepts of the equation $40 + 8y - 2z = -5x$.

27. *(25)* **Analyze** The first quartile of some test scores is 55 and the third quartile is 74.

a. Is it possible for the median of the test scores to be 77? Explain why or why not.

b. The maximum score is 93 and the range of the test scores is 55. What is the minimum test score?

c. What is the range of test scores that will place a student in the middle 50% of the class?

28. *(24)* **Multi-Step** **a.** Write an equation in two variables to represent that the sum of two numbers is 73.

b. Write an equation with the same variables to represent that twice the second number minus the first number is 2.

c. Write and solve a linear system to find the two numbers.

29. *(18)* Convert 61 yd^2 to square inches

***30.** *(32)* What is the inverse of the matrix $\begin{bmatrix} 2 & 2 \\ 4 & 5 \end{bmatrix}$?

LESSON 34

Graphing Linear Equations II

Warm Up

1. (13) **Vocabulary** The expression $\frac{\text{rise}}{\text{run}}$ is the __________ of a line.
2. (13) What is the y-intercept of the line whose equation is $y = 2x + 5$?
3. (7) One point on the line with equation $-x + 3y = 4$ is (___, 2).

New Concepts

A relation is a set of ordered pairs, usually denoted (x, y). A function is a relation in which there is exactly one value of y for every value of x. To determine whether a graph is the graph of a function, you can use the vertical line test.

A function with a constant rate of change is called a **linear function,** and its graph is a line. Every non-vertical line represents a linear function.

Example 1 Identifying Graphs of Linear Functions

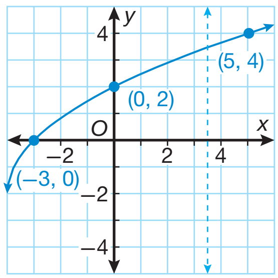
Determine whether each graph represents a linear function.

a. Rate of change from $(-3, 0)$ to $(0, 2)$:

$$\frac{2 - 0}{0 - (-3)} = \frac{2}{3}$$

Rate of change from $(0, 2)$ to $(5, 4)$:

$$\frac{4 - 2}{5 - 0} = \frac{2}{5}$$



SOLUTION No vertical line intersects the graph in more than one point, so it represents a function. However, it is not a linear function because it does not have a constant rate of change.

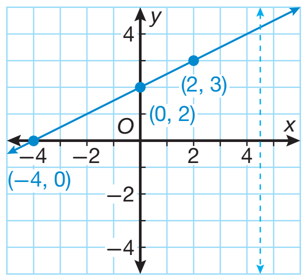
> **Hint**
>
> To be sure that a graph represents a linear function, you either need to know that it is a line, or you need to know its equation.

b. Rate of change from $(-4, 0)$ to $(0, 2)$:

$$\frac{2 - 0}{0 - (-4)} = \frac{2}{4} = \frac{1}{2}$$

Rate of change from $(0, 2)$ to $(2, 3)$:

$$\frac{3 - 2}{2 - 0} = \frac{1}{2}$$



SOLUTION No vertical line intersects the graph in more than one point, so it represents a function. And, it is a linear function because it has a constant rate of change.

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A linear function can be represented by a linear equation. There are three forms of a linear equation that are commonly used.

Forms of a Linear Equation		
Standard Form	$Ax + By = C$	A and B are not both zero.
Slope-Intercept Form	$y = mx + b$	m is the slope and b is the y-intercept.
Point-Slope Form	$y - y_1 = m(x - x_1)$	m is the slope and (x_1, y_1) is a point on the line.

Example 2 Graphing Linear Equations, Given Standard Form

Graph each linear equation.

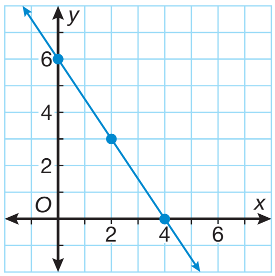
Hint

It is easy to compute with zeros, so it is often helpful to include one or both intercepts when making a table of values.

a. $3x + 2y = 12$

SOLUTION Make a table of ordered pairs, plot the ordered pairs, and draw a line through them.

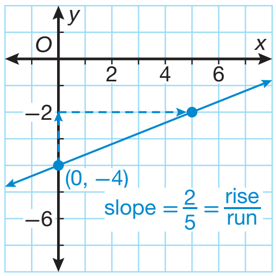
x	0	4	2
y	6	0	3



b. $-2x + 5y = -20$

SOLUTION Solve the equation for y to get slope-intercept form.

$$-2x + 5y = -20$$
$$5y = 2x - 20$$
$$y = \frac{2}{5}x - 4$$



The y-intercept is −4, so the line crosses the y-axis at (0, −4). Plot (0, −4). Then use the slope: from (0, −4), count 2 up and 5 right to plot another point on the line. Draw the line through the points.

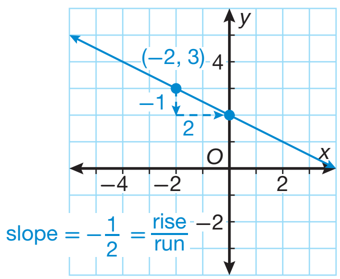
Example 3 Graphing a Linear Equation, Given Point-Slope Form

Hint

There are different ways to count using slope. For example, $-\frac{1}{2} = \frac{2}{-4} = \frac{\text{rise}}{\text{run}}$, so in Example 3, you can count 2 up and 4 left to plot another point on the line.

Graph $y - 3 = -\frac{1}{2}(x + 2)$.

SOLUTION $y - 3 = -\frac{1}{2}(x + 2)$ is in the point-slope form $y - y_1 = m(x - x_1)$. The slope is $m = -\frac{1}{2}$ and a point on the line is $(x_1, y_1) = (-2, 3)$. Plot (−2, 3) and then count 1 down and 2 right to plot another point on the line. Draw the line through the points.



The most basic linear function is $y = x$. It is called the *parent function* of all linear functions. All other linear functions are *transformations* of $y = x$. Reflections, shifts, stretches, and compressions are types of transformations.

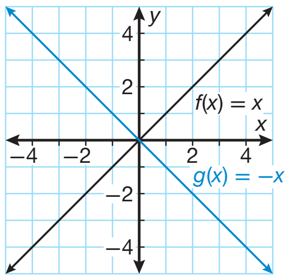
Transformations of $f(x) = x$
$-f(x)$ is a reflection over the x-axis.
$f(x) + c$ is a vertical shift c units up if c is positive.
$f(x) + c$ is a vertical shift c units down if c is negative.
$c \cdot f(x)$ is a vertical stretch by a factor of c if $c > 1$.
$c \cdot f(x)$ is a vertical compression by a factor of c if $0 < c < 1$.

Example 4 Graphing Transformations of $f(x) = x$

Graph each transformation of $f(x) = x$.

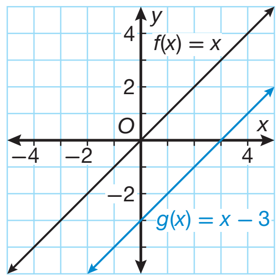
a. $g(x) = -x$

SOLUTION $g(x) = -x = -f(x)$, which is the reflection of $f(x)$ over the x-axis.



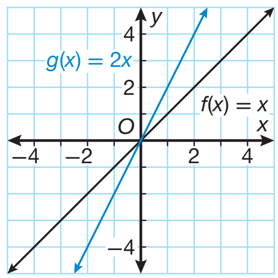
b. $g(x) = x - 3$

SOLUTION $g(x) = x - 3 = f(x) - 3$, which is a vertical shift of $f(x)$ down 3 units.



c. $g(x) = 2x$

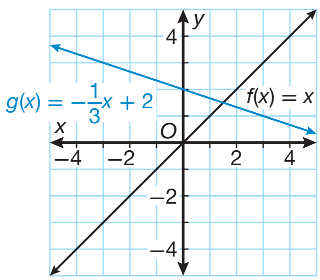
SOLUTION $g(x) = 2x = 2 \cdot f(x)$, which is a vertical stretch of $f(x)$ by a factor of 2.



d. $g(x) = -\frac{1}{3}x + 2$

SOLUTION $g(x) = -\frac{1}{3}x + 2 = -\frac{1}{3}f(x) + 2$, which consists of three transformations of $f(x)$:

- a vertical compression by a factor of $\frac{1}{3}$
- a reflection over the x-axis
- a vertical shift 2 units up



If a line has a slope of zero, then its equation $y = mx + b$ is $y = 0x + b$, or $y = b$.

Graphing Calculator Tip

To enter $g(x) = -\frac{1}{3}x + 2$, use these key strokes: Y=, then (−) 1 ÷ 3 X,T,θ,n + 2, then GRAPH. Adjust the viewing window as needed.

Horizontal and Vertical Lines

Let b be any constant.

$y = b$ is a linear function whose graph is a **horizontal line** with a slope of zero.

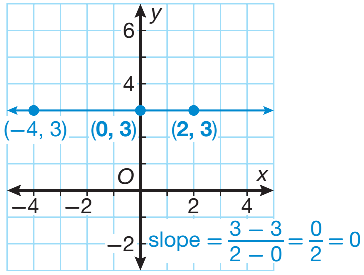
$x = b$ is not a function; its graph is a vertical line with an undefined slope.

Example 5 Graphing Horizontal and Vertical Lines

a. Graph $y = 3$.

SOLUTION Make a table of ordered pairs. Use 3 for y and any number for x.

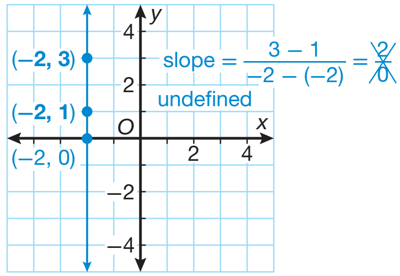
x	−4	0	2
y	3	3	3



b. Graph $x = -2$.

SOLUTION Make a table of ordered pairs. Use −2 for x and any number for y.

x	−2	−2	−2
y	3	1	0

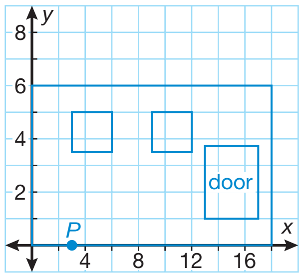


Math Language

An **undefined slope** is also called **no slope.** Remember that slope is a number. A zero in a denominator indicates no slope because there is no number that has zero in the denominator.

Example 6 Application: Wheelchair Accessibility

The Americans with Disabilities Act specifies that the maximum slope for a wheelchair ramp should be 1:12. A builder needs to install a wheelchair ramp from point P to the entrance door shown in the diagram. Graph a line that can be used to represent the ramp. Write an equation of the line in slope-intercept form.



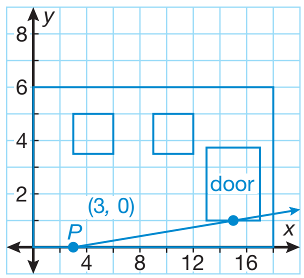
SOLUTION Draw a line through point P with a rise of 1 and a run of 12. Use the given point and given slope to write an equation of the line in point-slope form. Then solve for y to obtain slope-intercept form.

$$y - y_1 = m(x - x_1)$$

$$y - 0 = \frac{1}{12}(x - 3)$$

$$y = \frac{1}{12}(x - 3)$$

$$y = \frac{1}{12}x - \frac{1}{4}$$

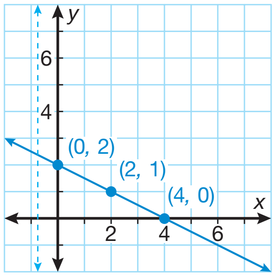


Check The graph is correct because the rise is 1, the run is 12, and it passes through $P(3, 0)$. The equation $y = \frac{1}{12}x - \frac{1}{4}$ is correct because its slope is $\frac{1}{12}$ and the ordered pair (3, 0) is a solution.

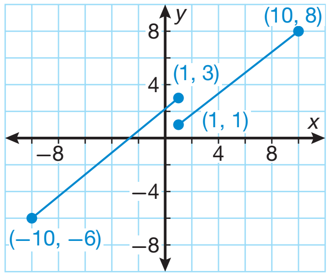
Lesson Practice

Determine whether the graph represents a linear function.
(Ex 1)

a.



b.



c. Graph $2x - 4y = 8$ using a table.
(Ex 2)

d. Graph $2x - 5y = 5$ by solving for y to obtain slope-intercept form.
(Ex 2)

e. Graph $y + 1 = \frac{3}{4}(x - 4)$.
(Ex 3)

Graph by transforming the parent function $f(x) = x$.
(Ex 4)

f. $g(x) = -6x$

g. $g(x) = x + 5$

h. $g(x) = 4x - 7$

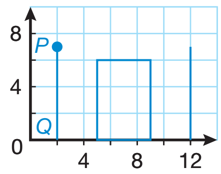
i. $g(x) = -3x + 1$

Graph.
(Ex 5)

j. $x = 4$

k. $y = -5$

l. The diagram shows two walls and a door of a shed. $\overline{PQ}$ represents a wall. Graph a line through point P to represent a roof with a slope of $\frac{3}{5}$. Write an equation of the line in slope-intercept form.
(Ex 6)



Practice Distributed and Integrated

1. *(7)* **Berries** Chuck and Nikita picked 173 quarts of berries. How many did each pick if Chuck picked 11 more quarts than Nikita?

2. *(23)* Factor: $-2ab + abx + abx^2$.

Evaluate.

3. *(2)* $x^2 - y^2(x - y)$ if $x = \frac{1}{2}$ and $y = \frac{1}{3}$.

4. *(2)* $ax - a(a - x)$ if $a = -\frac{1}{2}$ and $x = \frac{1}{4}$.

5. *(21)* Use substitution to solve: $\begin{matrix} x + 2y = 5 \\ 3x - y = 7 \end{matrix}$.

6. *(19)* Expand: $\frac{4x^{-2}y^{-2}}{z^2}\left(\frac{3x^2y^2z^2}{4} + \frac{2x^0y^{-2}}{z^2y^2}\right)$.

Simplify.

7. *(3)* $\frac{(0.0003 \times 10^8)(6000)}{(0.006 \times 10^{15})(2000 \times 10^5)}$

8. *(3)* $\frac{3m}{x} - \frac{2x^{-1}}{m^0m^{-1}} + \frac{5x^2m^2}{x^3m}$

9. Write the equation $y = \frac{2}{3}x + \frac{4}{5}$ in standard form.
(26)

10. Solve for C: $[2 \quad -29 \quad 5] + C = [31 \quad -18 \quad 14]$.
(5)

***11.** **Multiple Choice** Which statement is true about the graph of $y = 3x + 2$?
(34)

A The rate of change between any two points on the graph is 2.

B The rate of change between any two points on the graph is 3.

C The rate of change decreases along the graph from left to right.

D The rate of change increases along the graph from left to right.

***12.** **Analyze** Given the functions $f(x) = \frac{3x - 2}{x + 2}$, $g(x) = \frac{10x^2 + 20x}{3x^2 - 2x}$, and $h(x) = 10$, compare the function product $f(x) \cdot g(x)$ and the constant function $h(x) = 10$. Describe how they are related.
(31)

13. **Population** The table lists the population estimates, in hundreds of thousands, for Vermont from 2000 to 2002.
(13)

Year	2000	2001	2002
Population	6.1	6.13	6.16

a. Write the data above as ordered pairs where x is the number of years after 2000 and y is the population in hundreds of thousands.

b. **Estimate** Use slope and y-intercept to find the equation of the line representing this data.

14. **a.** Multiply the first equation in $\begin{matrix} 0.2x + 0.5y = -1 \\ x - y = 16 \end{matrix}$ by 10. How does this make solving the system easier?
(24)

b. Solve the system.

***15.** **Graphing Calculator** Use a graphing calculator to graph $y = (x + 2)^2$ and $y = x^2$ on the same coordinate axes. How do the graphs compare?
(30)

16. **River Current** A motor boat makes a 175-kilometer trip down the Mississippi River to New Orleans in 3.3 hours. But it takes an hour longer for the boat to return upstream to its starting point. How fast is the river's current?
(15)

***17.** **Probability** A circle has a radius of 4 and center (0, 0). Suppose a point is chosen at random inside the circle. What is the probability that the chosen point is in quadrant I and below the line $y = x$?
(34)

18. **Geometry** You are given the equation of a line on a coordinate grid. You are then given three other equations of lines that combine with the first to form a rectangle. How many of the three lines have an inconsistent solution with the original line? How many of the three lines have a consistent solution with the original line?
(21)

19. **Multi-Step** Find the inverse of matrix A. Then prove your answer is correct. Do not use a calculator. $A = \begin{bmatrix} 3 & 8 \\ 2 & 4 \end{bmatrix}$
(32)

20. (19) **Multiple Choice** Multiply $(x + 7)(x - 9)$
A $x^2 - 16x + 63$ **B** $x^2 - 2x + 63$ **C** $x^2 - 2x - 63$ **D** $x^2 + 2x - 63$

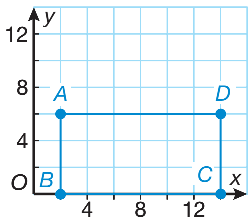
21. (7) **Verify** Is $x = 4$ the solution to the equation $6x - 4 = 20$?

22. (25) **Shipping Boxes** The table shows the measurements in inches of the side lengths of various shipping boxes available from the United States Postal Service.

Box	Measurement 1	Measurement 2	Measurement 3
1	11.875	3.375	13.625
2	11	8.5	5.5
3	7.5	5.125	14.375

Which measurement has the greatest range of side lengths? What is that range?

***23.** (34) **Roofing** The minimum slope (pitch) that is normally recommended for an asphalt shingle roof is $\frac{1}{3}$. Rectangle $ABCD$ represents the front wall of a building. Graph a line through point A to represent a roof with a slope of $\frac{1}{3}$. Write an equation of the line in slope-intercept form.



24. (33) **Passwords** A voicemail system password is 1 letter followed by a 3-digit number less than 600. How many different voicemail passwords are possible?

25. (15) **Analyze** Explain the difficulty you encounter when trying to use Cramer's Rule to solve the system
$$\begin{aligned} 4x - 12y &= -4 \\ -6x + 18y &= 6 \end{aligned}$$

26. (33) **Error Analysis** A student is choosing a four-letter password. She wants to know how many passwords are possible if letters may not be repeated. To find out, she adds: $26 + 25 + 24 + 23 = 98$. What is the error? Find the correct answer.

27. (23) **Analyze** Is $x^2 + 7x + 12$ a basic quadratic polynomial, a perfect square trinomial, or a difference of two squares? Explain how you can tell by looking at the polynomial.

***28.** (34) **Write** Describe how to obtain the graph of $y = \frac{3}{4}x - 3$ by transforming the graph of $y = x$.

29. (15) **Error Analysis** A student solved the linear system $\begin{aligned} x + y &= 3 \\ x - y &= 1 \end{aligned}$ and got 2 as the answer. Explain and correct the student's error.

30. (23) Use $x^2 + 16x + 64$. Rewrite the expression in the form of a perfect square trinomial.

LESSON 35

Solving Quadratic Equations I

Warm Up

1. **Vocabulary** A parabola is a graph of a __________ function. (27)

2. Factor $8x^2 + 4x$ completely. (23)

3. Factor $16x^2 - 20x - 6$ completely. (23)

New Concepts

Every quadratic function can be written in the standard form $f(x) = ax^2 + bx + c$, where $a \neq 0$. The zeros of a function $f(x)$ are the x-values that make $f(x) = 0$. To find the zeros of a quadratic function, solve the related equation $ax^2 + bx + c = 0$. Many quadratic equations can be solved by factoring and applying the Zero Product Property, which says that if a product is zero, then at least one of its factors is zero.

Hint

The Zero Product Property applies to any number of factors. For example, if $abc = 0$, then $a = 0$ or $b = 0$ or $c = 0$.

Zero Product Property
Let a and b be real numbers. If $ab = 0$, then $a = 0$ or $b = 0$.

Example 1 Finding the Zeros of Quadratic Functions

Find the zeros of each quadratic function.

a. $f(x) = 4x^2 - 25$

Hint

The function $f(x) = 4x^2 - 25$ is in standard form $f(x) = ax^2 + bx + c$ with $b = 0$.

SOLUTION

$4x^2 - 25 = 0$ Write the related quadratic equation.

$(2x + 5)(2x - 5) = 0$ Factor.

$2x + 5 = 0$ or $2x - 5 = 0$ Zero Product Property

$2x = -5$ $\quad$ $2x = 5$ Solve each equation.

$x = -\frac{5}{2}$ $\quad$ $x = \frac{5}{2}$

The zeros are $-\frac{5}{2}$ and $\frac{5}{2}$.

b. $f(x) = 2x^2 + x$

SOLUTION

$2x^2 + x = 0$ Write the related quadratic equation.

$x(2x + 1) = 0$ Factor.

$x = 0$ or $2x + 1 = 0$ Zero Product Property

$2x = -1$

$x = -\frac{1}{2}$

The zeros are 0 and $-\frac{1}{2}$.

Online Connection
www.SaxonMathResources.com

The **roots of an equation** are the solutions to the equation.

Example 2 Solving Quadratic Equations

Find the roots of the quadratic equation.

$x^2 = -2x + 15$

SOLUTION

$x^2 = -2x + 15$	Write the given equation.
$x^2 + 2x - 15 = 0$	Add $2x - 15$ to both sides to get zero on one side.
$(x + 5)(x - 3) = 0$	Factor.
$x + 5 = 0 \quad \text{or} \quad x - 3 = 0$	Zero Product Property
$x = -5 \qquad\qquad x = 3$	Solve each equation.

The roots are -5 and 3.

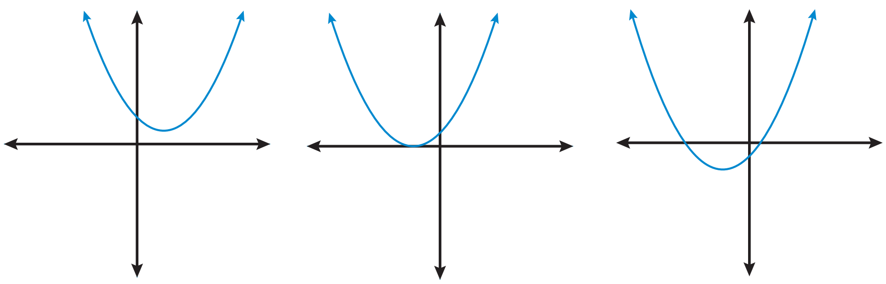
Math Language

Functions have zeros or x-intercepts.

Equations have solutions or roots.

The graph of a quadratic function shows how many zeros the function has.

Zeros of a Quadratic Function
A quadratic function has 0, 1, or 2 real zeros.



No Real Zeros — One Real Zero — Two Real Zeros

When a quadratic function has exactly one real zero, that zero is a **double root** of the related equation.

Math Reasoning

Generalize When a quadratic function has exactly one real zero, at how many points does the graph intersect the x-axis?

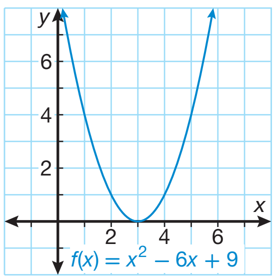
Example 3 Comparing Roots, Zeros, and x-Intercepts

Find the roots of each equation. Graph the related function and describe the relationship between the roots, the zeros, and the x-intercepts.

a. $x^2 - 6x + 9 = 0$

SOLUTION

$$x^2 - 6x + 9 = 0$$
$$(x - 3)(x - 3) = 0$$
$$x = 3$$



3 is the double root of the equation, the only zero of the related function $f(x) = x^2 - 6x + 9$, and the only x-intercept of the graph.

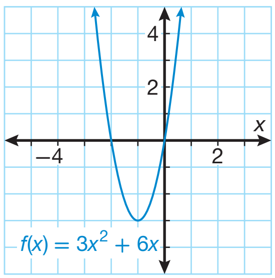
Graphing Calculator Tip

Verify your graph with a graphing calculator. To enter $f(x) = 3x^2 + 6x$, use these key strokes:

3 X,T,θ,n x^2 + 6 X,T,θ,n

Adjust the viewing window as needed.

b. $3x^2 + 6x = 0$



SOLUTION

$$3x^2 + 6x = 0$$

$$3x(x + 2) = 0$$

$$3x = 0 \quad \text{or} \quad x + 2 = 0$$

$$x = 0 \quad \text{or} \quad x = -2$$

0 and -2 are the roots of the equation, the zeros of the related function $f(x) = 3x^2 + 6x$, and the x-intercepts of the graph.

Example 4 Writing a Quadratic Function, Given its Zeros

Write a quadratic function that has zeros $-\frac{5}{3}$ and 1.

SOLUTION Reverse the process of solving a quadratic equation.

$\left(x + \frac{5}{3}\right)(x - 1) = 0$ Write the related equation in factored form.

$3\left(x + \frac{5}{3}\right)(x - 1) = 3(0)$ Multiply both sides by 3 to eliminate the fraction.

$(3x + 5)(x - 1) = 0$ Distribute the 3.

$3x^2 + 2x - 5 = 0$ Multiply the binomials.

The quadratic function $f(x) = 3x^2 + 2x - 5$ has zeros $-\frac{5}{3}$ and 1.

Hint

Velocity is a vector quantity, which means it has both magnitude and direction. If an object is thrown straight down at a speed of 20 feet per second, its initial velocity is -20 feet per second.

Example 5 Application: Vertical Motion

The height of a free-falling object is given by the function $h(t) = -16t^2 + v_0t + h_0$, where h_0 is the initial height in feet, v_0 is the initial velocity in feet per second, and $h(t)$ is the height in feet at time t seconds. How long does it take for an object to hit the ground after it is thrown straight down at a speed of 20 feet per second from a height of 84 feet?

SOLUTION

$h(t) = -16t^2 + v_0t + h_0$

$0 = -16t^2 - 20t + 84$ Substitute given values for v_0 and h_0.

$0 = -4(4t^2 + 5t - 21)$ Factor out -4.

$0 = -4(4t - 7)(t + 3)$ Factor the trinomial.

$4t - 7 = 0 \quad \text{or} \quad t + 3 = 0$ Use the Zero Product Property.

$t = 1.75 \quad \text{or} \quad t = -3$ Ignore $t = -3$. Time cannot be negative.

It takes the object 1.75 seconds to hit the ground.

Lesson Practice

Find the zeros of each quadratic function.
(Ex 1)

a. $f(x) = 25x^2 - 1.$ **b.** $f(x) = x^2 - 7x.$

c. Find the roots of $3x^2 - x = 2$.

Find the roots of each equation. Graph the related function and describe the relationship between the roots, the zeros, and the *x*-intercepts.
(Ex 3)

d. $x^2 + 8x + 16 = 0$ **e.** $f(x) = x^2 + 2x - 8$

(Ex 4) **f.** Write a quadratic function that has zeros -5 and $\frac{3}{4}$.

(Ex 5) **g.** The height of a free-falling object is given by the function $h(t) = -16t^2 + v_0t + h_0$, where h_0 is the initial height in feet, v_0 is the initial velocity in feet per second, and $h(t)$ is the height in feet at time t seconds. How long does it take for an object to hit the ground after it is thrown straight *up* at a speed of 8 feet per second from a height of 80 feet?

Practice Distributed and Integrated

Simplify.

1. *(3)* $\dfrac{(b^2c^{-2})^{-3}c^{-3}}{(b^2c^0b^{-2})^4}$

2. *(3)* $\dfrac{(2x^2ya)^{-3}ya^3}{x^2y(ay)^{-2}y}$

Solve.

3. *(7)* $3(-2x - 3) - 2^2 = -(-3x - 5) - 2$

4. *(23)* $x^2 + 26x - 56 = 0$

5. *(26)* Find the equation of the line that passes through $(-3, 4)$ and $(3, -2)$.

Use elimination to solve.

6. *(24)* $\begin{aligned} 4x - 3y &= -1 \\ 2x + 5y &= 19 \end{aligned}$

7. *(24)* $\begin{aligned} 5x + \tfrac{1}{2}y &= -4 \\ -x + y &= 3 \end{aligned}$

8. *(5)* Solve for a and b: $\begin{bmatrix} a \\ 64 \end{bmatrix} = \begin{bmatrix} 102 \\ a - b \end{bmatrix}$.

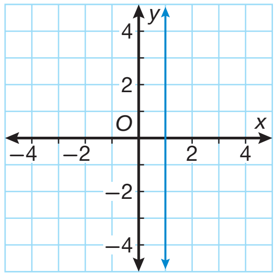
9. *(20)* Find $(f \cdot g)(x)$, given $f(x) = 4x - 3$ and $g(x) = 8x + 9$.

10. *(22)* **Formulate** Describe a plan for determining if a function is continuous.

***11.** *(35)* **Multiple Choice** What are the zeros of the function $f(x) = x^2 - 5x + 6$?
A 0 and 6 **B** 0 and -6 **C** 2 and 3 **D** 2 and -3

12. *(32)* **Probability** Suppose you roll a pair of number cubes twice. What is the probability of rolling a sum greater than 10 on both rolls?

*13. **Error Analysis** A student described this line as having no slope, or zero slope. What is wrong with this description?
(34)



14. If a polynomial of degree 3 is added to a polynomial of degree 4, what are all possible values for the degree of the result?
(11)

15. **Analyze** The product $\frac{x^2-4}{x^2-3x-4} \cdot \frac{x-4}{x+2}$ in simplest form is $\frac{x-2}{x+1}$, as shown:
(31)

$\frac{x^2-4}{x^2-3x-4} \cdot \frac{x-4}{x+2} = \frac{\cancel{(x+2)}(x-2)}{(x+1)\cancel{(x-4)}} \cdot \frac{\cancel{(x-4)}}{\cancel{(x+2)}} = \frac{x-2}{x+1}$. When 4 is substituted for x in $\frac{x-2}{x+1}$, the value is $\frac{2}{5}$. Explain why the value of $\frac{x^2-4}{x^2-3x-4} \cdot \frac{x-4}{x+2}$ is not $\frac{2}{5}$ when $x = 4$.

16. **Write** A square is drawn on a coordinate grid with one side along the x-axis and one vertex at $(a - 1, 0)$. The area of the square is $a^2 - 8a + 16$. Determine an expression for an adjacent vertex. Explain.
(23)

*17. **Justify** Explain why this paragraph describes a method of graphing the equation: $y + 1 = \frac{1}{4}(x - 2)$: Plot the ordered pair $(2, -1)$. Then from that point, count 1 unit up and 4 units to the right to plot another point. Then draw a line through the two plotted points.
(34)

18. Solve and graph the compound inequality: $4 < 2(x - 2) \leq 16$.
(10)

19. **Average Velocity** If an object is thrown straight down with initial velocity 8 feet per second, the number of feet it travels in t seconds is given by the expression $16t^2 + 8t$ (neglecting air resistance).
(31)

a. Write a rational expression for the average velocity of the object before it hits the ground over the time interval from 0 seconds to t seconds. Then simplify the expression. (Average velocity equals distance traveled divided by elapsed time.)

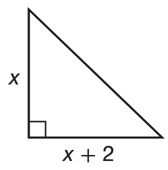
b. Evaluate the expression to find the average velocity of the object over the time interval from 0 seconds to 2.5 seconds.

20. **Chemistry** How many liters of a 15% acid solution should be mixed with a 35% acid solution to make 7 liters of a 20% acid solution?
(24)

*21. **Write** State all possible numbers of real zeros a quadratic function can have. Explain what information the number of real zeros provides about the point(s) of intersection of the graph and the x-axis.
(35)

22. Is the equation $y = \sqrt{2}x$ a direct or inverse variation?
(12)

*23. **Geometry** The area of the right triangle is 24 square units. Find the value of x.
(35)



*24. **Currency Exchange** During 2006, the average exchange rate for euros and U.S. dollars was $1.2563 per euro. This relationship is given by the function $y = 1.2563x$, where y represents dollars and x represents euros. Describe how to transform the graph of the parent function $y = x$ to obtain the graph of $y = 1.2563x$.
(34)

25. (22) **Multiple Choice** An example of a set of numbers that can be listed is:

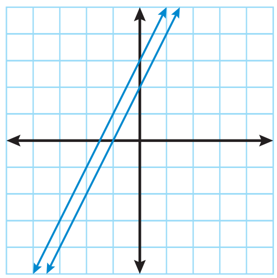
A real numbers **B** integers

C rational numbers **D** irrational numbers

***26.** (26) **Graphing Calculator** When you're traveling in a car that comes to an abrupt halt, the car continues forward even though the brakes have been pressed. The distance traveled is called the "stopping distance." The table below shows the stopping distance at three different speeds for a car traveling on wet pavement. Use your graphing calculator and the linear regression capabilities to write an equation in slope-intercept form for the stopping distance.

Initial Traveling Speed (mile/hour)	Stopping Distance (feet)
35	53.389
49	114.159
62	202.678

27. (21) **Coordinate Geometry** Which set of equations could be represented by this graph?



A $y = 2x + 4$
$2y - 2x = 8$

B $y = 2x + 4$
$12 - 3y = -6x$

C $2y = 4x - 12$
$y = 2x + 6$

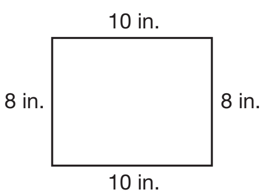
D $-4x = 8 - 2y$
$y = 2x + 6$

28. (32) **Nutrition** A spaghetti entrée consists of spaghetti, ground beef, and tomato sauce. The entrée contains 118.68 grams of carbohydrates, 40.12 grams of fat, and 41.2 grams of protein. How much of each ingredient is in the entrée? To find the answer, write a system of equations and use an inverse matrix to solve it. Use the table below.

	Spaghetti	Ground Beef	Tomato Sauce
Carbohydrates	81%	0%	87%
Fat	5%	60%	3%
Protein	14%	40%	10%

***29.** (35) **Flags** There is no required size for a United States flag, but the ratio of width to length that is recommended in most cases is 1 : 1.9. Write and solve a quadratic equation to find the length and width of a United States flag that has an area of 17.1 square feet.

30. (18) What is the perimeter of this rectangle in feet?



LESSON 36

Using Parallel and Perpendicular Lines

Warm Up

1. (13) **Vocabulary** If you know the slope of a line and the coordinates of a point on the line, you can use the ________ formula to write the equation of the line.

2. (13) Find the slope of the line that crosses (1, 4) and (5, 9).

3. (13) Write the equation $3x - 7y = 28$ in slope-intercept form.

New Concepts

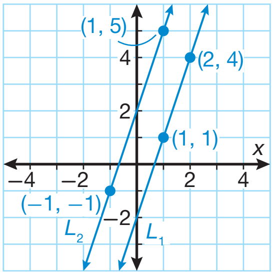
From geometry you know that parallel lines do not intersect and that perpendicular lines meet at right angles. When parallel and perpendicular lines are graphed on an coordinate plane, the equations of these lines have special properties.

Parallel lines have identical slopes and different y-intercepts. Perpendicular lines have slopes that are reciprocals of each other and have opposite signs. The product of the slopes of two perpendicular lines is -1.

> **Hint**
>
> For parallel lines: $m_1 = m_2$.
> For perpendicular lines: $m_1m_2 = -1$

Example 1 Graphs of Parallel and Perpendicular Lines

a. Show that L1 and L2 are parallel.



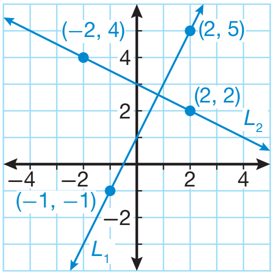
SOLUTION

Use the slope formula to find the slope of each line.

L_1	L_2
$m_1 = \frac{y_2 - y_1}{x_2 - x_1}$	$m_2 = \frac{y_2 - y_1}{x_2 - x_1}$
$m_1 = \frac{4 - 1}{2 - 1}$	$m_2 = \frac{5 - (-1)}{1 - (-1)}$
$m_1 = 3$	$m_2 = 3$

Since the slopes are identical and the lines do not have the same y-intercepts, the lines are parallel.

b. Show that L_1 and L_2 are perpendicular.



SOLUTION

Use the slope formula to find the slope of each line.

L_1	L_2
$m_1 = \frac{y_2 - y_1}{x_2 - x_1}$	$m_2 = \frac{y_2 - y_1}{x_2 - x_1}$
$m_1 = \frac{5 - (-1)}{2 - (-1)}$	$m_2 = \frac{4 - 2}{2 - 2}$
$m_1 = 2$	$m_2 = -0.5$

Since $2(-0.5) = -1$, the lines are perpendicular.

> **Math Reasoning**
>
> **Generalize** For a linear equation in standard form, $Ax + By = C$, what is the slope-intercept form in terms of the coefficients A, B, and C?

Example 2 Writing Equations of Parallel and Perpendicular Lines

a. Find the equation of the line parallel to $y = 3x + 4$ that crosses (2, 3).

SOLUTION

Step 1:

The equation of the line parallel to $y = 3x + 4$ has the same slope: $m = 3$

Step 2:

Use the point-slope formula to find the equation of the line.

$$m = \frac{y - y_1}{x - x_1}$$

$$3 = \frac{y - 3}{x - 2}$$

Substitute the coordinate point (2, 3) for (x_1, y_1) and 3 for the slope m.

$$3(x - 2) = y - 3$$

$$y = 3x - 6 + 3$$

Solve for y by multiplying each side by $(x - 2)$, then adding 3 to both sides.

$$y = 3x - 3$$

b. Find the equation of the line that passes through the point (−1, 3) and is perpendicular to the line $y = -0.5x - 1$.

SOLUTION

Step 1:

Find the slope that is the opposite reciprocal of −0.5.

$$m = -\left(-\frac{1}{0.5}\right) = 2$$

Step 2:

Use the point-slope formula to find the equation of the line.

$$m = \frac{y - y_1}{x - x_1}$$

$$2 = \frac{y - 3}{x - (-1)}$$

Substitute the coordinate point (−1, 3) for (x_1, y_1) and 2 for the slope m.

$$2\big(x - (-1)\big) = y - 3$$

$$y = 2x + 2 + 3$$

Solve for y by multiplying each side by $\big(x - (-1)\big)$, then adding 3 to both sides.

$$y = 2x + 5$$

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Example 3 Determining if Two Lines Are Perpendicular or Parallel

a. Given two equations, determine if they are perpendicular or parallel.

$$5x + 3y = 27 \qquad 15x + 9y = -72$$

SOLUTION

Write each equation in slope-intercept form.

$$\begin{aligned} 5x + 3y &= 27 \\ 3y &= -5x + 27 \\ y &= -\frac{5}{3}x + 9 \end{aligned} \qquad \begin{aligned} 15x + 9y &= -72 \\ 9y &= -15x - 72 \\ y &= -\frac{5}{3}x - 9 \end{aligned}$$

Because the slopes are equal, the lines are parallel.

b. The points A, B, and C are collinear. The points D, E, and F are also collinear. Compare the equations of the lines through those sets of points.

	A	*B*	*C*	*D*	*E*	*F*
x	1	3	4	2	4	6
y	0.5	5.5	8	0.7	−0.1	−0.9

Math Reasoning

Formulate Use the point-slope formula to find the equations of the two lines in Example 3. Use the coordinates for *C* and *F*.

SOLUTION

Step 1:

Use the slope formula with points A and B and D and E.

$$m_1 = \frac{0.5 - 5.5}{1 - 3} \qquad m_2 = \frac{0.7 - (-0.1)}{2 - 4}$$

$$m_1 = \frac{-5}{-2} \qquad m_2 = \frac{0.8}{-2}$$

$$m_1 = \frac{5}{2} \qquad m_2 = -0.4$$

Step 2:

To test if the lines are perpendicular, multiply the two slopes.

$$\begin{aligned} m_1m_2 &= \left(\frac{5}{2}\right)(-0.4) \\ &= -\frac{2}{2} \\ &= -1 \end{aligned}$$

Because the product of the slopes is −1, the lines are perpendicular.

Example 4 Finding the Equations of Horizontal and Vertical Lines

a. Find the equation of the line perpendicular to the graph of $y = 2$ that passes through the point $(4, -1)$.

SOLUTION

The graph of $y = 2$ is a horizontal line. The equation of a line perpendicular to $y = 2$ is a vertical line of the form $x = c$ for some constant c.

Use the x-coordinate of the point $(4, -1)$ to find the equation.

The equation $x = 4$ is perpendicular to the graph of $y = 2$ and passes through the point $(4, -1)$.

b. Find the equation of the line perpendicular to the graph of $x = 15$ that passes through the point $(7, -1)$.

SOLUTION

The graph of $x = 15$ is a vertical line. The equation of a line perpendicular to $x = 15$ is a horizontal line of the form $y = c$ for some constant c.

Use the y-coordinate of the point $(7, -1)$ to find the equation.

The equation $y = -1$ is perpendicular to the graph of $x = 15$ and passes through the point $(7, -1)$.

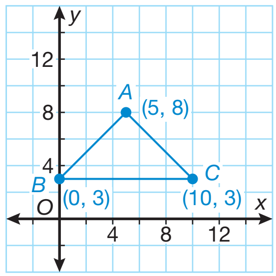
Reading Math

$L_1 \parallel L_2$ is read "L_1 is parallel to L_2".

$L_1 \perp L_2$ is read "L_1 is perpendicular to L_2".

Example 5 Application: Right Triangles

Geometry Use what you have learned about the properties of perpendicular lines to prove that $\triangle ABC$ is a right triangle.



SOLUTION

If $\triangle ABC$ is a right triangle, then $\angle BAC$ is a right angle and $\overline{AB} \perp \overline{AC}$. Use the slope formula with points A and B and with A and C to see if $\overline{AB}$ and $\overline{AC}$ are perpendicular.

$$m\overline{AB} = \frac{8 - 3}{5 - 0} \qquad m\overline{AC} = \frac{8 - 3}{5 - 10}$$

$$m\overline{AB} = \frac{5}{5} \qquad m\overline{AC} = \frac{5}{-5}$$

$$m\overline{AB} = 1 \qquad m\overline{AC} = -1$$

Since the product of the slopes is -1, then $\angle BAC$ is a right angle. $\triangle ABC$ is a right triangle.

Lesson Practice

a. (Ex 1) The points (1, −4) and (2, −1) are collinear. The points (3, 19) and (5, 25) are also collinear. What can you say about the lines that cross these points?

b. (Ex 1) The points (1, 16.88) and (3, 16.63) are collinear. The points (2, 24) and (4, 40) are also collinear. What can you say about the lines that cross these points?

c. (Ex 2) Write the equation of the line that is parallel to the graph of $y = -13x + 10$ and crosses the point (0, 15).

d. (Ex 2) Write the equation of the line that is perpendicular to the graph of $y = 25x + 99$ and crosses the point (1, 37).

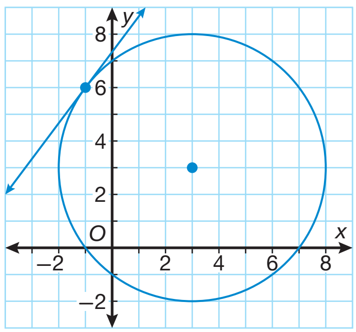
e. (Ex 3) What is the relationship between the graphs of $y = -27x + 15$ and $y = -27x + 5$.

f. (Ex 3) What is the relationship between the graphs of $y = -0.125x + 10$ and $y = 8x + 1$.

g. (Ex 4) Find the equation of the line perpendicular to the graph of $y = 5$ that passes through the point (21, 1).

h. (Ex 4) Find the equation of the line perpendicular to the graph of $x = 7$ that passes through the point (−3, 5).

i. (Ex 5) The graph of $y = \frac{4}{3}x + \frac{22}{3}$ is tangent to the circle at (−1, 6). The center of the circle is at (3, 3). Use what you know about perpendicular lines to show that the tangent is perpendicular to the diameter at that point.



Practice Distributed and Integrated

Use the slope formula to find the slope of each line.

***1.** (26) (−3, 7)(1, 0)

***2.** (26) (2, −5)(0, −6)

Determine the kind of variation, if any, for each equation.

3. (12) $y = mx + b$, for $m, b \neq 0$

4. (12) $y = mx + b$, for $m \neq 0$ and any value of b

Simplify. Write answers with all exponential expressions in the numerator.

5. (3) $\dfrac{xx^2(x^0y^{-1})^2}{x^2x^{-5}(y^2)^5}$

6. (3) $\dfrac{m^2p^0(m^{-2}p)^2}{m^{-2}p^{-1}(m^{-3}p^2)^3}$

Solve.

7. (27) $x^2 + 2 = 4x + 7$

8. (24) $\begin{aligned} 12x - 5y &= 24 \\ 20y - 4x &= 3 \end{aligned}$

9. Write the equation in standard form: $y = \frac{1}{11}x + \frac{1}{7}$.
(26)

10. **Analyze** Explain how to create a rational expression with four excluded values. Then give an example.
(28)

11. **Error Analysis** Carmen factored the polynomial $x^2 + 5x - 6$ into $(x - 5)(x - 1)$. Describe her error.
(23)

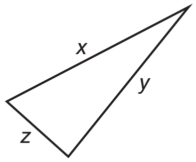
12. **Verify** that the graph $y = 3x - 6$ is a continuous function.
(22)

***13.** **Graphing Calculator** Calculate the value of x in the matrix below. Substitute the value of x and use a graphing calculator to verify your answer. $\begin{vmatrix} -4 & 3 \\ 5 & x \end{vmatrix} = -x$
(14)

14. **Multiply** Assume that all expressions are defined. $\frac{x}{15} \cdot \frac{x^7}{2x} \cdot \frac{20}{x^4}$
(31)

***15.** **Olympic Swimming Pool Size** An Olympic swimming pool has an area of 1250 square meters, and its length is 25 meters more than its width. Write and solve a quadratic equation to find the length and width of an Olympic swimming pool.
(35)

16. **Geometry** In the triangle, x is 5 greater than y, and y is 21 less than 3 times z, and the perimeter is 180. What is the value of each variable? To find the answer, write a system of equations and use an inverse matrix to solve it.
(32)



17. **Data Analysis** The table represents the number of ice creams sold at an ice cream shop for the months of January (01), May (05), and October (10).
(29)

Month	Sales
01	124
05	4,476
10	3,796

Let x be the month and y be the sales. Use the data in the table to create a system of three equations to find a quadratic equation of the form $y = ax^2 + bx + c$ that models the data.

***18.** **Cab Fares** The normal fares for taxicab rides in New York City in the year 2006 satisfied a linear function. The table shows the fares for rides of several different distances.
(34)

Number of Miles (x)	0.2	0.4	1	5
Fare in Dollars (y)	2.90	3.30	4.50	12.50

Graph the function, state the rate of change, and describe what it means.

19. A helicopter takes off with a horizontal speed of 5 ft/s and a vertical speed of 20 ft/s. Write parametric equations to show the motion of the helicopter.
(Inv 2)

*20. **Error Analysis** Two students are finding the equation of the line parallel to $y = 5x - 3$ that crosses $(1, 4)$ but get different results. Which student made the mistake? Explain.
(36)

Student A	Student B
$5 = \frac{y - 4}{x - 1}$	$5 = \frac{y - 1}{x - 4}$
$5x - 5 = y - 4$	$5x - 20 = y - 1$
$y = 5x - 1$	$y = 5x - 19$

21. **Multiple Choice** Which data set gives a median of 8 and a mode of 4?
(25)
A {4, 5, 6, 8, 8, 9, 12} **B** {4, 4, 8, 9, 10}
C {2, 3, 4, 8, 8, 9} **D** {2, 3, 4, 4, 8, 8}

*22. **Write** Is the triangle defined by vertices $(-4, 6)$, $(4, 4)$, and $(1, -8)$ a right triangle? How do you know?
(36)

23. **Small Business** The profits, for x weeks, in one year for a small business can be modeled by the function $y = 0.45(x - 22)^2 - 840.50$. Does the graph of the function have a maximum or a minimum? What is that value? During what week did it occur?
(30)

*24. Find the roots of $x^2 + 7x + 6 = 0$.
(35)

25. **Justify** In $f(x) = ax^2 + bx + c$, why is a the only constant that is not allowed to be zero?
(27)

*26. **Multiple Choice** There are 8 runners in a race. How many ways can runners finish in first, second, and third place?
(33)
A 21 **B** 24 **C** 336 **D** 512

27. **Multi-Step** The matrix $\begin{bmatrix} a & c \\ b & d \end{bmatrix}$ represents a parallelogram, with vertices at $(0, 0)$, (a, b), (c, d), and $(a + c, b + d)$.
(14)
a. Graph this parallelogram in the first quadrant, putting (a, b) clockwise from (c, d).
b. To find the area of this parallelogram, draw a rectangle around it and subtract off the areas of triangles and rectangles.
c. Show that the area of the parallelogram equals the determinant of the matrix.

*28. **Women's Basketball** There are 12 teams in the Atlantic Coast Conference of college women's basketball. How many ways can teams finish in first, second, and third place?
(33)

29. **Write** Describe how symmetry can be used to graph $y = (x - 2)^2 + 2$.
(30)

30. **Multiple Choice** What is the value of $\frac{x^2 + x}{x^2 - 9} \cdot \frac{3x - x^2}{3x + 3}$ if $x = 6$?
(31)
A $-\frac{4}{3}$ **B** $-\frac{4}{9}$ **C** $\frac{4}{9}$ **D** $\frac{4}{3}$

LESSON 37

Adding and Subtracting Rational Expressions

Warm Up

1. (1) **Vocabulary** In the expression $\frac{a}{b}$, if a and b are integers and $b \neq 0$, then $\frac{a}{b}$ is a ___________ number.

2. (1) Add the fractions. $\frac{2}{7} + \frac{4}{5}$

3. (23) Factor the expression. $12x^2 - 28x + 15$

New Concepts

Both rational numbers and rational expressions are expressed as ratios.

Rational Number	Rational Expression
$\frac{a}{b}$, for integers a and b, $b \neq 0$	$\frac{f(x)}{g(x)}$, for polynomials $f(x)$ and $g(x)$, $g(x) \neq 0$, $g(x)$ of degree ≥ 1

Recall that a rational expression is undefined for values of the variable that cause the denominator to equal zero. In this lesson assume that all rational expressions are defined unless otherwise indicated.

Math Reasoning

Write If $f(x)$ is a polynomial of degree 1, and not equal to zero, explain why $\frac{1}{f(x)}$ is a rational expression.

Example 1 Combining Expressions with Like Denominators

a. Simplify the following expression.

$$\frac{1}{2x} + \frac{3}{2x}$$

SOLUTION

When two rational expressions have the same denominator, add the numerators.

$$\frac{1}{2x} + \frac{3}{2x} = \frac{4}{2x} = \frac{2}{x}$$

b. Simplify the following expression.

$$\frac{x}{x^2 - 2x + 1} - \frac{1}{x^2 - 2x + 1}$$

SOLUTION

When two rational expressions have the same denominator, subtract the numerators.

$$\frac{x}{x^2 - 2x + 1} - \frac{1}{x^2 - 2x + 1} = \frac{x - 1}{x^2 - 2x + 1}$$

Note that the denominator can be factored. $\frac{x-1}{x^2 - 2x + 1} = \frac{x-1}{(x-1)^2}$

Both the numerator and the denominator have an $(x - 1)$ term in common, which can be further simplified. $\frac{x-1}{(x-1)^2} = \frac{1}{x-1}$

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c. Simplify the following expression.

$$\frac{2x}{x^2 - 4x + 3} - \frac{x + 2}{x^2 - 4x + 3} + \frac{-1}{x^2 - 4x + 3}$$

SOLUTION

When two or more rational expressions have the same denominator, add or subtract the numerators. With binomial expressions in the numerator, distribute the sign to both terms.

$$\frac{2x}{x^2 - 4x + 3} - \frac{x + 2}{x^2 - 4x + 3} + \frac{-1}{x^2 - 4x + 3} = \frac{2x - x - 2 - 1}{x^2 - 4x + 3}$$

$$= \frac{x - 3}{x^2 - 4x + 3}$$

Note that the denominator can be factored. $\frac{x - 3}{x^2 - 4x + 3} = \frac{x - 3}{(x - 1)(x - 3)}$

Both the numerator and the denominator have an $(x - 3)$ term in common, which can be further simplified. $\frac{x - 3}{(x - 1)(x - 3)} = \frac{1}{x - 1}$

Example 2 Finding the LCD of Two Rational Expressions

a. Find the least common denominator of the two rational expressions.

$$\frac{1}{2x} \qquad \frac{1}{7x}$$

Math Language

The **least common denominator** is the least common multiple of the denominators of the fractions.

SOLUTION

Finding the LCD of the two rational expressions, in this case, means finding the LCM of $2x$ and $7x$.

$2x \quad 4x \quad 6x \quad 8x \quad 10x \quad 12x \quad 14x$

$7x \quad 14x$

The LCD is $14x$, resulting in the rational expressions $\frac{7}{14x}$ and $\frac{2}{14x}$.

b. Find the least common denominator of the two rational expressions.

$$\frac{1}{x^2 - 4} \qquad \frac{1}{x^2 + 5x + 6}$$

SOLUTION

With more complicated rational expressions, follow these steps.

Step 1: Write each term in factored form.

$$\frac{1}{(x - 2)(x + 2)} \qquad \frac{1}{(x + 2)(x + 3)}$$

Step 2: Multiply by the *least* number of factors they do not have in common:

$$\frac{(x + 3)}{(x - 2)(x + 2)(x + 3)} \qquad \frac{(x - 2)}{(x - 2)(x + 2)(x + 3)}$$

Example 3 Undefined Values of a Rational Expression

Find any values of x for which the following expression is undefined:

$$\frac{1}{2x^2 + 4x - 6}$$

SOLUTION

A rational expression $\frac{f(x)}{g(x)}$ is undefined when $g(x) = 0$. Follow these steps to find these undefined values:

Step 1: Write each term in factored form.

$$\frac{1}{(2x - 2)(x + 3)}$$

Step 2: Determine the values where each factor is zero.

$2x - 2 = 0$ $\quad\quad$ $x + 3 = 0$

$x = 1$ $\quad\quad$ $x = -3$

So, the rational expression is undefined for $x = 1$ and $x = 3$.

Math Reasoning

Analyze Suppose that $\frac{1}{ax^2 + bx + c}$ has no restrictions on x. What can you can conclude about this expression?

Example 4 Combining Expressions with Unlike Denominators

a. Add the two rational expressions.

$$\frac{1}{3x^3} + \frac{1}{6x^2}$$

SOLUTION

Find the least common denominator by finding the LCM.

$$\frac{1 \cdot 2}{3x^3 \cdot 2} + \frac{1 \cdot x}{6x^2 \cdot x}$$

$$= \frac{2 + x}{6x^3}$$

b. Subtract the two rational expressions.

$$\frac{1}{x^2 - 1} - \frac{1}{x^2 + 3x + 2}$$

SOLUTION

Factor each denominator to find the LCM.

$$\frac{1}{(x - 1)(x + 1)} - \frac{1}{(x + 1)(x + 2)}$$

$$= \frac{(x + 2)}{(x - 1)(x + 1)(x + 2)} - \frac{(x - 1)}{(x + 1)(x + 2)(x - 1)}$$

$$= \frac{3}{(x - 1)(x + 1)(x + 2)}$$

Example 5 Application: Travel Time

On a roundtrip by car, it took 2 hours longer on the return trip than it did on the first leg of the trip. How much faster did the car go on the first part of the trip?

SOLUTION

Math Language

The **equation for speed** is a variation of the equation for distance: $d = rt$, where r is the rate, t is the time, and d is the distance. Solving the equation for rate gives $r = d \div t$.

Use the equation for speed:

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

First part of the trip	Return trip
$s_1 = \frac{d}{t}$	$s_2 = \frac{d}{t+2}$

$$s_1 - s_2 = \frac{d}{t} - \frac{d}{t+2}$$

$$= \frac{d(t+2)}{t(t+2)} - \frac{dt}{t(t+2)} = \frac{2d}{t(t+2)}$$

Lesson Practice

a. (Ex 1) Simplify the expression. $\frac{7}{5x^3} + \frac{3}{5x^3}$

b. (Ex 1) Simplify the expression. $\frac{x}{x^2-1} - \frac{1}{x^2-1}$

c. (Ex 1) Simplify. $\frac{4x}{3x^2+19x-14} + \frac{1}{3x^2+19x-14} - \frac{x+3}{3x^2+19x-14}$

d. (Ex 2) Find the LCD: $\frac{1}{6x^3}$ $\frac{1}{7x^2}$

e. (Ex 2) Find the LCD: $\frac{1}{7x^2+32x-15}$ $\frac{1}{2x^2+3x-35}$

f. (Ex 3) For what values of x is the denominator zero? $\frac{1}{7x^2-66x+27}$

g. (Ex 4) Add the two rational expressions. $\frac{1}{5x^4} + \frac{1}{7x^5}$

h. (Ex 4) Subtract the two rational expressions.

$$\frac{1}{8x^2-22x+15} - \frac{1}{2x^2+13x-24}$$

i. (Ex 5) On a canoe trip, it took 45 minutes longer going upstream than it did downstream. How much slower was the upstream part of the trip, in miles per hour?

Practice Distributed and Integrated

1. (24) Use elimination to solve the system $\begin{matrix} 5x + 2y = 70 \\ 3x - 2y = 10 \end{matrix}$.

2. (26) Find the equation of the line that has a slope of $-\frac{3}{8}$ and passes through (4, 4).

***3.** (37) Add: $\frac{k^2}{2p} + c - \frac{4}{p^2c}$.

***4.** (36) Find the equation of the line that passes through the point (3, 5) and is parallel to the line $y = \frac{1}{6}x - 2$.

5. (20) Expand: $\frac{x^2y^{-2}}{z^2}\left(\frac{z^2}{y^2\,(2x^{-2})^{-1}} - \frac{4x^2\,y^0}{z^{-2}}\right)$.

6. (9) Multiply: $\begin{bmatrix} 4 & -1 \\ 3 & 7 \end{bmatrix} \cdot \begin{bmatrix} 2 & 4 & 3 \\ 1 & 0 & 8 \end{bmatrix}$.

Factor completely.

7. (23) $x^2 + x - 6$

8. (23) $x^3 - 9x$

***9.** (37) Find the difference. $\frac{1}{1 + \frac{1}{x+1}} - \frac{1}{1 + \frac{1}{x-1}}$

10. (13) Given $f(x) = 2x - 7$ and $g(x) = -5x + 8$, graph $f(x)$ and $g(x)$ on the same grid.

11. (17) **Write** Determine the four resulting inequalities that are used to solve $|x| > |2 + 7x|$.

12. (27) **Multi-Step** **a.** Tell how to determine the values of a, b, and c for any quadratic function written in standard form.

b. Write a quadratic equation in standard form where $a = -1$, $b = -2$, and $c = 3$.

***13.** (37) **Aviation** Because of the jet stream, on a roundtrip flight from Washington, D.C. to California, the first part of the trip will take half an hour longer than the return trip. How much faster is the return trip?

14. (31) **Error Analysis** A student multiplied rational expressions as follows:

$$\frac{\cancel{x^2} - 1}{\cancel{x^2} + 2x + 1} \cdot \frac{\cancel{x} + 1}{\cancel{x} - 3} = \frac{-1}{2x+1} \cdot \frac{1}{-3} = \frac{1}{3(2x+1)}.$$

What is the error? Find the correct product.

15. (15) **Multiple Choice** Which is the solution to the linear system $\begin{matrix} y = x \\ y = 3x - 2 \end{matrix}$?

A (3, 3) **B** (2, 4) **C** (1, 1) **D** There is no solution.

16. **Error Analysis** Find and correct the error a student made below.
(24)

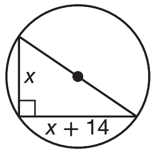
$$\begin{aligned} 2x + 2y &= 6 \\ -2x + y &= -12 \end{aligned} \longrightarrow \begin{aligned} 2x + 2y &= 6 \\ + -2x + y &= -12 \\ \hline 2y &= -6 \\ y &= -3 \end{aligned} \longrightarrow \begin{aligned} -2x + y &= -12 \\ -2x - 3 &= -12 \\ -2x &= -9 \\ x &= 4.5 \end{aligned}$$

The solution is (4.5, –3).

***17.** **Economics** The exchange rate from dollars to different currencies is shown in the table. A company charges a fee of \$3 for amounts up to \$50 and \$5 for amounts above \$50. Write the equations for exchanging Euros to dollars. What do the graphs of the equations have in common?
(36)

Currency	US $
1 euro =	1.377
1 Canadian dollar =	0.9472
1 British pound =	2.0284

18. **Probability** The right triangle is inscribed in the circle. The area of the triangle is 120 square units. Suppose a point is chosen at random inside the circle. What is the probability that the chosen point is inside the triangle?
(35)



19. **Multi-Step** **a.** Graph the equations $y = \frac{1}{2}x - 2$ and $y = \frac{1}{2}x + 3$.
(34)

b. What is the slope of each line?

c. State a property of lines and their slopes that is illustrated by the graphs.

***20.** **Write** What must be true if the LCM of rational expressions $\frac{f(x)}{g(x)}$ and $\frac{h(x)}{i(x)}$ is $g(x)i(x)$?
(37)

***21.** **Graphing Calculator** Use a graphing calculator to solve.
(14)

$$\det \begin{bmatrix} 2 & -3 & 5 \\ 2 & 0 & -1 \\ 2 & 3 & 1 \end{bmatrix}$$

22. **Analyze** A card is chosen from a standard 52-card deck. Consider these four events: *Choose a red card. Choose a face card. Choose the jack of spades. Choose a black jack.* How many pairs of mutually exclusive events can be made from the events? List all the pairs.
(33)

23. **Basketball** The length of a regulation high school basketball court is represented by the expression $(x + 24)$ feet. The width of a regulation high school basketball court is represented by the expression $(x - 10)$ feet.
(19)

a. Write an expression that represents the area of a regulation high school basketball court.

b. Evaluate the area of the court if $x = 60$.

***24.** **Justify** Suppose that $y_1 = m_1x + b_1$ and $y_2 = m_2x + b_2$ are each perpendicular to $y_3 = m_3x + b_3$. Justify the statement $y_1 \parallel y_2$.
(36)

25. (21) **Verify** Solve the system of equations by solving the first equation for y then substituting. Verify your answer by solving the second equation for y, then substituting.

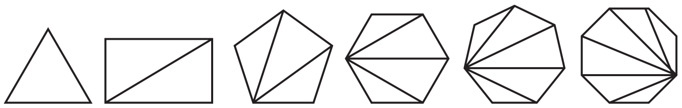
$2y - 3x = -10$

$4x = 7 - y$

26. (32) **Item Costs** An office worker ordered breakfast items for coworkers as shown below. Monday: 6 muffins, 6 bagels, and 6 fruit cups; total bill \$24.00
Tuesday: 4 muffins, 12 bagels, and 4 fruit cups; total bill \$22.80
Wednesday: 8 muffins, 12 bagels, and 10 fruit cups; total bill \$39.30

What was the cost of each item? To find the answer, write a system of equations and use an inverse matrix to solve it.

27. (22) **Geometry** Examine the polygons given below. Each polygon has been divided up into triangles. Complete the table. Graph the function. State the domain and range. Determine if the graph is continuous, discontinuous, and/or discrete.



Number of sides of the polygon	Number of triangles

28. (31) **Multi-Step** Given the rational functions $f(x) = \frac{2x - 1}{x + 5}$ and $g(x) = \frac{14x - 7}{14x^2 + 70x}$:

a. Find the function quotient $f(x) \div g(x)$ in simplest form.

b. Find the value of $f(x) \div g(x)$ if $x = 10$.

***29.** (37) What is the sum of $\frac{1}{x^2 + x - 72}$ and $\frac{1}{x^2 - x - 56}$?

A $\dfrac{2}{x^2 + 16x + 63}$

B $\dfrac{2}{x^2 + x - 72}$

C $\dfrac{2}{x^2 - x - 56}$

D $\dfrac{2x + 16}{x^3 + 8x^2 - 65x - 504}$

30. (16) **Construction** A contractor is replacing the tile in two bathrooms. For one bathroom, she purchased 416 large tiles and 256 small tiles at a cost of \$233.60. For the other bathroom, she purchased 400 large tiles and 512 small tiles at a cost of \$251.20. How much does each size tile cost?

LESSON 38

Dividing Polynomials Using Long Division

Warm Up

1. (10) **Vocabulary** An expression that contains the symbols < or > is called a(n) ______.

2. (39) Multiply: $(x + 3)(x + 7)$.

3. (4) Determine $f(-5)$ if $f(x) = 2x^2 - 3x$.

New Concepts

Polynomial long division is a similar process to integer long division: The quotient can have a remainder of zero or not.

$$
\begin{array}{r l}
x - 1 & \leftarrow \text{Quotient} \\
(x+1)\overline{)\,x^2 + 0 - 1} & \leftarrow \text{Dividend} \\
-(x^2 + x) & \\
\hline
0 - x - 1 & \\
-(-x - 1) & \\
\hline
0 & \leftarrow \text{Remainder}
\end{array}
$$

Divisor: $(x + 1)$

Math Reasoning

Analyze Suppose m and n are two different integers. If m is a prime number and $n > 1$, how do you know that $\frac{m}{n}$ has a nonzero remainder?

Example 1 Dividing a Polynomial by a Monomial

Divide: $(12x^3 - 6x^2 + 5x + 4)$ by $3x$.

SOLUTION

Use polynomial long division.

$$
\begin{array}{r}
4x^2 - 2x + 1 \\
3x\overline{)\,12x^3 - 6x^2 + 5x + 4} \\
-(12x^3) \qquad\qquad\qquad \\
\hline
0 - 6x^2 + 5x + 4 \\
-(-6x^2) \qquad\quad \\
\hline
0 + 5x + 4 \\
-(3x) \quad \\
\hline
2x + 4
\end{array}
$$

The quotient and remainder can be written as:

$$(4x^2 - 2x + 1) + \frac{2x + 4}{3x}$$

Check

$$\left(4x^2 - 2x + 1 + \frac{2x + 4}{3x}\right)(3x) = (12x^3 - 6x^2 + 3x) + (2x + 4)$$

$$= 12x^3 - 6x^2 + 5x + 4 \quad ✓$$

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Example 2 Dividing a Polynomial by a Linear Polynomial

Divide: $(x^4 + 2x^3 - 13x^2 - 38x - 24)$ by $(x + 4)$.

> **Hint**
>
> Distribute the −1 to each of the numbers when subtracting the product of the quotient and the divisor.

SOLUTION

$$
\begin{array}{r}
x^3 - 2x^2 - 5x - 18 \\
(x+4)\overline{)x^4 + 2x^3 - 13x^2 - 38x - 24} \\
-(x^4 + 4x^3) \qquad\qquad\qquad\qquad \\
\hline
0 - 2x^3 - 13x^2 - 38x - 24 \\
-(-2x^3 - 8x^2) \qquad\qquad\quad \\
\hline
0 - 5x^2 - 38x - 24 \\
-(-5x^2 - 20x) \qquad \\
\hline
0 - 18x - 24 \\
-(-18x - 72) \\
\hline
0 + 48
\end{array}
$$

The quotient and remainder are

$$(x^3 - 2x^2 - 5x - 18) + \frac{48}{x+4}$$

Check

$$\left(x^3 - 2x^2 - 5x - 18 + \frac{48}{x+4}\right)(x+4) = (x^4 + 2x^3 - 13x^2 - 38x - 72) + 48$$

$$= x^4 + 2x^3 - 13x^2 - 38x - 24 \quad \checkmark$$

Example 3 Dividing a Polynomial by a Lower-Degree Polynomial

Divide: $(4x^4 - x^3 - 11x - 484)$ by $(x^2 + 11)$.

SOLUTION

Even though there is no quadratic term in the dividend, include a place for it.

$$
\begin{array}{r}
4x^2 - x - 44 \\
(x^2+11)\overline{)4x^4 - x^3 + 0x^2 - 11x - 484} \\
-(4x^4 \qquad + 44x^2) \qquad\qquad\quad \\
\hline
0 - x^3 - 44x^2 - 11x - 484 \\
-(-x^3 \qquad\qquad - 11x) \qquad\quad \\
\hline
0 \quad - 44x^2 + 0 \quad - 484 \\
-(-44x^2 \qquad\qquad - 484) \\
\hline
0
\end{array}
$$

The quotient is $(4x^2 - x - 44)$. There is no remainder.

Example 4 Testing if One Polynomial Is a Factor of Another

a. Is $(x + 2)$ a factor of $2x^3 - x^2 - 7x + 6$?

SOLUTION

Use polynomial long division.

$$
\begin{array}{r}
2x^2 - 5x + 3 \\
(x + 2)\overline{)\,2x^3 - x^2 - 7x + 6} \\
\underline{-(2x^3 + 4x^2)} \\
0 - 5x^2 - 7x + 6 \\
\underline{-(-5x^2 - 10x)} \\
0 + 3x + 6 \\
\underline{-(3x + 6)} \\
0
\end{array}
$$

Because the remainder is zero, $(x + 2)$ is a factor.

b. Is $(x + 3)$ a factor of $6x^3 - 6x^2 - 6x + 6$?

SOLUTION

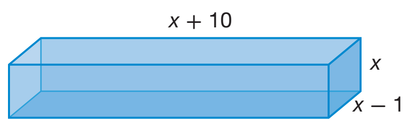
Use polynomial long division.

$$
\begin{array}{r}
6x^2 - 24x + 66 \\
(x + 3)\overline{)\,6x^3 - 6x^2 - 6x + 6} \\
\underline{-(6x^3 + 18x^2)} \\
0 - 24x^2 - 6x + 6 \\
\underline{-(-24x^2 - 72x)} \\
66x + 6 \\
\underline{(66x + 198)} \\
204
\end{array}
$$

Because the remainder is not zero, $(x + 3)$ is not a factor.

Example 5 Application

What is the ratio of the volume to the surface area for the rectangular prism?



SOLUTION

$$\frac{\text{volume}}{\text{surface area}} = \frac{x(x + 10)(x - 1)}{2x(x + 10) + 2(x - 1)(x + 10) + 2x(x - 1)}$$

$$= \frac{x(x + 10)(x - 1)}{6x^2 + 36x - 20}$$

Since the numerator and denominator do not share any factors, the ratio has a nonzero remainder.

Math Reasoning

Verify Show that the ratio of the volume to the surface area has no common factors.

Lesson Practice

a. Divide $12x^3 - 9x^2 - 237x + 40$ by $3x$.
(Ex 1)

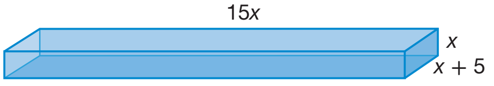
b. Divide $6x^4 - 23x^3 - 34x^2 + 207x - 180$ by $x - 3$.
(Ex 2)

c. Divide $2x^4 + 7x^3 - 241x^2 - 192x + 3024$ by $x^2 - 5x - 36$.
(Ex 3)

d. Is $x - 6$ a factor of $2x^3 + x^2 - 177x + 594$?
(Ex 4)

e. Is $x + 27$ a factor of $7x^3 + 178x^2 - 107x - 78$?
(Ex 4)

f. What is the ratio of the volume to surface area in the figure below? Does this expression have a nonzero remainder?
(Ex 5)



Practice Distributed and Integrated

Use the descriptions below to write the vertex form of a quadratic equation.

1. The parent function is shifted 10 units left.
(30)

2. The parent function is shifted 1 unit right and 3 units down.
(30)

***3.** Divide $x^3 - 10x^2 - 69x + 30$ by $(x + 5)$.
(38)

4. Given $A = \begin{bmatrix} 4 & -2 \\ 8 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$, solve $AX = B$.
(32)

A number cube is rolled. Determine if the following events are mutually exclusive.

5. *Roll an odd number* or *roll a 6*
(33)

6. *Roll an even number* or *roll a number less than 3*
(33)

***7.** Find the difference. $\frac{1}{x+3} - \frac{1}{x+2} - \frac{1}{x+1}$
(37)

8. Use Cramer's Rule to solve $\begin{matrix} 3x - 12y = -15 \\ x + 2y = 7 \end{matrix}$.
(16)

Find the determinants of the following matrices.

9. What is $|A|$ if $A = \begin{bmatrix} 2 & -12 \\ 7 & 13 \end{bmatrix}$?
(14)

10. What is $|B|$ if $B = \begin{bmatrix} -1 & -24 \\ 0.25 & 0.5 \end{bmatrix}$?
(14)

***11.** What are the slope and y-intercept of $3x + 10y = -1$?
(38)

12. **Geometry** As the diameter (d) of circle increases in size, the circumference (C) increases. Likewise, as the diameter decreases in size, so does the circumference. The constant of variation between C and d is π. Describe the kind of variation between the circumference and diameter in a circle. Write the equation.
(12)

***13.** (35) **Write** State all the possible numbers of y-intercepts a quadratic function graph can have. Explain.

***14.** (38) **Justify** Consider the polynomial $x^2 + ax + bx + ab$. What is the remainder when this polynomial is divided by $x + a$? How do you know?

15. (29) **Hourly Pay** An employee makes three different rates of pay based on the type of work. The rates of pay are \$16 per hour, \$24 per hour, and \$32 per hour. The combined number of hours worked at the rate of \$16 per hour and \$24 per hour was 10 times the hours worked at the rate of \$32 per hour. If in one week the employee earned \$808 for working 44 hours, how many hours were worked at each rate of pay?

***16.** (38) **Error Analysis** Two students are dividing $-14x^3 + 46x^2 + 44x - 16$ by $x - 2$ but get different results. Which student made the mistake?

Student A

$$\begin{array}{r} -14x^2 + 18x + 80 \\ (x-2)\overline{)-14x^3 + 46x^2 + 44x - 16} \\ \underline{-(-14x^3 + 28x^2)} \\ 0 + 18x^2 + 44x - 16 \\ \underline{-(18x^2 - 36x)} \\ 0 + 80x - 16 \\ \underline{-(80x - 160)} \\ 144 \end{array}$$

Student B

$$\begin{array}{r} -14x^2 + 18x - 8 \\ (x-2)\overline{)-14x^3 + 46x^2 + 44x - 16} \\ \underline{-(-14x^3 + 28x^2)} \\ 0 + 18x^2 + 44x - 16 \\ \underline{-(18x^2 - 36x)} \\ 0 + 8x - 16 \\ \underline{-(8x - 16)} \\ 0 \end{array}$$

***17.** (27) **Graphing Calculator** Use a graphing calculator to determine the minimum of the quadratic equation $f(x) = x^2 - 4x - 2$.

***18.** (36) **Multiple Choice** Which is the equation of the line perpendicular to $y = 4x + 10$ that crosses the (1, 1)?

A $y = 4x + \frac{5}{4}$ **B** $y = -\frac{1}{4}x + \frac{5}{4}$ **C** $y = -4x + \frac{5}{4}$ **D** $y = \frac{1}{4}x + \frac{5}{4}$

19. (34) **Multi-Step** Graph the equations $y = \frac{3}{2}x - 1$ and $y = -\frac{2}{3}x + 2$ on the same coordinate system.

a. What are the slopes of the lines?

b. What is the product of the slopes?

c. State a property of lines and their slopes that is illustrated by the graphs.

20. (28) **Geography** Crater Lake in southern Oregon lies inside a volcanic basin and is nearly circular in shape. Write the area to circumference ratio of the lake as a simplified rational expression. Then find the ratio given that the radius of the lake is about 3 miles.

21. (33) **Error Analysis** A card is drawn at random from a standard 52-card deck. How many outcomes are in the event *draw an ace or a spade*? A student solved this problem by reasoning as follows: There are 4 aces and 13 spades, so there are 4 + 13 = 17 outcomes in the event. What is the error? Find the correct answer.

22. (24) **Multi-Step** **a.** Rewrite the system $\begin{aligned} y &= 5 - 7x \\ 10 &= 14x + 2y \end{aligned}$ so that both equations are in standard form.

b. By inspection, what do you think the solution is? Why?

c. Solve and classify the system.

23. (32) **Analyze** Students at a high school were asked if they visited a museum, public library, or historical site during the last month. The table shows data about the results of the survey. How many students in each grade were surveyed? To find the answer, write a system of equations and use an inverse matrix to solve it.

	Ninth Graders	Tenth Graders	Eleventh Graders	Total
Museum	20%	30%	25%	143
Public Library	40%	50%	45%	258
Historical Site	20%	20%	35%	142

24. (30) **Generalize** Describe shifts, stretches, and compressions of parabolas and discuss how those changes appear in the equations of parabolas.

25. (23) **Write** Explain in words how the Zero Product Property is useful in solving polynomial equations.

***26.** (37) **Aviation** Because of a strong head wind the return trip on an airplane takes 1.5 hours longer than the first part of the trip. How much slower is the return trip?

27. (27) **Justify** Graph $f(x) = -x^2 + 3$. Tell why the range of the function is the set of real numbers less than or equal to 3.

***28.** (38) **Multiple Choice** Which of the following is a factor of $3x^3 + 8x^2 - 31x + 20$?

A $(x + 2)$ **B** $(3x + 4)$ **C** $(x - 1)$ **D** $(x + 6)$

29. (26) **Medicine** The Centers for Disease Control has developed growth charts that pediatricians use to see if a child's height is within a certain average. The chart below shows data for a boy's height in the 75th percentile. The three points are on a line. Use the data to write the equation of the line in slope-intercept form.

Age (years)	Height (inches)
3	38.5
4	41.5
5	44.5

***30.** (37) **Probability** Two six-sided number cubes have labels $\frac{1}{x}$, $\frac{1}{2x}$, $\frac{1}{3x}$, $\frac{1}{4x}$, $\frac{1}{5x}$, and $\frac{1}{6x}$. If both number cubes are rolled, what is the probability that the sum of the expressions is $\frac{1}{x}$?

LESSON 39

Graphing Linear Inequalities in Two Variables

Warm Up

1. (13) **Vocabulary** The equation $y = \frac{1}{4}x - 2$ is written in ________ form.
2. (13) Find the slope of the line that passes through (9, 3) and (−7, 5).
3. (13) The slope of a vertical line is ________.
4. (13) True or False: The y-intercept of $2x - y = -4$ is 4.

New Concepts

When two expressions have the same value, they can be joined by an equal sign. If two expressions do not have the same value, they can be joined by one of the inequality signs shown below.

Less than	Greater than	Less than or equal to	Greater than or equal to
$<$	$>$	$\leq$	$\geq$

A **linear inequality in two variables** relates two variables, often x and y, with an inequality sign. A solution (x, y) makes the inequality true when the values of x and y are substituted into the inequality.

Example 1 Determining if an Ordered Pair is a Solution of a Linear Inequality

Determine if each point is a solution of the inequality.

a. $y > -3x + 5$, (2, 9)

SOLUTION

$9 \overset{?}{>} -3(2) + 5$

$9 > -1$ True; (2, 9) is a solution.

b. $3y - x < -18$, (−3, −7)

SOLUTION

$3(-7) - (-3) \overset{?}{<} -18$

$-18 < -18$ False; (−3, −7) is not a solution.

c. $-2y \leq 1 - x$, (−3, 4)

SOLUTION

$-2(4) \overset{?}{\leq} 1 - (-3)$

$-8 \leq 4$ True; (−3, 4) is a solution.

Caution

A number can not be less than (or greater than) itself. In Example 2b, the point would be a solution if the inequality symbol were $\leq$ rather than $<$.

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Exploration Finding Solutions of Linear Inequalities on the Coordinate Plane

Graph $y = \frac{4}{3}x - 2$. Then graph each of the following points and tell if the point is *above the line, on the line,* or *below the line.*

(6, 2), (−3, −6), (1, 4), (−5, −5), (0, −2), (4, −1)

Which of the points satisfies $y < \frac{4}{3}x - 2$? How are these solutions related to the graph?

Which satisfy $y \geq \frac{4}{3}x - 2$? How are these solutions related to the graph?

Reading Math

A dashed line indicates that the points on the line are not solutions of the inequality. A solid line indicates that they are solutions.

The graph of a linear inequality in two variables is the set of all points that satisfy the inequality. To graph a linear inequality, first graph the related linear equation (just suppose the inequality sign is an equal sign). Make the line dashed when the symbol is $<$ or $>$ and solid when the symbol is $\leq$ or $\geq$. This line is the **boundary line.** It separates the plane into two **half-planes.** Shade the half-plane that includes the solutions of the inequality.

To determine which half-plane contains the solutions of the inequality, use a test point. If the test point satisfies the inequality, then all the points in the half-plane that contains the test point will also satisfy the inequality. The test point should not be a point on the boundary line. The point (0, 0) makes a good test point when it is not on the boundary.

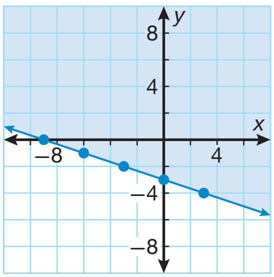
Example 2 Using a Table of Values to Graph a Linear Inequality in Two Variables

Graph $3y + x \geq -9$ by making a table of values.

SOLUTION

Step 1: Find ordered pairs that satisfy the equation $3y + x = -9$. Use the x- and y-intercepts as well as other points.

x	0	−9	−6	−3	3
y	−3	0	−1	−2	−4



Step 2: Connect the points with a solid line.

Step 3: Use the test point (0, 0) to determine which half-plane to shade.

$$3y + x \geq -9$$
$$3(0) + 0 \overset{?}{\geq} -9$$
$$0 \geq -9 \quad \text{True}$$

The point (0, 0) is a solution. Shade the half-plane that contains (0, 0).

When the inequality is written in slope-intercept form, you can choose which half-plane to shade by looking at the direction of the inequality sign. Shade above the line for $>$ and $\geq$, and below the line for $<$ and $\leq$.

Example 3 Using Slope-Intercept Form to Graph a Linear Inequality in Two Variables

a. Graph $5y + x < 20$ using slope-intercept form.

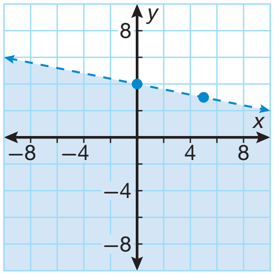
SOLUTION

Step 1: Write the inequality in slope-intercept form.

$$5y + x < 20$$
$$5y < -x + 20$$
$$y < -\frac{1}{5}x + 4$$

Step 2: Graph $y = -\frac{1}{5}x + 4$ by plotting (0, 4) and a second point by moving 1 unit down and 5 units right. Connect the points with a dashed line to indicate that the points on the line are not solutions.

Step 3: Shade below the line because the inequality symbol is <.



The test point (0, 0) verifies this.

$$5y + x < 20$$
$$5(0) + 0 \overset{?}{<} 20$$
$$0 < 20 \quad \text{True}$$

b. Graph $-2y + 10x < -2$ using slope-intercept form.

SOLUTION

Step 1: Write the inequality in slope-intercept form. Don't forget to switch the direction of the inequality sign when multiplying or dividing by a negative number.

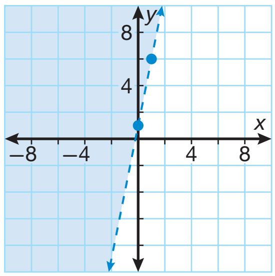
$$-2y + 10x < -2$$
$$-2y < -10x - 2$$
$$y > 5x + 1$$

Step 2: Plot (0, 1) and find a second point by moving 5 units up and 1 unit right. Connect the points with a dashed line.

Step 3: Shade above the line because the inequality symbol is > in slope-intercept form.

The test point (0, 0) verifies this.

$$-2y + 10x < -2$$
$$-2(0) + 10(0) \overset{?}{<} -2$$
$$0 < -2 \quad \text{False}$$



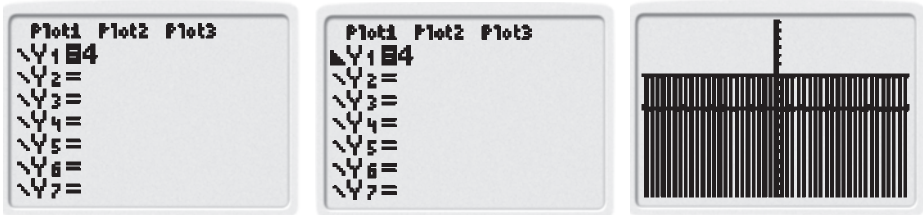
Hint

Look at the y-axis to help decide which half-plane is above the line.

Example 4 Using Technology to Graph a Linear Inequality in Two Variables

a. Graph $y \leq 4$ on a graphing calculator.

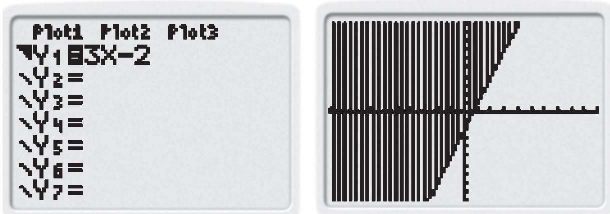
SOLUTION In the **[Y=]** editor, after y, enter 4. To indicate shading below the line, use the arrow key to highlight the symbol to the left of Y_1. Press enter until the symbol shows the lower left portion of the square shaded, then graph.



b. Graph $3x - y \leq 2$ on a graphing calculator.

SOLUTION First write the inequality in slope-intercept form.

$$3x - y \leq 2$$
$$-y \leq -3x + 2$$
$$y \geq 3x - 2$$



Example 5 Application: Food Preparation

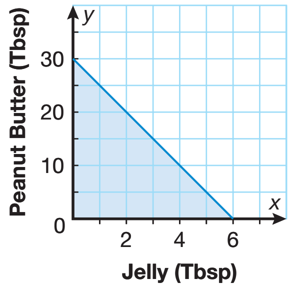
Nellie is making a peanut butter and jelly snack for a school function. Each tablespoon of jelly has 15 grams of carbohydrates and each tablespoon of peanut butter has 3 grams of carbohydrates. She wants the snack to have no more than 90 grams of carbohydrates. Graph the inequality that represents the possible amounts of each ingredient.

SOLUTION Let x represent the tablespoons of jelly and y the tablespoons of peanut butter. The phrase *no more than* indicates *less than or equal to*. Then the inequality is $15x + 3y \leq 90$.

Write $15x + 3y \leq 90$ in slope-intercept form.

$$15x + 3y \leq 90$$
$$3y \leq -15x + 90$$
$$y \leq -5x + 30$$

Graph $y = -5x + 30$ with a solid line. Shade below the line.



The inequality could also be graphed by plotting the points that contain the intercepts (0, 30) and (6, 0). The test point (0, 0) makes the original inequality true and should be in the solution set.

Math Reasoning

Analyze Do all the solutions make sense in context of the application? Explain.

Lesson Practice

Determine if each point is a solution of the inequality.

a. *(Ex 1)* $y \geq 7 - 4x$, $(-2, 1)$

b. *(Ex 1)* $-2y + x < 0$, $(0, 0)$

c. *(Ex 1)* $4y - 6x > 15$, $(4, 13)$

d. *(Ex 2)* Graph $2y - 6x \leq -12$ by making a table of values.

e. *(Ex 3)* Graph $4y + 4 > x$ using slope-intercept form.

f. *(Ex 3)* Graph $4 - 2y > -x$ using slope-intercept form.

g. *(Ex 4)* Graph $y \geq 0$ on a graphing calculator.

h. *(Ex 4)* Graph $3y + 9 \geq 3x$ on a graphing calculator.

i. *(Ex 5)* A school chef is making a vegetable side dish for lunch. Each cup of beans has 35 grams of protein and each cup of peas has 48 grams of protein. He wants the dish to have more than 840 grams of protein. Graph the inequality that represents this.

Practice Distributed and Integrated

Determine if the given points are solutions of the inequalities.

***1.** *(39)* $y < 6x - 2$, $(3, 18)$

***2.** *(39)* $y + 2x \geq -4$, $(-1, -2)$

3. *(20)* Add: $m + \frac{x}{c} + \frac{c}{x^2b}$

4. *(29)* Solve. $\begin{aligned} -8x - 22 + 2y &= 2z \\ -6x + 2y &= 16 + z \\ 6 + 3y + z &= -x \end{aligned}$

Solve by substitution:

5. *(21)* $\begin{aligned} x &= y + 1 \\ 3x + 2y &= 8 \end{aligned}$

6. *(21)* $\begin{aligned} 3x - y &= 22 \\ 2x + 3y &= -11 \end{aligned}$

7. *(21)* $\begin{aligned} -7x + y &= 8 \\ 5x - y &= -6 \end{aligned}$

8. *(38)* Divide $6x^3 + 23x^2 - 20x - 9$ by $3x + 1$.

9. *(28)* Find the excluded values of $\frac{x^2 - 25}{3x^2 + 15x}$.

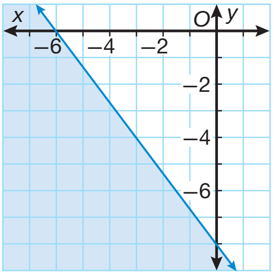
10. *(23)* Factor the polynomial $2x^4 + 4x^3 - 4x$.

11. *(28)* **Verify** Show why the x's cannot be divided out in $\frac{x + 2}{x + 5}$.

***12.** *(39)* **Multiple Choice** The graph of which inequality is shown?

A $-3y - 4x \geq 24$ **B** $-3y - 4x \leq 24$

C $-3y - 4x > 24$ **D** $-3y - 4x < 24$



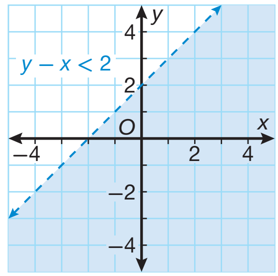
*13. **Statistics** The fraction of students that are interested in a new brand of sneakers (37) is twice the fraction of those that are not. The fraction of students who are not interested is $\frac{1}{x}$. Those who expressed no opinion in the sneakers were 10% of the students who were interested. What percent of students were interested?

14. **Baseball Equipment** Last month, a baseball coach bought 4 bats and 9 balls for $355. (16) This month, the coach bought 2 bats and a dozen balls for $290. The prices did not change from last month to this month. How much did each bat and ball cost?

a. Set up equations relating the monetary values of the bats and balls last month and this month. Let T represent the cost (in dollars) per bat, and B represent the cost per ball.

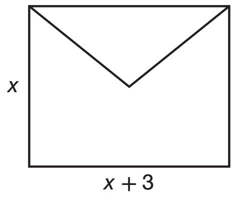
b. Use Cramer's Rule to solve the system of equations.

*15. **Verify** Using the graph, choose a point in the shaded region and (39) test it. Choose a point in the unshaded region and test it. Is the graph correct? Explain.



*16. **Graphing Calculator** Graph $y = 4$ on your graphing calculator. Determine if this (22) relation is a function. Determine if the graph is continuous, discontinuous, and/or discrete.

*17. **U.S. Postal Service** The United States Postal Service uses size, shape, and weight (35) to determine pricing for first class mail. One category of first class mail is large envelope. The maximum length of a large envelope is 3 inches greater than its maximum width, and its maximum area is 180 square inches. Write and solve a quadratic equation to find the maximum length and maximum width of a large envelope.



18. **Multi-Step** Use $x^2 + 8x - 48$. (23)

a. Find the factors of -48.

b. Which of the factors have a sum of 8?

c. What is the factored form of the polynomial?

19. **Multiple Choice** If the graph of $y = x$ is reflected over the x-axis, and then the (34) resulting graph is translated 4 units up, which equation is represented by the final graph?

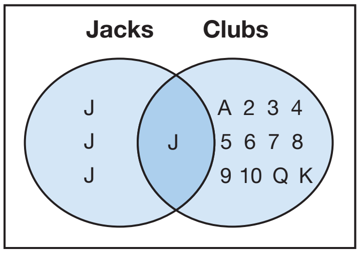
A $y = -\frac{1}{4}x$ **B** $y = -4x$ **C** $y = x - 4$ **D** $y = -x + 4$

20. **Multi-Step** A store sells granola and dried fruit by the pound. Dillon bought (21) 4 pounds of granola and 2 pounds of dried fruit for $22. Carmen bought 2 pounds of granola and 4 pounds of dried fruit for $26.

a. Write a system of equations that represents the price of the granola, g, and the price of the dried fruit, f.

b. How much should one pound of granola and one pound of dried fruit cost individually and together?

***21.** (33) **Model** The figure at the right is a Venn diagram. It shows that jacks and clubs are not mutually exclusive events when choosing a card from a standard 52-card deck. Draw a Venn diagram to show that kings and hearts are not mutually exclusive events when choosing a card from a standard 52-card deck.



22. (32) **Analyze** What system of equations is represented by the matrix equation shown below? Solve the system mentally.

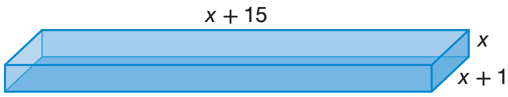
$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 12 \\ 50 \end{bmatrix}$$

23. (28) **Error Analysis** A student divided rational expressions as follows: $\frac{20x^2}{y} \div \frac{y}{x^3} = \frac{20\cancel{x^2}}{\cancel{y}} \div \frac{\cancel{y}}{x \cdot \cancel{x^2}} = \frac{20}{x}$. What error did the student make? Find the correct quotient.

***24.** (38) **Model** Show that the ratio of an odd number to an even number, when each is expressed as a polynomial, has a nonzero remainder.

25. (30) **Verify** Suppose a rocket is fired straight into the air such that the graph of the function representing the height of the rocket after x seconds has its vertex at (3.75, 225) and passes through the point (5, 200). Verify that the value of a in the equation written in vertex form is -16.

*** 26.** (38) **Geometry** What is the ratio of the volume to the surface area for the rectangular prism shown? Does the ratio have a remainder?



27. (29) **Investing** A person invests \$50,000 for one year; some is invested at 7%, some at 8%, and the remainder at 12%. The combined interest earned at the end of the year from these investments was \$4,770. The amount invested at 8% is \$4,000 more then the amount invested at 7% and 12% combined. Find the amount of money invested at each rate.

***28.** (39) **Money** Nelson is building a tower of coins with half-dollars and dimes. He wants the tower to be less than 15 centimeters high. Each half dollar is 2.15 mm thick and each dime is 1.35 mm thick. Write an inequality that represents how many of each coin are needed. Then show how to use the inequality to see if using 30 half-dollars and 70 dimes is a possibility.

29. (27) **Business** Mr. Jing makes and sells carved animals. His monthly profit function can be modeled by $f(x) = -0.15x^2 + 7x$ where x represents the number of carved animals he sells each month.

a. For how many animals does he make the maximum profit? How much profit is it?

b. At what point would Mr. Jing begin to lose money?

***30.** (36) Write the equation of the line in slope intercept form that is parallel to $y = 5x - 3$ and goes through (1, 4).

LESSON

40 Simplifying Radical Expressions

Warm Up

1. (25) **Vocabulary** $\sqrt{16} = 4$. 4 is the ________ of 16.
2. (19) Multiply $(2x - 5)(2x + 5)$.
3. (2) Combine like terms: $3x^2 + 5x^3 - 8x^3 + x^2 - 5x^2$
4. (3) Simplify $7^3 \cdot 7^2$
5. (3) Simplify $(3^2)^3$

New Concepts

Because $4^2 = 16$, 4 is a **square root** of 16. However, $(-4)^2 = 16$, so -4 is also a square root of 16. To indicate that the positive, or **principal,** square root is desired, the **radical symbol** is used. Therefore, $\sqrt{16}$ equals 4 and only 4. The expression under a radical sign is the **radicand.** $\sqrt{16}$ is a **radical,** or radical expression, and 16 is the radicand.

Math Reasoning

Analyze Why is -2 not a cube root of 8?

There are roots other than square roots. For example, because $2^3 = 8$, 2 is a cube root of 8. Because $3^4 = 81$ and $(-3)^4 = 81$, -3 and 3 are fourth roots of 81.

***n* Roots**
Given that a and b are real numbers and n is an integer greater than 1, if $a^n = b$, then a is an nth root of b.
Given that a and b are not negative and n is an even integer greater than 1, or given that a and b are real numbers and n is an odd integer greater than 1, if $a^n = b$, then $\sqrt[n]{b} = a$.

For $\sqrt[n]{b}$, n is the **index.** When n is not shown, it is assumed to be 2.

Example 1 Simplifying Radicals

Simplify each expression.

a. $\sqrt[5]{-32}$

SOLUTION $\sqrt[5]{-32} = -2$ because $(-2)^5 = -32$.

b. $\sqrt[3]{27}$

SOLUTION $\sqrt[3]{27} = 3$ because $3^3 = 27$.

c. $\sqrt[4]{16}$

SOLUTION $\sqrt[4]{16} = 2$ because $2^4 = 16$. Although -2 is a fourth root of 16, the radical sign indicates that only the positive root is to be given.

Online Connection
www.SaxonMathResources.com

The Product Rule for Radicals
Given that a and b are real numbers and n is an integer greater than 1, $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$ and $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$

The product rule states that $\sqrt{10}$ can be written as $\sqrt{2} \cdot \sqrt{5}$. This rule is helpful when one of the radicals in the product can be simplified. For instance, $\sqrt{20} = \sqrt{4} \cdot \sqrt{5}$. Now, $\sqrt{4}$ can be simplified: $\sqrt{20} = 2\sqrt{5}$.

Types of Real Roots
If n is even, $\sqrt[a]{x^n} = x^{\frac{n}{a}}$.   If n is odd, $\sqrt[a]{x^n} = x^{\frac{n-1}{a}} \cdot \sqrt{x}$

Example 2 Using the Product Rule for Radicands

Simplify each expression.

a. $\sqrt{8} \cdot \sqrt{5}$

SOLUTION Either multiply and then simplify, or simplify and then multiply.

Option 1: $\sqrt{8} \cdot \sqrt{5} = \sqrt{40} = \sqrt{4} \cdot \sqrt{10} = 2\sqrt{10}$

Option 2: $\sqrt{8} \cdot \sqrt{5} = \sqrt{4} \cdot \sqrt{2} \cdot \sqrt{5} = 2\sqrt{2} \cdot \sqrt{5} = 2\sqrt{10}$

Caution

Simplifying before multiplying does not always mean that the product will not need to be simplified further.

b. $\sqrt[3]{54}$

SOLUTION

$\sqrt[3]{54} = \sqrt[3]{27} \cdot \sqrt[3]{2}$ Find a factor of 54 that has a cube root.

$= 3\sqrt[3]{2}$ The cube root of 27 is 3.

c. $\sqrt{567}$

SOLUTION

$$\begin{aligned} \sqrt{567} &= \sqrt{9 \cdot 63} \\ &= 3\sqrt{9 \cdot 7} \\ &= 3 \cdot 3\sqrt{7} \\ &= 9\sqrt{7} \end{aligned}$$

When radical expressions have the same radicand and root, they are **like radicals** and can be combined. This is analogous to combining like terms.

Combine like terms: $2x + 3x = 5x$

Combine like radicals: $2\sqrt{x} + 3\sqrt{x} = 5\sqrt{x}$

Example 3 Combining Radical Expressions

Simplify each expression.

a. $12\sqrt[3]{8} - \sqrt[3]{8} + 5\sqrt{8} + 9\sqrt{8}$

SOLUTION

$12\sqrt[3]{8} - \sqrt[3]{8} + 5\sqrt{8} + 9\sqrt{8}$

$11\sqrt[3]{8} + 14\sqrt{8}$ Combine like radicals.

$11 \cdot 2 + 14\sqrt{4} \cdot \sqrt{2}$ $\sqrt[3]{8} = 2$ and $\sqrt{8} = \sqrt{4}\sqrt{2}$.

$22 + 28\sqrt{2}$

b. $\sqrt{242} + \sqrt{72} - \sqrt{48}$

SOLUTION Simplify each term to identify any like terms.

$\sqrt{242} + \sqrt{72} - \sqrt{48}$

$\sqrt{121} \cdot \sqrt{2} + \sqrt{36} \cdot \sqrt{2} - \sqrt{16} \cdot \sqrt{3}$ Use the Product Rule.

$11\sqrt{2} + 6\sqrt{2} - 4\sqrt{3}$ Simplify.

$17\sqrt{2} - 4\sqrt{3}$ Combine like terms.

c. $\sqrt[4]{32} + \sqrt[4]{2} + \sqrt[4]{162}$

SOLUTION

$\sqrt[4]{32} + \sqrt[4]{2} + \sqrt[4]{162}$

$\sqrt[4]{16} \cdot \sqrt[4]{2} + \sqrt[4]{2} + \sqrt[4]{81} \cdot \sqrt[4]{2}$ Use the Product Rule.

$2\sqrt[4]{2} + \sqrt[4]{2} + 3\sqrt[4]{2}$ Take the fourth roots of 16 and 81.

$6\sqrt[4]{2}$ Combine like terms.

Hint

Make a list of square, cube, and fourth roots to look at while simplifying radicals.

All the concepts in this lesson can be extended to variables. Keep in mind, however, that for a positive root to be taken, a variable must represent a positive number. For example, the product rule can be used to expand a variable representing positive values as shown below:

$$\sqrt{x^6} = \sqrt{x^2} \cdot \sqrt{x^2} \cdot \sqrt{x^2} = x \cdot x \cdot x = x^3$$

Example 4 Simplifying Radicals with Variables

Simplify each expression. All variables represent non-negative real numbers.

a. $\sqrt{x^4}$

SOLUTION $\sqrt{x^4} = x^2$ because $(x^2)^2 = x^4$.

b. $\sqrt{18h^{17}}$

SOLUTION

$$\sqrt{18h^{17}} = \sqrt{9} \cdot \sqrt{2} \cdot h^{\frac{17-1}{2}} \cdot \sqrt{h}$$
$$= 3 \cdot \sqrt{2} \cdot h^8 \cdot \sqrt{h}$$
$$= 3h^8\sqrt{2h}$$

Hint

With conjugates, the middle two terms of the product will always be opposites.

c. $\sqrt[4]{48x} + \sqrt[4]{3x} + \sqrt[4]{243x}$

SOLUTION Simplify each term to identify any like terms.

$\sqrt[4]{48x} + \sqrt[4]{3x} + \sqrt[4]{243x}$	
$\sqrt[4]{16} \cdot \sqrt[4]{3x} + \sqrt[4]{3x} + \sqrt[4]{81} \cdot \sqrt[4]{3x}$	Use the Product Rule.
$2\sqrt[4]{3x} + \sqrt[4]{3x} + 3\sqrt[4]{3x}$	Take the fourth roots of 16 and 81.
$6\sqrt[4]{3x}$	Combine like terms.

d. $\sqrt[4]{16x^2} + \sqrt{4x} + \sqrt[3]{27}$

SOLUTION

$\sqrt[4]{16x^2} + \sqrt{4x} + \sqrt[3]{27}$	
$\sqrt[4]{16} \cdot \sqrt[4]{x^2} + \sqrt{4} \cdot \sqrt{x} + 3$	Use the Product Rule.
$2\sqrt[4]{x^2} + 2\sqrt{x} + 3$	Take the indicated roots of 16 and 4.
$2x^{\frac{2}{4}} + 2\sqrt{x} + 3$	Take the 4th root of x^2.
$2\sqrt{x} + 2\sqrt{x} + 3$	Simplify
$4\sqrt{x} + 3$	Combine like terms.

Example 5 Application: Physics

The pendulum in the Pantheon in Paris makes one complete swing in $\sqrt{\frac{110\pi^2}{4}}$ seconds. Simplify this expression.

The length of a pendulum is given by the equation $l = \frac{8s^2}{\pi^2}$, where l is the length in feet and s is the time in seconds the pendulum takes to make a complete swing. Using your simplified expression from the first part of the problem, write an expression to find the length of the pendulum and simplify it to find the length of the pendulum.

SOLUTION Simplify the expression using the product rule for radicals.

$\sqrt{\frac{110\pi^2}{4}} = \sqrt{\pi^2} \cdot \sqrt{\frac{1}{4}} \cdot \sqrt{110}$	Use the Product Rule.
$\pi \cdot \frac{1}{2} \cdot \sqrt{110}$	Take the indicated roots.
$\frac{\pi\sqrt{110}}{2}$	Simplify.

Now, plug this expression into the formula given for the length of a pendulum.

$$l = \frac{8s^2}{\pi^2}$$

$$l = \frac{8\left(\frac{\pi\sqrt{110}}{2}\right)^2}{\pi^2} = \frac{8\left(\frac{\pi^2 110}{4}\right)}{\pi^2} = \frac{2\pi^2 110}{\pi^2} = 2 \cdot 110 = 220$$

So the pendulum is 220 feet long.

Lesson Practice

Simplify each expression. Assume all variables represent non-negative real numbers.

a. $\sqrt[3]{-125}$
(Ex 1)

b. $-\sqrt{9}$
(Ex 1)

c. $\sqrt[5]{-1}$
(Ex 1)

d. $\sqrt{45} \cdot \sqrt{2}$
(Ex 2)

e. $\sqrt[5]{64}$
(Ex 2)

f. $\sqrt{50} - \sqrt{32}$
(Ex 3)

g. $\sqrt{98w^{10}}$
(Ex 4)

h. $-\sqrt[4]{y} + 2\sqrt{y} - 7\sqrt[4]{y} - \sqrt{y}$
(Ex 4)

i. Refer to the formula given in example 5. If a pendulum makes one complete swing in $\sqrt{2\pi^4} \cdot \sqrt[4]{2}$ seconds, write a simplified expression for the length of the pendulum.
(Ex 5)

Practice Distributed and Integrated

***1.** Make a table of values to satisfy the inequality $2x - 4y \geq 12$.
(39)

Write the equation in standard form for the line that crosses the two points.

2. $(4, 5.5), (-1, 2.5)$
(26)

3. $(3, 5), (6, 4)$
(26)

Multiply then simplify.

***4.** $\sqrt{30} \cdot \sqrt{6}$
(40)

***5.** $\sqrt{180} \cdot \sqrt{10}$
(40)

Factor:

6. $5x^2y^2 - 2xy + 10xy^2$
(23)

7. $x^2y^3m^5 + 12x^3ym^4 - 3x^2y^2m^2$
(23)

8. Given the rational functions $f(x) = \frac{1}{x}$ and $g(x) = \frac{x^2}{x+4}$, find the value of $f(x) \cdot g(x)$ if $x = -2$.
(20)

Solve:

9. $-x + x^2 = 12$
(35)

10. $-48x = -2x^2 - x^3$
(35)

***11. Multi-Step** **a.** Expand the expression as much as possible. Do not simplify: $\sqrt{x^8}$
(40)

b. Explain how can you rewrite your answer from part a so the expression can be easily simplified.

c. Simplify the expression as much as possible. The variable represents non-negative real numbers.

12. Error Analysis Explain and correct the error a student made below.
(28)

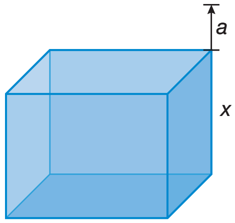
$$\frac{x^2}{x^5} = x^{5-2} = x^3$$

13. *(32)* **Multiple Choice** If $A = \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 \\ 5 & 0 \end{bmatrix}$, what is the solution to the matrix equation $AX = B$?

A $X = \begin{bmatrix} 19 & -8 \\ -13 & 6 \end{bmatrix}$ **B** $X = \begin{bmatrix} 11 & -8 \\ -13 & 6 \end{bmatrix}$

C $X = \begin{bmatrix} 19 & 8 \\ -13 & 6 \end{bmatrix}$ **D** $X = \begin{bmatrix} 19 & -8 \\ 13 & 6 \end{bmatrix}$

***14.** *(38)* **Chemistry** Water has a unique property that causes it to increase in volume as it changes from liquid to ice. In the illustration, water just fills a cube-shaped container. The water is frozen, and increased volume of ice changes in one direction. What is the ratio of the new volume to the original volume?



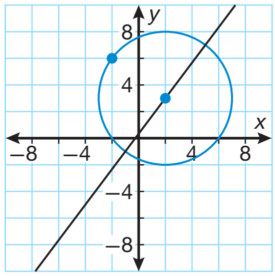
15. *(37)* **Multiple Choice** Subtract. $\frac{2}{x^2 - 4x - 45} - \frac{1}{x^2 + 9x + 20}$

A $\frac{x + 17}{x^2 - 61x - 180}$ **B** $\frac{x + 17}{x^3 - 61x - 180}$ **C** $\frac{1}{x^3 - 61x - 180}$ **D** $\frac{1}{x^2 - 61x - 180}$

16. *(25)* **Error Analysis** A chemist weighs samples obtained from a production run. The weights of the samples are 12 g, 14 g, 65 g, 11 g, 15 g, 12 g, 13 g, 15 g, 14 g, and 11 g. She eliminated the outlier and calculated the mean of the data. What is her error?

$$\bar{x} = \frac{12 + 14 + 11 + 15 + 12 + 13 + 15 + 14 + 11}{10} = \frac{117}{10} = 11.7$$

17. *(36)* **Geometry** For the circle shown, the line $y_1 = \frac{4}{3}x + \frac{1}{3}$ passes through the center of the circle at (2, 3). How do you know that the equation of the tangent at (−2, 6) is parallel to y_1?



18. *(23)* **Predict** Use your knowledge of the factoring formulas to find the product of $(5x^6 + 3)$ and $(5x^6 - 3)$ without multiplying the polynomials. Explain your prediction.

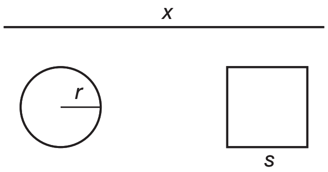
19. *(34)* **Analyze** Does the table of values represent a linear function? Explain your answer without graphing.

x	−4	0	2	8
y	11	5	2	−7

20. *(33)* **Sports** There are 4 teams in the National Football Conference East division. How many ways can the teams finish in first, second, third, and fourth place?

21. *(39)* **Write** Describe how to graph $y - 1 \leq 5x$ on a graphing calculator.

22. (31) **Multi-Step** Consider this question: Which is the larger area that a piece of rope of length x will enclose, a circle or a square?



a. Write a simplified expression for the area of the circle in terms of x.

b. Write a simplified expression for the area of the square in terms of x.

c. Which area is larger? Find the ratio of the areas.

d. Express the larger area as a percent of the smaller area. Round to the nearest percent.

23. (35) **Write** For the function $f(x) = x^2 + 12x + 36$, describe the relationship among the zero(s) of $f(x)$, the x-intercept(s) of $f(x)$, and the root(s) of $f(x) = 0$.

***24.** (27) **Graphing Calculator** Determine the vertex of the function $f(x) = x^2 - 6x + 1$ using a graphing calculator.

***25.** (40) **Biology** The metabolism rate of an 18 pound animal is approximately $73\sqrt[4]{625}$ Calories per day. Simplify the metabolism rate.

26. (24) **Work** In one month, Angela worked a total of 20 hours, some of it spent babysitting and the rest spent washing cars. She earned a total of \$96.25. If she gets paid \$5.50 per hour for babysitting and \$2.75 per hour for washing cars, how many hours of each type of work did she do?

27. (39) **Sports** In American football, a touchdown with a conversion is worth 7 points and a field goal is worth 3 points. Suppose the only scoring in a game was due to touchdowns with conversions and field goals. Write the inequality that represents how many of each could have occurred if the total number of points earned in the game was less than or equal to 105. Give a solution that would be on the boundary line if the inequality were graphed.

28. (21) **Coordinate Geometry** You are given equations of three lines on a coordinate grid that intersect to form a triangle. Explain in words how you could find the vertices of the triangle.

***29.** (40) **Verify** Supply two examples to show that b and the value of $\sqrt[n]{b}$ can be negative when n is odd.

***30.** (26) **Aviation** The Federal Aviation Administration tracks airline delays for airports around the country. From the beginning of the year to the end of March, there were 6,000 delays at Atlanta-Hartsfield Airport. By the end of May, there were 10,000 cumulative delays. Use this information to write an equation in slope-intercept form that could be used to predict the total delays throughout the year. Let x equal the month of the year as a number and y equal the cumulative delays.

INVESTIGATION

4

Understanding Cryptography

Computer Internet security is a top priority for governments and private companies. For information to be secure, it needs to be encrypted when sent out over the Web and decrypted when received. This way only the sender and receiver know the contents of the message. In the area of cryptography, different methods of keeping messages secure have been developed.

One of the simplest ways of encrypting a message is to use a substitution cipher. A number replaces each letter of the alphabet, so that all messages consist of a string of numbers. Without the cipher, it is difficult to decipher the message.

Use a spreadsheet to create a substitution cipher.

1. Use the cipher in the table below and transfer it to a spreadsheet. Input the letters A–Z in cells A1 to A26, and input the numeric cipher in B1 to B26.

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1	3	5	7	9	11	13	15	17	19	21	23	25	2	4	6	8	10	12	14	16	18	20	22	24	26

The cipher is the key to encrypting and decrypting messages. A spreadsheet automates the process of using such a cipher.

Math Reasoning

Generalize Suppose you wanted the cipher to include word spaces and special characters. How would you change the cipher?

2. Input the letters of the words SECRET MESSAGE in cells D1 through D13. The space between the words is not encrypted.
3. The VLOOKUP formula searches for a value in the leftmost column of the table (A) and then returns a value in the same row from a column you specify (B). Input this formula into cell E1: **=VLOOKUP(D1,$A$1:$B$26,2).** Then copy and paste the formula into cells E2 to E13.

	A	B	C	D	E
1	A	1		S	=VLOOKUP(D1,$A$1:$B$26,2)
2	B	3		E	
3	C	5		C	
4	D	7		R	
5	E	9		E	
6	F	11		T	
7	G	13		M	
8	H	15		E	
9	I	17		S	
10	J	19		S	
11	K	21		A	
12	L	23		G	
13	M	25		E	
14	N	2			
15	O	4			
16	P	6			
17	Q	8			
18	R	10			
19	S	12			
20	T	14			
21	U	16			
22	V	18			
23	W	20			
24	X	22			
25	Y	24			
26	Z	26			

4. Write the encrypted message as a string of numbers.

5. Use the spreadsheet to encrypt the following messages. List the message as a string of numbers. If necessary, copy and paste the spreadsheet formula to more cells.

a. QUADRATIC
b. ALGEBRA

The spreadsheet you created can be used to encrypt messages. A variation of this spreadsheet can be used to decrypt messages. Suppose your teacher has sent you the following message using the same encryption cipher.

8 16 17 26 14 15 17 12 11 10 17 7 1 24

You need to create a decryption spreadsheet.

6. **Multi-Step** Create a new spreadsheet.

a. Input the numeric cipher *in ascending order* to cells A1 to A26, and input the alphabet to B1 to B26. The letters will no longer be in alphabetical order.
b. Input the encrypted message above to cells D1 to D14.
c. Input this formula into cell E1: **=VLOOKUP(D1,$A$1:$B$26,2).** Then copy and paste the formula into cells E2 to E14.

	A	B	C	D	E
1	1	A		8	=VLOOKUP(D1,$A$1:$B$26,2)
2	2	N		16	
3	3	B		17	
4	4	O		26	
5	5	C		14	
6	6	P		15	
7	7	D		17	
8	8	Q		12	
9	9	E		11	
10	10	R		10	
11	11	F		17	
12	12	S		7	
13	13	G		1	
14	14	T		24	
15	15	H			
16	16	U			
17	17	I			
18	18	V			
19	19	J			
20	20	W			
21	21	K			
22	22	X			
23	23	L			
24	24	Y			
25	25	M			
26	26	Z			

d. What is your teacher's message?

An additional level of security can be added to the substitution cipher. This involves using matrix multiplication.

Online Connection
www.SaxonMathResources.com

7. Encrypt the following message using the cipher you previously produced.
TOP SECRET MESSAGE

8. Write the string of numbers in two 3×3 cipher matrices. The first number goes in the first row first column, the second number goes in the second row first column. The first matrix is started. Complete the two matrices.

$$\begin{bmatrix} 14 & 12 & \_\_ \\ 4 & \_\_ & \_\_ \\ 6 & \_\_ & \_\_ \end{bmatrix} \begin{bmatrix} 25 & \_\_ & \_\_ \\ 9 & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix}$$

9. **Graphing Calculator** Multiply the cipher matrices by the encoding matrix shown below.

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 0 & -5 \\ 2 & -3 & 2 \end{bmatrix}$$

10. Write the newly encrypted message as a string of numbers.

11. **Analyze** Why is this encryption method more secure than a substitution cipher?

Decrypting a message using the cipher-matrix system requires that you know both the encrypted and the encoding matrices. Suppose your teacher has a message encoded in the encrypted matrices:

$$\begin{bmatrix} 38 & -50 & -8 \\ 39 & -51 & -71 \\ 19 & -20 & -9 \end{bmatrix} \begin{bmatrix} 23 & -28 & -24 \\ 41 & -57 & -10 \\ 37 & -47 & 9 \end{bmatrix} \begin{bmatrix} 9 & -9 & 18 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

12. **Graphing Calculator** Follow these steps to decrypt the message.

a. Store the encoding matrix in your calculator as matrix [A].
b. Store the encrypted matrices as separate matrices in the calculator.
c. Multiply each encrypted matrix by the inverse of the encoding matrix.
d. Write the partially decrypted message as a string of numbers.
e. Use the decryption spreadsheet to decipher the message. What is the message?

Investigation Practice

Graphing Calculator **Use the cipher-matrix system to decrypt these messages. Use the encoding matrix from problem 9.**

a. $$\begin{bmatrix} 16 & -17 & -30 \\ 25 & -30 & -45 \\ 25 & -29 & -18 \end{bmatrix} \begin{bmatrix} 31 & -43 & -87 \\ 11 & -12 & -25 \\ 33 & -46 & -20 \end{bmatrix} \begin{bmatrix} 9 & -9 & 18 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

b. $$\begin{bmatrix} 38 & -50 & 22 \\ 55 & -75 & 65 \\ 17 & -21 & -34 \end{bmatrix} \begin{bmatrix} 38 & -52 & -12 \\ 27 & -37 & -91 \\ 51 & -68 & 63 \end{bmatrix} \begin{bmatrix} 22 & -22 & 44 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$