LESSON 1

Points, Lines, and Planes

Warm Up

Start off each lesson by practicing prerequisite skills and math vocabulary that will make you more successful with today's new concept.

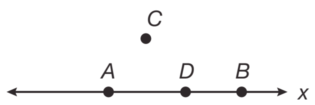
1. (SB 13) **Vocabulary** The __________ plane contains the x-axis and the y-axis.
2. (SB 4) Kira needs to buy a piece of pipe that is 40% longer than the 7-inch piece she already has. What length of pipe does Kira need?
3. (SB 6) Simplify $\sqrt[4]{81}$.
4. (SB 14) Evaluate $\frac{4(n+6)}{2n}$ for $n = 2$.

New Concepts

In geometry, a **definition** of a term is a statement that defines a mathematical object. Definitions usually reference other mathematical terms. A basic mathematical term that is not defined using other mathematical terms is called an **undefined term**. In geometry, points, lines, and planes are undefined terms that are the building blocks used for defining other terms.

A **point** names a location and has no size. It is represented by a dot and labeled using a capital letter, such as P.

A **line** is a straight path that has no thickness and extends forever. There are an infinite number of points on a line. A line is named using either a lowercase letter or any two points on the line. Two possible names for the line shown in the diagram are $\overleftrightarrow{AB}$ and line x.

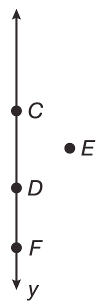


Hint

A ruler can be used to determine if points are collinear or noncollinear. A ruler can always connect two points, so two points are always collinear. Three points are only collinear if you can use the ruler to draw a line passing through all three of them.

Any set of points that lie on the same line are called **collinear** points. In the diagram, A, B, and D are collinear. If points do not lie on the same line, they are **noncollinear**. Points A, B, and C are noncollinear.

Example 1 Identifying Lines and Collinear Points



a. Give two different names for the line.

SOLUTION

Two possible names for the line are line y and $\overleftrightarrow{CD}$. The order of the points does not matter, so $\overleftrightarrow{DC}$ would also be correct.

b. Name three collinear points and three noncollinear points.

SOLUTION

Points C, D, and F are collinear. Points C, D, and E are noncollinear.

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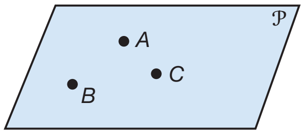
Math Reasoning

Model What are some common objects that are planes?

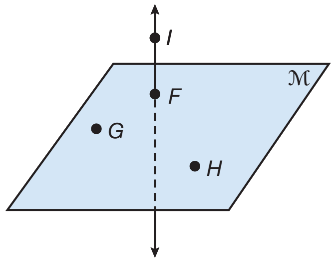
A **plane** is a flat surface that has no thickness and extends forever. A plane is named using either an uppercase letter or three noncollinear points that lie in the plane. The plane in the diagram below could be called plane $\mathcal{P}$ or plane ABC.



Lines or points that are in the same plane are said to be **coplanar**. If there is no plane that contains the lines or points, then they are **noncoplanar**. Space is the set of all points. Therefore, space includes all lines and all planes.

Example 2 Identifying Planes

What are two different names for this plane?



SOLUTION

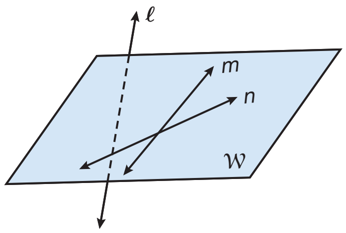
Two possible names for the plane are plane FGH or plane $\mathcal{M}$.

Each day brings you a **New Concept** where a new topic is introduced and explained through thorough **Examples** — using a variety of methods and real-world applications.

You will be reviewing and building on this concept throughout the year to gain a solid understanding and ensure mastery on the test.

Example 3 Identifying Coplanar Lines

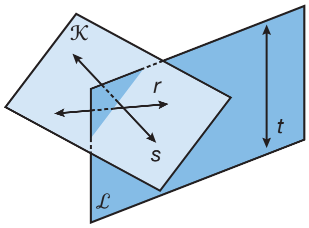
a. Identify the coplanar and noncoplanar lines in the diagram.



SOLUTION

Lines m and n are coplanar. Line ℓ is noncoplanar with lines m and n.

b. Identify the coplanar and noncoplanar lines in the diagram.



SOLUTION

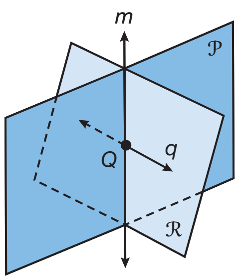
Lines r and s are coplanar. Line t is noncoplanar with lines r and s.

Math Reasoning

Model Can two planes have no intersections at all? What common objects illustrate what this might look like?

An **intersection** is the point or set of points in which two figures meet. When two lines intersect, their intersection is a single point. When two planes intersect, their intersection is a single line. If a line lies in a plane, then their intersection is the line itself. If the line does not lie in the plane, then their intersection is a single point.

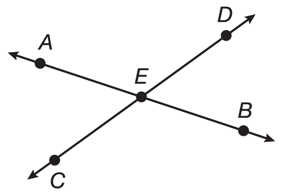
Lines q and m intersect at point Q. Plane $\mathcal{R}$ intersects plane $\mathcal{P}$ at line m. The intersection of plane $\mathcal{R}$ and line m is line m. Line q intersects planes $\mathcal{P}$ and $\mathcal{R}$ at point Q.



In some lessons, **Explorations** allow you to go into more depth with the mathematics by investigating math concepts with manipulatives, through patterns, and in a variety of other ways.

Example 4 Intersecting Lines and Planes

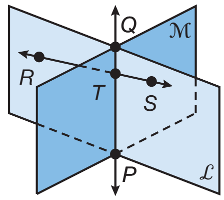
a. What is the intersection of $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$?



SOLUTION

The intersection of $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ is point E.

b. What is the intersection of $\overleftrightarrow{PQ}$ and $\overleftrightarrow{RS}$? What is the intersection of planes $\mathcal{M}$ and $\mathcal{L}$?



SOLUTION

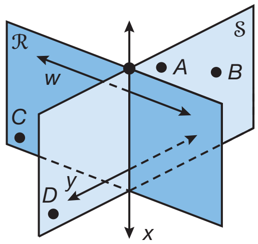
The intersection of $\overleftrightarrow{PQ}$ and $\overleftrightarrow{RS}$ is point T. The intersection of the planes $\mathcal{M}$ and $\mathcal{L}$ is $\overleftrightarrow{PQ}$.

The **Lesson Practice** lets you check to see if you understand today's new concept.

The italic numbers refer to the Example in this lesson in which the major concept of that particular problem is introduced. You can refer to the lesson examples if you need additional help.

Lesson Practice

Identify each of the following from the diagram.



a. All of the lines. *(Ex 1)*

b. A pair of collinear points. *(Ex 1)*

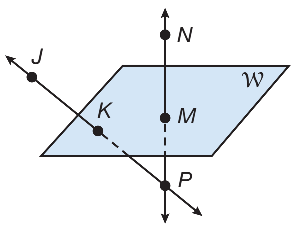
c. All of the planes. *(Ex 2)*

d. Three coplanar points. *(Ex 2)*

e. Two coplanar lines. *(Ex 3)*

f. A pair of noncoplanar lines. *(Ex 3)*

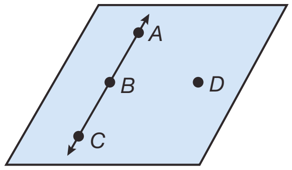
Use the diagram to answer each question.



g. What is the intersection of $\overleftrightarrow{JK}$ and $\overleftrightarrow{NM}$? *(Ex 4)*

h. What is the intersection of $\overleftrightarrow{JK}$ and plane $\mathcal{W}$? *(Ex 4)*
What is the intersection of $\overleftrightarrow{NP}$ and plane $\mathcal{W}$?

Practice Distributed and Integrated



1. *(1)* In the diagram, which set of three points are collinear? Which point cannot be included in a collinear set of three points?

2. *(1)* Can two points be noncollinear?

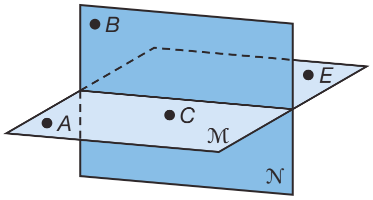
3. *(1)* Can two noncoplanar lines intersect?

4. *(1)* Name three undefined basic figures of geometry.

5. *(1)* What term describes two lines that have a point in common?

6. *(1)* **Generalize** Can three points be noncoplanar? Explain.

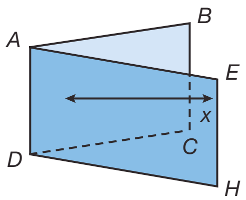
7. *(1)* Name the coplanar points shown on plane $\mathcal{M}$.



The ***italic numbers*** refer to the lesson(s) in which the major concept of that particular problem is introduced. You can refer to the examples or practice in that lesson, if you need additional help.

8. *(1)* **Write** The floor, ceiling, and walls of a room are all parts of planes. How many planes intersect the plane of the floor in your classroom? What geometric figures are formed where the planes intersect? Explain.

Use the diagram to answer problems 9–11.



9. *(1)* What is the intersection between the planes?

10. *(1)* **Justify** Are $\overleftrightarrow{AD}$, $\overleftrightarrow{CD}$, and $\overleftrightarrow{CH}$ coplanar? Explain.

11. *(1)* State the intersection point of $\overleftrightarrow{BC}$ and line x.

12. *(1)* **Multiple Choice** Which statement is true?
A Any two lines are coplanar.
B Any three lines are coplanar.
C Any two intersecting lines are coplanar.
D Any two perpendicular lines are noncoplanar.

13. *(1)* How many different points can a line contain?

14. *(1)* How many different lines can a plane contain?

15. **Multiple Choice** Which statement is true?
(1)
A The intersection of two lines forms a one-dimensional figure.
B The intersection of two planes forms a two-dimensional figure.
C The intersection of two planes forms a one-dimensional figure.
D The intersection of two lines forms a two-dimensional figure.

16. **Error Analysis** Miri used two points to name a plane. What mistake did Miri make in naming the plane?
(1)

17. Evaluate: $5 - (7 + 8) \div 5 + (-2)^3$
(SB 1)

18. Name the property of addition shown by this equation:
(SB 2)
$(+3) + (-4) = (-4) + (+3)$

19. **Error Analysis** Jacob said that -3 is a rational number. Aaron said that -3 is an irrational number. Who is correct? Explain.
(SB 3)

20. **Justify** Is $0.\overline{3}$ irrational? Why or why not?
(SB 3)

21. **Baseball** A ball player is at bat 55 times and hits the ball 33 times. What ratio of his times at bat does he not hit the ball?
(SB 41)

22. Evaluate: $(-3)^3 - \left(\frac{1}{3}\right)^{-3}$
(SB 1)

23. Simplify: $2\sqrt{12} + 6\sqrt{27}$
(SB 6)

24. **Construction** A concrete pad has dimensions 9 feet by 9 feet by 4 inches. How many cubic yards of concrete does it contain?
(SB 9)

25. **Meteorology** Determine the mean and median values for the weekly rainfall data.
(SB 11)

Day	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Rainfall (mm)	0.5	2	0	4	2.5	5	7

26. **Physics** On average, the acceleration due to gravity is $9.807\ m/s^2$. A science student measured it as $9.760\ m/s^2$. To the nearest hundredth of a percent, what was the student's percent error?
(SB 10)

27. **Coordinate Geometry** Plot these points on the coordinate plane: (2, 1), (−1, −1), (0, 0), and (3, −1).
(SB 13)

28. **Algebra** Evaluate the expression $xy^{-2} + \frac{x}{y}$, where $x = -2$, $y = \frac{1}{2}$.
(SB 14)

29. **Algebra** Transform the formula $I = Prt$ to solve for r.
(SB 16)

30. **Algebra** State the slope of the line $3x + 4y - 15 = 0$.
(SB 19)

In the **Practice,** you will review today's new concept as well as math you learned in earlier lessons. By practicing problems from many lessons every day, you will begin to see how math concepts relate and connect to each other and to the real world.

Also, because you practice the same topic in a variety of ways over several lessons, you will have "time to learn" the concept and will have multiple opportunities to show that you understand.

The mixed set of Practice is just like the mixed format of your state test. You'll be practicing for the "big" test every day!

LESSON 2 Segments

Warm Up

1. (1) **Vocabulary** Points that lie on the same line are called ________ points.
2. (SB 15) Solve for x: $5x + 6 = 2x - 5$
3. (SB 1) Simplify: $5(2x - 6) + 3x - 7$

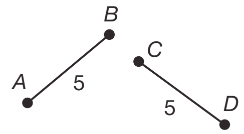
New Concepts

A **line segment** is a part of a line consisting of two **endpoints** and all points between them.

> **Math Language**
>
> Point C is **between** points A and B if A, B, and C are collinear and $AC + CB = AB$.

The diagram above depicts a line segment with endpoints A and B. A segment is named by its two endpoints in either order with a straight segment drawn over them. This segment could be called either $\overline{AB}$ or $\overline{BA}$.

Two geometric objects that have the same size and shape are **congruent**. **Congruent segments** have the same length.

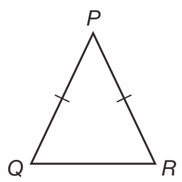


> **Caution**
>
> When comparing segments or other geometric figures, congruence statements are used. When the length of segments are being compared, or any other measurements that can be expressed as numbers, an equal sign is used.

In this figure, $\overline{AB}$ and $\overline{CD}$ are congruent. As shown on the diagram, they both have a **length** of 5 units.

A **congruence statement** shows that two segments are congruent. The symbol $\cong$ is read "is congruent to." The congruence statement for the segments above is $\overline{AB} \cong \overline{CD}$.

In a diagram, congruent segments are shown with **tick marks**. The diagram below shows congruent segments indicated by tick marks.



> **Hint**
>
> For more on the Reflexive, Symmetric, and Transitive Properties of Equality, see the Skills Bank at the back of this textbook.

The following properties apply to all congruent segments.

Reflexive Property of Congruence	$\overline{AB} \cong \overline{AB}$
Symmetric Property of Congruence	If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$.
Transitive Property of Congruence	If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.

Example 1 Using Properties of Equality and Congruence

Identify the property that justifies each statement.

a. $\overline{WX} \cong \overline{YZ}$, so $\overline{YZ} \cong \overline{WX}$

SOLUTION

Symmetric Property of Congruence

b. $\overline{PQ} \cong \overline{RS}$ and $\overline{RS} \cong \overline{TU}$, so $\overline{PQ} \cong \overline{TU}$

SOLUTION

Transitive Property of Congruence

c. $\overline{GH} \cong \overline{GH}$

SOLUTION

Reflexive Property of Congruence

Reading Math

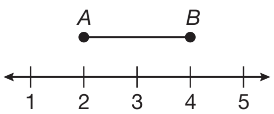
The length of $\overline{AB}$ is denoted AB.

A ruler can be used to measure the lengths of segments. The points on a ruler correspond with the points on a line segment. This concept is presented in the Ruler Postulate. A **postulate** is a statement that is accepted as true without proof.

Postulate 1: Ruler Postulate
The points on a line can be paired in a one-to-one correspondence with the real numbers such that: **1.** Any two given points can have coordinates 0 and 1. **2.** The distance between two points is the absolute value of the difference of their coordinates.

Distance is the measure of the segment connecting two points. The distance between two points can be represented by those two points with no segment symbol. For example, AB means "the distance between A and B."

Distance is always positive, so absolute values are used to calculate distances.



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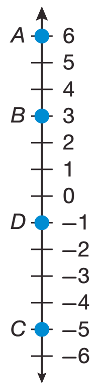
$|\text{Point } A - \text{Point } B| = |2 - 4| = |-2| = 2$

The distance from point A to point B is 2.

Example 2 Finding Distance on a Number Line

Find each distance.



a. AB

SOLUTION

$$AB = |6 - 3|$$
$$= |3|$$
$$= 3$$

b. BC

SOLUTION

$$BC = |3 - (-5)|$$
$$= |8|$$
$$= 8$$

c. CD

SOLUTION

$$CD = |(-5) - (-1)|$$
$$= |-4|$$
$$= 4$$

d. AC

SOLUTION

$$AC = |6 - (-5)|$$
$$= |11|$$
$$= 11$$

Hint

Postulates are statements that are accepted as true without proof. See the Postulates and Theorems section in the back of this book for a complete list of postulates in this program.

In the example above, notice that $AC = AB + BC$. This is not a coincidence.

Postulate 2: Segment Addition Postulate
If B is between A and C, then $AB + BC = AC$.

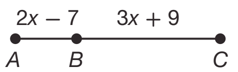
Example 3 Using the Segment Addition Postulate

a. Point S lies on $\overline{RT}$ between R and T. $RS = 12$ and $RT = 31$. Find ST.

SOLUTION

$RT = RS + ST$	Segment Addition Postulate
$31 = 12 + ST$	Substitute.
$19 = ST$	Subtract 12 from both sides.

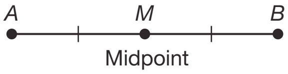
b. Find AC in terms of x.



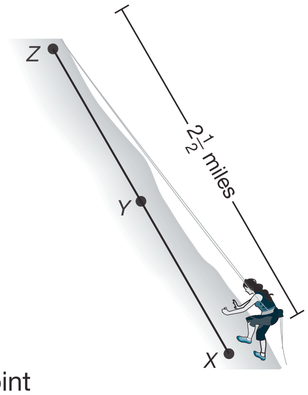
SOLUTION

$AC = AB + BC$	Segment Addition Postulate
$AC = (2x - 7) + (3x + 9)$	Substitute.
$AC = 5x + 2$	Simplify.

The **midpoint** of a segment is the point that divides the segment into two congruent parts. If M is the midpoint of $\overline{AB}$, then $AM = MB$.



Example 4 Application: Hiking



A hiker is traveling up a mountain towards the summit. The distance from the base of the mountain to the summit is 2.5 miles, as shown. How far will she have traveled when she reaches the midpoint (Y) of the hike?

SOLUTION

$XZ = XY + YZ$	Segment Addition Property
$XY = YZ$	Definition of midpoint
$XZ = XY + XY$	Substitute XY for YZ.
$2.5 = 2(XY)$	Substitute.
$1.25 = XY$	Divide both sides by 2.

The hiker will have traveled 1.25 miles when she reaches the midpoint.

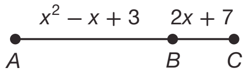
> **Hint**
>
> Notice in Examples 3 and 4 that each step of the solution is explained to the right. This will help prepare you for performing algebraic and geometric proofs later in the program.

Lesson Practice

a. Identify the property that justifies the statement, $\overline{KL} \cong \overline{MN}$, so $\overline{MN} \cong \overline{KL}$.
(Ex 1)

b. Find the distance between the points A and B.
(Ex 2)

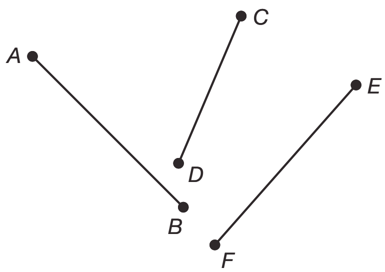
c. Find AC in terms of x.
(Ex 3)



d. The drive from Seattle to San Francisco is 811 miles. How many miles is the midpoint from either city?
(Ex 4)

Practice Distributed and Integrated

1. Measure the line segments at right to determine which segments are congruent.
(2)



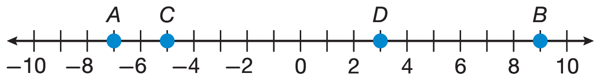
2. Estimate $\sqrt{32}$ to the nearest tenth.
(SB 6)

3. What property is illustrated by this statement?
(SB 2)

If a and b are real numbers, then $a + b = b + a$.

4. **Multiple Choice** What is the least number of points that can determine a plane?
(1)
A 1 **B** 2
C 3 **D** 4

Calculate the length of each segment using the diagram below.



5. $\overline{AD}$
(2)

6. $\overline{BC}$
(2)

7. $\overline{DA}$
(2)

8. $\overline{AC}$
(2)

9. **Data Analysis** The numbers in this set are the math test scores for a class. Find the mode.
(SB 11)
$\{62, 53, 74, 55, 66, 72, 80, 83, 83, 86, 92, 93, 40, 51, 61, 71\}$

10. Evaluate $3m^2 - 5m + 17$ for $m = -3$.
(SB 14)

11. In what quadrant is point S if its coordinates are $(2, -4)$?
(SB 13)

12. Find the median of the set $\{2, 2, 3, 5, 6, 7, 7, 8, 9\}$.
(SB 11)

13. **Algebra** A, B, and C are collinear, $AB = 5x - 19$, and $BC = 3x + 4$. Find an expression for AC if B is between A and C.
(2)

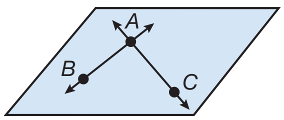
14. **City Planning** The city is planning to install streetlights and wants five lights along a walkway of 60 yards. If there is a light at the beginning and at the end of the walkway and the lights are evenly spaced, what is the distance between each light?
(2)

15. **Error Analysis** Sunil stated that three points determine a unique plane. Explain Sunil's error and and give a corrected statement.
(1)

16. **Algebra** Suppose $AB = 3x$, $BC = 2y + 16$, $AC = 60$, and B is the midpoint of AC. Find the values of x and y.
(2)

17. **Analyze** Points D, E, and F are collinear with E between D and F. $DE = 15$, $EF = x + 17$, and $DF = 3x - 10$. Find EF and DF.
(2)

18. **Write** Describe how equality and congruence are used to describe two line segments and their lengths.
(1, 2)



19. Are points A, B, and C collinear, coplanar, both, or neither?
(1)

20. How many points are required to determine a line?
(1)

21. **Multiple Choice** Which of these statements is false?
(1)
A Two distinct points determine a line.
B Three noncollinear points determine a plane.
C Two noncoplanar lines determine a space.
D Four noncoplanar points determine a space.

22. *(1)* Lines *MN* and *PQ* intersect at point *E*. Name two sets of three collinear points.

23. *(SB 18)* Factor: $2x^2 - 16x - 66$

24. *(SB 18)* Expand: $(x - 4)(x + 7)$

25. *(SB 9)* **Carpentry** A piece of wood that is 8 feet long needs to be 7 feet 4 inches long. How much has to be cut off? Express your answer in feet.

26. *(SB 6)* Simplify: $\frac{-36x^{-4}y^5}{12x^2y^{-3}}$. Express your answer with positive exponents.

27. *(SB 4)* **Sales Tax** If sales tax is 6%, estimate the sales tax on an item that retails for \$48.99.

28. *(SB 5)* Estimate the sum to the nearest whole number.
$1.8 + 2.345 + 0.65 + 13.56$

29. *(SB 9)* **Multi-Step** There are 1000 liters in a cubic meter and approximately 3.85 liters in one gallon. A swimming pool measures 8 meters wide by 4 meters long by 1.5 meters deep. Approximately how many gallons of water can the pool hold?

30. *(SB 9)* **Physics** In a free fall, acceleration due to gravity is approximately 9.8 m/s^2. Convert this rate into ft/s^2.

LESSON 3

Angles

Warm Up

1. (2) **Vocabulary** Two figures that have the same size and shape are called __________ figures.

2. (SB 1) Simplify: $\frac{1}{2}\left(\frac{2^2}{2} - 2\right)$

3. (SB 3) Convert $\frac{22}{7}$ into a decimal number. Is it a terminating decimal number, repeating decimal number, or non-terminating and non-repeating decimal number?

New Concepts

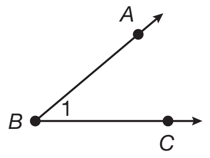
A **ray** is a part of a line that starts at an endpoint and extends infinitely in one direction.

A ray is named by its endpoint and any other point on the ray. For example, the ray in the diagram is called $\overrightarrow{AB}$, which is read "ray *AB*."

Two rays that have a common endpoint and form a line are called **opposite rays**.

Rays $\overrightarrow{YX}$ and $\overrightarrow{YZ}$ are opposite rays.

An **angle** is a figure formed by two rays with a common endpoint. The common endpoint is the angle's **vertex**. The rays are the **sides** of the angle. The sides of this angle are $\overrightarrow{BA}$ and $\overrightarrow{BC}$. The vertex is *B*.

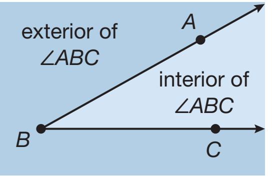


Caution

An angle can be named by its vertex only if it is clear that there is only one angle at the vertex.

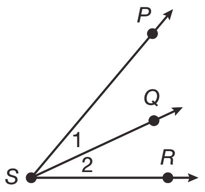
An angle can be named in several different ways: by its vertex, by a point on each ray and the vertex, or by a number. For example, the angle in the diagram could be called $\angle B$, $\angle ABC$, $\angle CBA$, or $\angle 1$.

The exterior of an angle is the set of all points outside the angle. The interior of an angle is the set of all points between the sides of an angle.



Online Connection
www.SaxonMathResources.com

Example 1 Naming Angles and Rays



a. Name three rays in the diagram.

SOLUTION

$\overrightarrow{SP}$, $\overrightarrow{SQ}$, and $\overrightarrow{SR}$

b. Name three angles in the diagram.

SOLUTION

$\angle PSQ$ or $\angle 1$, $\angle QSR$ or $\angle 2$, and $\angle PSR$.

c. Could $\angle PSQ$ also be referred to as $\angle S$?

SOLUTION

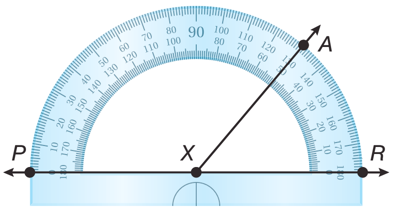
No, there are three different angles with S as a vertex.

A **protractor** is a tool used to measure angles. Unlike segments, angles are measured in **degrees**. One degree is a unit of angle measure that is equal to $\frac{1}{360}$ of a circle.

Postulate 3: Protractor Postulate

Given a point X on $\overleftrightarrow{PR}$, consider rays $\overrightarrow{XP}$ and $\overrightarrow{XR}$, as well as all the other rays that can be drawn with X as an endpoint, on one side of $\overleftrightarrow{PR}$. These rays can be paired with the real numbers from 0 to 180 such that:

1. $\overrightarrow{XP}$ is paired with 0, and $\overrightarrow{XR}$ is paired with 180.
2. If $\overrightarrow{XA}$ is paired with a number c and $\overrightarrow{XB}$ is paired with a number d then $m\angle AXB = |c - d|$.



Math Language

Analyze Rays that form a **straight angle** are called opposite rays.

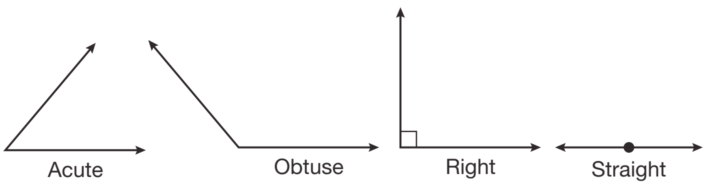
Angles are classified according to their angle measure.

An **acute angle** measures greater than 0° and less than 90°.

An **obtuse angle** measures greater than 90° and less than 180°.

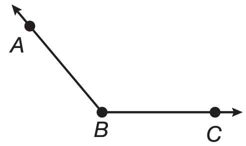
A **right angle** measures exactly 90°. A box drawn at the vertex of an angle shows that it is a right angle, as shown in the diagram.

A **straight angle** measures exactly 180°.



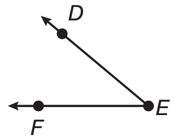
Example 2 Measuring and Classifying Angles

a. Use a protractor to measure $\angle ABC$, then classify the angle.



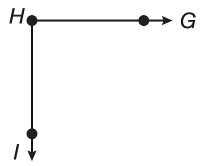
SOLUTION $\angle ABC$ measures 130°, so it is an obtuse angle.

b. Use a protractor to measure $\angle DEF$, then classify the angle.



SOLUTION $\angle DEF$ measures 40°, so it is an acute angle.

c. Use a protractor to measure $\angle GHI$, then classify the angle.



SOLUTION $\angle GHI$ measures 90°, so it is a right angle.

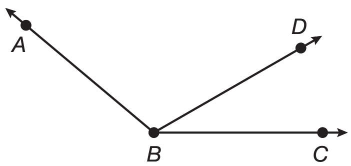
Angles can be added in the same way that segments are added.

> **Reading Math**
>
> The measure of $\angle ABD$ is represented by adding a lowercase 'm' to the notation, that is $m\angle ABD$.

Postulate 4: The Angle Addition Postulate

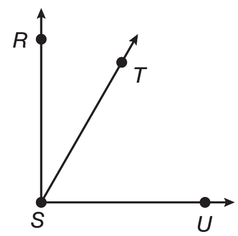
If point D is in the interior of $\angle ABC$, then

$m\angle ABD + m\angle DBC = m\angle ABC$.



Example 3 Using the Angle Addition Postulate

The measure of $\angle RST = 22°$ and $m\angle TSU = 69°$. Find $m\angle RSU$. Classify the angle.



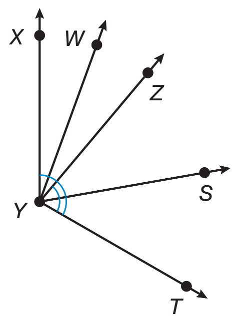
SOLUTION

$m\angle RST + m\angle TSU = m\angle RSU$	Angle Addition Postulate
$22° + 69° = m\angle RSU$	Substitute.
$91° = m\angle RSU$	Simplify.

$\angle RSU$ is an obtuse angle.

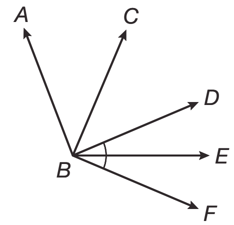
To **bisect** a figure is to divide it into two congruent parts. An **angle bisector** is a ray that divides an angle into two **congruent angles**. Congruent angles have the same measure. They are marked with **arc marks**, as shown in the diagram.

$\angle XYW \cong \angle WYZ$ and $\angle ZYS \cong \angle SYT$.



Example 4 Using Angle Bisectors and Congruence Marks

The measure of $\angle ABC = 44°$. $\overrightarrow{BC}$ bisects $\angle ABD$. The measure of $\angle EBF = 23°$. Find the measure of $\angle CBE$.



SOLUTION Since $\overrightarrow{BC}$ bisects $\angle ABD$, it divides $\angle ABD$ into two congruent angles. So, $\angle ABC \cong \angle CBD$ and $m\angle ABC = m\angle CBD$. Since $m\angle ABC = 44°$, $m\angle CBD = 44°$.

Using the congruence marks in the diagram, $\angle DBE \cong \angle EBF$, so $m\angle DBE = m\angle EBF$. Since $m\angle EBF = 23°$, $m\angle DBE = 23°$.

$m\angle CBE = m\angle CBD + m\angle DBE$	Angle Addition Postulate
$= 44° + 23°$	Substitute.
$= 67°$	Add.

The measure of $m\angle CBE$ is 67°.

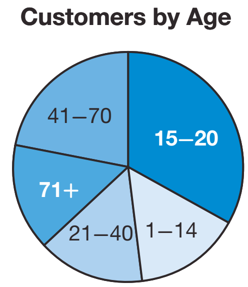
Example 5 Application: Interpreting Statistics

> **Hint**
>
> For more about displaying data using circle graphs, see the Skills Bank at the back of this textbook.

Louis runs a restaurant. He knows that he has about 900 customers a day. The circle graph in the diagram shows what percentage of his customers fall into the given age brackets. He wants to know exactly how many of his customers are between ages 15 and 20. Use a protractor to measure the angle and find the number of Louis's customers that fall into the 15–20 age bracket.

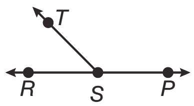
SOLUTION

Measure the angle of the sector that represents 15–20-year-old customers. The sector has an angle measure of 120°. Since an entire circle is 360°, this is $\frac{120}{360} = \frac{1}{3}$ of the circle. One third of Louis's customers is $\left(\frac{1}{3}\right)(900) = 300$ customers.

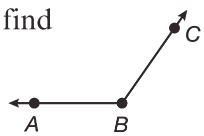


Lesson Practice

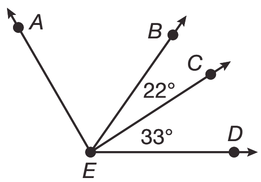
a. Name three rays and three angles in the diagram.
(Ex 1)



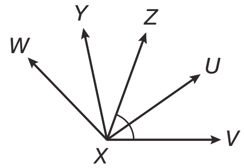
b. Classify $\angle ABC$ and use a protractor to find the measure of it.
(Ex 2)



c. Determine $m\angle AEB$ if $m\angle AED = 120°$.
(Ex 3)



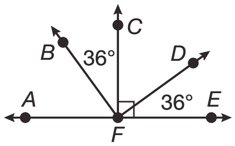
d. *(Ex 4)* The measure of $\angle WXY = 32°$. $\overline{XY}$ bisects $\angle WXZ$. The measure of $\angle UXV = 35°$. Find the measure of $\angle YXU$.



e. *(Ex 5)* A survey shows that 10% of students in a class did not eat lunch. What would be the degree measure of an angle indicating these students on a circle graph?

Practice Distributed and Integrated

Use the diagram to classify and find the measure of each angle.



1. *(3)* $\angle AFC$

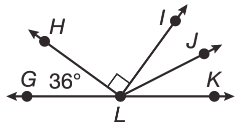
2. *(3)* $\angle CFD$

3. *(3)* $\angle BFD$

4. *(3)* $\angle AFD$

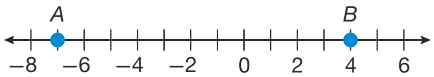
5. *(3)* **Multiple Choice** $\overrightarrow{LJ}$ bisects $\angle ILK$. What is the measure of $\angle GLJ$?

A 27° **B** 117°
C 54° **D** 153°



6. *(3)* What is the measure of a straight angle?

7. *(2)* Find *AB*.



8. *(SB 4)* Which of the following triplets are equivalent?

A $\frac{1}{2}$, 50%, 5:10 **B** $\frac{20}{20}$, 100%, 7:7

C $\frac{30}{36}$, $83.\overline{3}$%, 5:6 **D** All of the above

9. *(1)* What is the fewest number of points that can determine a line?

10. *(SB 7)* How many significant digits are there in 10,001?

11. *(2)* **Error Analysis** Jackson said, "Congruent segments are the same as equal segments." Explain how he has misunderstood congruence and equality.

xy^2 **12.** *(3)* **Algebra** Ray *BD* bisects $\angle ABC$. If $m\angle ABD = (x^2 + x + 12)°$ and $m\angle DBC = (x^2 + 3x + 4)°$, find x and $m\angle ABC$.

13. *(2)* **Landscaping** A homeowner wants to put up a fence around her square yard. She has 8 posts, which must be equally spaced. If the perimeter of her yard is 16 meters, how far apart must the posts be?

14. (2) **Verify** If $\overline{MN} \cong \overline{NH}$ and $\overline{NH} \cong \overline{ES}$, then $\overline{MN} \cong \overline{ES}$. What property justifies this?

15. (2) Points A, B, and C are collinear. Point B is between points A and C. $AB = 12$, $AC = 7x + 5$ and $BC = 4x - 1$. Find x.

16. (2) Find the midpoint of the segment connecting points with coordinates -142 and 53 on a number line.

17. (1) **Write** If four points are collinear, are they also coplanar? Explain.

18. (SB 4) The ratio of boys to girls at the camp is 14:17. If there are 186 children at the camp altogether, how many are boys?

19. (SB 18) **Algebra** Factor: $10x^2 + x - 21$

20. (SB 11) The mean of these six numbers is 20.38. What is the missing number?

25.14, 17.22, _____, 23.04, 20.21, 21.27

21. (SB 13) **Surveying** A surveyor measured a building lot. She recorded the corner points of the lot as $M(-80, 210)$, $N(-80, -120)$, $P(130, 210)$, and $Q(130, -120)$. The coordinates represent the distances, in feet, from the axes of a coordinate grid. Calculate the area of the lot.

22. (SB 22) **Farming** A farmer wishes to plant a field with flax. According to his budget, the total cost of planting and harvesting cannot exceed $12,000. The planting and harvesting costs are $0.015/square yard for flax. What is the maximum area the farmer could plant with flax while keeping within the budget?

23. (SB 17) How does the graph of $f(x)$ compare to the graph of $f(x) + 3$?

24. (SB 16) Solve the equation $\sqrt{6g} = (j + y)$ for g.

25. (SB 22) A stone is dropped from a cliff into a river. The function $h(t) = 82 - 4.9t^2$ gives the height h of the stone in meters t seconds after it is dropped. What is the height of the stone, to the nearest tenth of a meter, after 1.7 seconds?

26. (SB 7) **Solar System** Sirius, the brightest visible star in the sky, is about 8.6 light years away from Earth. One light year is 9.46×10^{15} kilometers. Find the distance between Earth and Sirius in kilometers. Express your answer in scientific notation with 3 significant figures.

27. (SB 4) The instructions on a juice concentrate container read "Five parts water, 1 part concentrate." How much juice can you make if you have 500 mL of concentrate?

28. (SB 3) To what sets of numbers does the number -8 belong?

29. (SB 3) Is the square root of any prime number a real number?

30. (SB 1) **Chemistry** Liquid nitrogen is extremely cold. The freezing point for nitrogen is approximately $-346°F$ and the boiling point for nitrogen is approximately $-320°F$. Use this data to find the range.

LAB

1

Congruent Segments and Angles

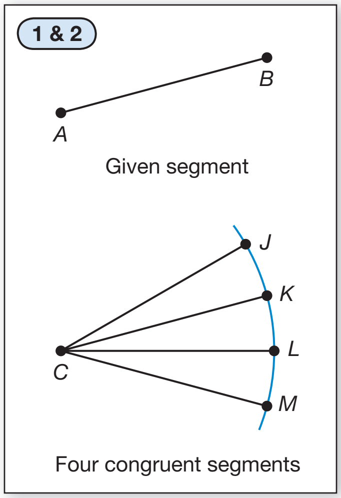
Construction Lab 1 *(Use with Lesson 3)*

Math Language

Construction is a method of creating a figure that is considered to be mathematically precise.

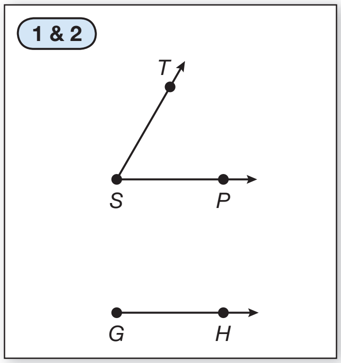
You can use a compass and a straightedge to construct figures. In Lessons 2 and 3, you learned about congruent segments and angles. The first figure you will construct using tools of geometry is a segment congruent to a given segment.

1. Begin with a given segment. Label the endpoints of the segment *A* and *B*. Set the compass to have radius *AB*.
2. Draw point *C* not on the segment and sweep an arc from that point using the compass setting from step 1. Because congruency does not depend on orientation, any of the points on the arc may serve as the other endpoint of the second segment.

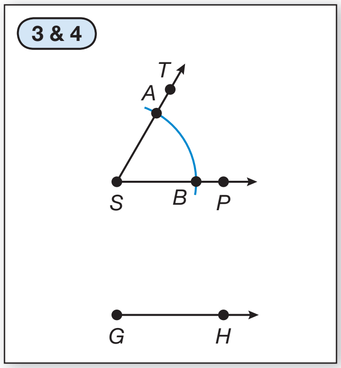


You can use the same tools you used to construct a segment to construct an angle congruent to a given angle.

1. To construct congruent angles, begin by drawing an angle of any measure and label it ∠*TSP*.
2. Draw $\overrightarrow{GH}$. This ray will be one side of the congruent angle.

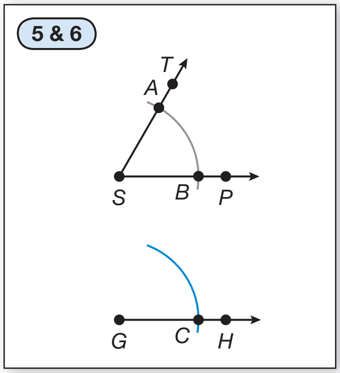


3. Use a compass to draw an arc across ∠*TSP* centered at *S*. Be sure that the arc intersects both $\overrightarrow{ST}$ and $\overrightarrow{SP}$.
4. Label the points of intersection *A* and *B*.



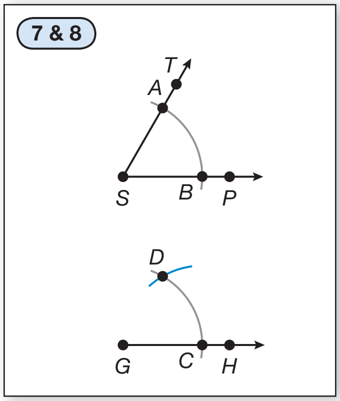
5. Use the same compass setting to draw an arc centered at *G*.

6. Label the point of intersection of this arc and $\overrightarrow{GH}$ as *C*.



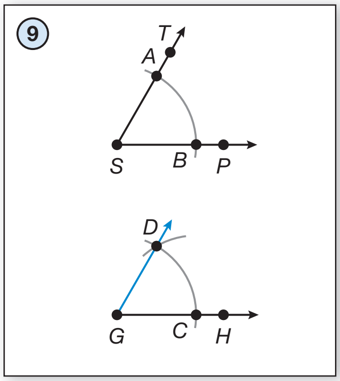
7. Adjust the compass setting to the distance between points *A* and *B*. Use this setting to draw an arc centered at *C* which intersects the first arc you drew on $\overrightarrow{GH}$.

8. Label the point of intersection of the two arcs as point *D*.



9. Draw $\overrightarrow{GD}$.

$\angle DGH \cong \angle TSP$



Math Reasoning

Verify Use a protractor to verify that the given angle and the angle you constructed are congruent.

Lab Practice

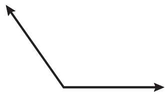
Practice constructing congruent segments and angles in pairs. Sketch a segment and an angle, then trade sketches with a partner. Next, using your compass, construct a segment and angle congruent to the ones given.

LESSON 4

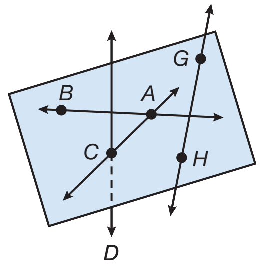
Postulates and Theorems About Points, Lines, and Planes

Warm Up

1. (2) **Vocabulary** The ______________ states that $a = a$ for any value of a.
2. (3) Classify the angle at right.



3. (1) **Multiple Choice** Which of the following is not a line on the plane ABC?
   A $\overleftrightarrow{AB}$ **B** $\overleftrightarrow{CD}$
   C $\overleftrightarrow{BC}$ **D** $\overleftrightarrow{GH}$



New Concepts

The postulates below, all of which help define the properties of points, lines, and planes, are essential to proving and understanding many of the most important theorems of geometry.

Postulate 5
Through any two points there is exactly one line.

Hint

Unlike postulates, theorems must be proved to be accepted as true. See the Postulates and Theorems section at the back of this textbook for a complete list of theorems and their proof locations.

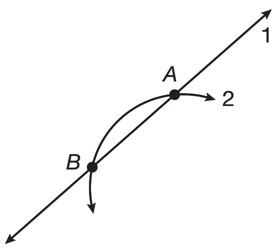
Postulate 5 simply states that, given a point A, and a point B, there is only one line that can be drawn through both points. The line $\overleftrightarrow{AB}$ is therefore unique.

Theorem 4-1
If two lines intersect, then they intersect at exactly one point.

Postulate 5 can be used to prove the theorem above, which states that when two lines intersect, they must intersect at only one point.

Example 1 Using Postulates and Theorems

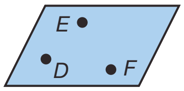
Does this diagram show two distinct lines through points A and B? Use Postulate 5 and Theorem 4-1 in your answer.



SOLUTION

Postulate 5 says that through any two points, there can be only one line. In the diagram, two "lines" pass through points A and B. However, the curve in this diagram is not straight and therefore is not an example of a line. Theorem 4-1 provides further evidence that the curve is not a line, since we know that two lines must intersect at only one point, whereas the line and the curve in this diagram intersect in two points.

Postulate 6
Through any three noncollinear points there exists exactly one plane.

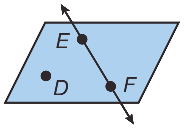


Math Reasoning

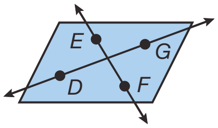
Model For three points to define a plane, they must be noncollinear. Draw three collinear points. Then draw two planes, both of which contain all three points.

If three points are collinear, then there are an infinite number of planes that can be drawn through them.

Theorem 4-2
If there is a line and a point not on the line, then exactly one plane contains them.



Theorem 4-3
If two lines intersect, then there exists exactly one plane that contains them.



Postulate 6, Theorem 4-2, and Theorem 4-3 give conditions for determining exactly one plane. Any of the following are sufficient: three noncollinear points, a line and a point not on the line, or two intersecting lines.

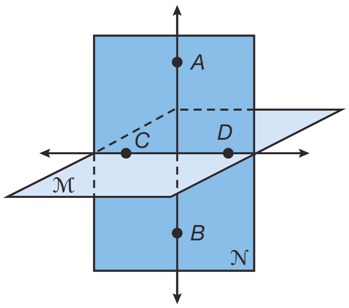
Example 2 Identifying Points and Lines in Planes

Name the following:

a. four points

b. two lines

c. two planes



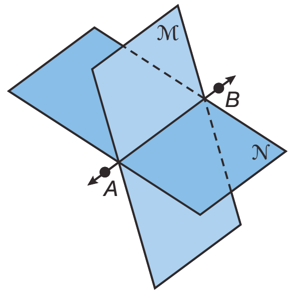
SOLUTION

a. Points A, B, C and D

b. $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$

c. Planes $\mathcal{M}$ and $\mathcal{N}$

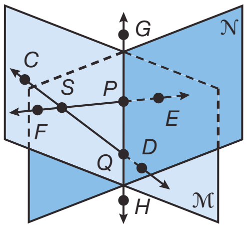
Just as two lines intersect in exactly one point, two planes intersect in exactly one line.

Postulate 7
If two planes intersect, then their intersection is a line. Planes $\mathcal{M}$ and $\mathcal{N}$ intersect at $\overleftrightarrow{AB}$. 

Math Reasoning

Generalize Is it possible to name three noncoplanar points in the diagram? Is it ever possible to name three noncoplanar points? Is it possible to name four noncoplanar points in the diagram?

Example 3 Identifying Intersections of Lines and Planes



a. Identify the intersection of planes $\mathcal{M}$ and $\mathcal{N}$.

SOLUTION

The intersection of two planes is a line. Though there are several lines in the diagram, only one of them lies in both plane $\mathcal{M}$ and plane $\mathcal{N}$. The intersection is $\overleftrightarrow{GH}$.

b. Identify all points of intersection of lines on plane $\mathcal{M}$.

SOLUTION

Lines *CD*, *FE*, and *AB* all intersect in plane $\mathcal{M}$. The points of intersection are points *P*, *Q*, and *S*.

Any two points in a plane are contained by a line on the same plane.

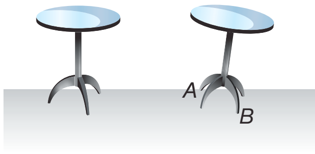
Postulate 8
If two points lie on a plane, then the line containing the points lies in the plane.

Postulate 9 gives the minimum number of points needed to define exactly one line, one plane, and space.

Postulate 9
A line contains at least 2 points. A plane contains at least 3 noncollinear points. Space contains at least 4 noncoplanar points.

Example 4 Application: Carpentry

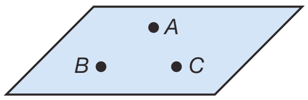
Often small tables have only three legs. They have the advantage of not tipping if placed on an uneven surface. Use postulates given in this lesson to describe why this is true and why a three-legged table will wobble less than a four-legged table.



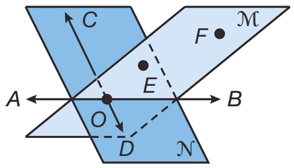
SOLUTION The legs of a three-legged table make a single plane, as shown in Postulate 7. The legs are noncollinear points. Even if they are uneven, the table will be stable. If the table has four legs and one of the legs is higher than the other three, the table will tip. Four points can be noncoplanar, so the table can tip between them.

Lesson Practice

a. *(Ex 1)* Can another plane be drawn that contains points A, B, and C? Justify your answer using a postulate.



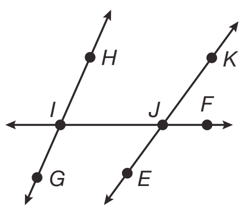
b. *(Ex 2, Ex 3)* Identify a line in plane $\mathcal{M}$. What is the intersection of $\overleftrightarrow{CD}$ and $\overleftrightarrow{AB}$? What is the intersection of planes $\mathcal{M}$ and $\mathcal{N}$? Are points E and F coplanar?



c. *(Ex 4)* Often camping supplies are made with three legs. For example, chairs, stoves, and small tables often have three legs. Explain why they might be made this way and use postulates to explain your reasons.

Practice Distributed and Integrated

* **1.** *(4)* **Multiple Choice** Which of the following is a set of noncollinear points?



A H, I, G **B** I, J, F
C H, I, K **D** K, J, E

* **2.** *(4)* **Write** Describe the difference between collinear points and coplanar lines.

* **3.** *(4)* Find the theorem or postulate that justifies the following statement.

If two lines are on the same plane, they will only intersect at one point.

Draw each of the following.

* **4.** Two planes that intersect at a line.
(4)

* **5.** Two lines that intersect on a plane at point O.
(4)

* **6.** Is this statement always true, never true, or sometimes true?
(4)
If two lines intersect then they are both contained in a single plane.

Use the figure to answer problems 7–9.

* **7.** Name two rays.
(3)

* **8.** What is the measure of $\angle ABC$?
(3)

* **9.** What name is given to this type of angle?
(3)

***10.** The base of a crane at a construction site makes an angle of 30° with the ground. As it lifts its load to clear an obstruction, the base makes an additional angle of 13°. What is the base's angle with the ground after lifting the load?
(3)

***11.** If $m\angle ABC = 62°$ and $\overline{DB}$ bisects $\angle ABC$, what is the measure of $\angle ABD$?
(3)

***12.** **Write** Describe how to locate the quarter points of a line segment, $\overline{AB}$.
(2)

***13.** **Algebra** Find the expression to represent AC if $AB = 2x + 5$, $BC = 5x - 16$ and A, B and C are collinear. B is between A and C.
(2)

14. **Probability** Handel flips a fair coin seven times and it comes up heads seven times. What is the probability his next flip will be heads? Why?
(SB 12)

***15.** **Traffic** There are three traffic lights on a street. If the second traffic light is 75 yards from the first, and all three traffic lights span 120 yards, how far is the second traffic light from the third one?
(2)

***16.** **Multiple Choice** Which statement is true? Make a drawing to show why each other statement is not always true.
(1)
A Any two lines are coplanar.
B Any two intersecting lines are coplanar.
C Any three intersecting lines are coplanar.
D Any two perpendicular lines are noncoplanar.

***17.** How many points are between any two points on a line?
(1)

***18.** What word describes two coplanar lines that never meet?
(1)

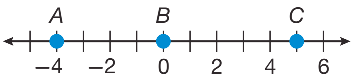
19. What is the parent function of $\frac{15 - 6x^2}{2}$?
(SB 17)

20. **Analyze** Graph $y = |x - 3|$. Is it a function?
(SB 17)

21. **Algebra** Solve for j. $\frac{4j - 12x^2}{3} = j$
(SB 16)

22. (SB 15) **Algebra** Solve for m. $-4m + 2 < 26$

***23.** (2) Determine AC.



24. (SB 10) Jaime measures the volume of a container to be 1.15 liters. If its actual volume is 1.25 liters, what was Jaime's percent error?

25. (SB 9) Convert 18 centimeters to inches. Round to the nearest tenth.

26. (SB 9) How many inches equal one-half of a mile?

27. (SB 7) Express 23,000,000 in scientific notation.

28. (SB 6) Evaluate 3^5.

29. (SB 5) Round 34,016.45 to the nearest ten.

30. (SB 2) What property of arithmetic is shown by $(a)(0) = 0$?

LESSON 5

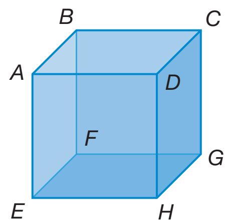
More Theorems About Lines and Planes

Warm Up

1. (1) **Vocabulary** Lines that lie in the same plane are ________.
2. (4) What geometric figure is formed when two planes intersect?
3. (1) **Multiple Choice** Which of these is not an undefined term?
   A line **B** plane
   C point **D** perpendicular
4. (4) How many points are needed to define a line?
5. (4) How many noncollinear points are needed to define a plane?

New Concepts

Lines and planes are classified by whether or not they intersect and how they intersect. When lines intersect to form a right angle, they are called **perpendicular** lines. The symbol to show that two lines are perpendicular is $\perp$. In the diagram, $\overline{AE}$ and $\overline{EH}$ are perpendicular, or simply: $\overline{AE} \perp \overline{EH}$.

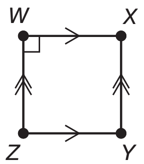


Math Language

Planes, segments, and rays can also be perpendicular to one another if they intersect at 90° angles.

Coplanar lines that do not intersect are called **parallel lines**. The symbol to show that two figures are parallel is $\parallel$. In the diagram, $\overline{AB} \parallel \overline{DC}$. Planes that do not intersect are **parallel planes**. In the diagram, the plane *ABC* is parallel to the plane *EFG*.

To indicate that lines are perpendicular on a diagram, it is only necessary to indicate that two segments intersect at a right angle, as $\overline{WX}$ and $\overline{WZ}$ do in the diagram of the square *WXYZ*. To indicate that lines are parallel in a diagram, arrowheads are drawn on them. In the diagram, the corresponding arrowheads indicate that $\overline{WX} \parallel \overline{ZY}$ and $\overline{WZ} \parallel \overline{XY}$.



If lines are not in the same plane and do not intersect, they are **skew lines**. In the cube shown above, the lines that contain $\overline{DH}$ and $\overline{EF}$ are skew.

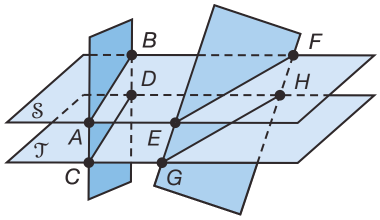
Theorem 5-1 can be used to identify parallel lines.

Theorem 5-1
If two parallel planes are cut by a third plane, then the lines of intersection are parallel.

Online Connection
www.SaxonMathResources.com

Example 1 Identifying Parallel Lines

In the figure, planes $\mathcal{S}$ and $\mathcal{T}$ are parallel. Identify two pairs of parallel lines.



SOLUTION

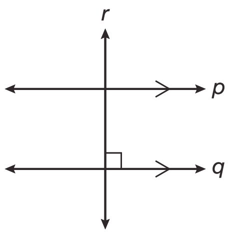
Since planes $\mathcal{S}$ and $\mathcal{T}$ are parallel, their intersections with any plane that crosses them form a pair of parallel lines. Therefore, $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ and $\overleftrightarrow{EF} \parallel \overleftrightarrow{GH}$.

Theorems 5-2 and 5-3 can be used to determine if coplanar lines are parallel or perpendicular to each other.

Theorem 5-2
If two lines in a plane are perpendicular to the same line, then they are parallel to each other.

Theorem 5-3
In a plane, if a line is perpendicular to one of two parallel lines, then it is perpendicular to the other one.

In the diagram, for example, if $p \parallel q$ and $r \perp q$, then r is also perpendicular to p by Theorem 5-3.



Lines are considered perpendicular when their intersection creates a single right angle, but all perpendicular lines actually create four right angles. Theorems 5-4, 5-5, and 5-6 describe the angles formed by perpendicular lines.

Math Reasoning

Formulate Recall that a circle is 360°. How could you use this information to show that four congruent adjacent angles must all be right angles?

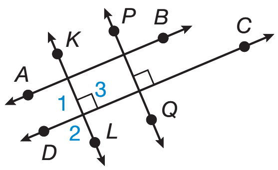
Theorem 5-4
If two lines are perpendicular, then they form congruent adjacent angles.

Theorem 5-5
If two lines form congruent adjacent angles, then they are perpendicular.

Theorem 5-6
All right angles are congruent.

Example 2 Classifying Pairs of Lines

In the figure, $\overleftrightarrow{AB} \parallel \overleftrightarrow{DC}$, $\overleftrightarrow{KL} \perp \overleftrightarrow{DC}$, and $\overleftrightarrow{PQ} \perp \overleftrightarrow{DC}$.



a. What is the relationship between $\overleftrightarrow{KL}$ and $\overleftrightarrow{AB}$?

SOLUTION

Since $\overleftrightarrow{AB}$ and $\overleftrightarrow{DC}$ are parallel, and $\overleftrightarrow{KL}$ and $\overleftrightarrow{DC}$ are perpendicular, $\overleftrightarrow{KL}$ and $\overleftrightarrow{AB}$ are perpendicular by Theorem 5-3.

b. What is the relationship between $\overleftrightarrow{KL}$ and $\overleftrightarrow{PQ}$?

SOLUTION

Since both $\overleftrightarrow{KL}$ and $\overleftrightarrow{PQ}$ are perpendicular to $\overleftrightarrow{DC}$, $\overleftrightarrow{KL}$ and $\overleftrightarrow{PQ}$ are parallel to each other by Theorem 5-2.

c. What is the measure of $\angle 1$? What is the measure of $\angle 2$?

SOLUTION

Since $\overleftrightarrow{KL} \perp \overleftrightarrow{DC}$, they form pairs of congruent adjacent angles. Therefore, $\angle 1$ and $\angle 2$ are congruent, and $\angle 1$ is congruent to $\angle 3$, which is known to be 90°. By the definition of congruency, $\angle 1$ and $\angle 2$ must also measure 90°.

For any given line, there are an infinite number of other lines that are parallel to it. However, there is only one line that is parallel to another through a given point, as stated in the Parallel Postulate.

Postulate 10: The Parallel Postulate
Through a point not on a line, there exists exactly one line through the point that is parallel to the line.

Example 3 Using the Parallel Postulate

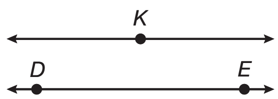
Hint

The distance between two parallel lines is the same at every point.

Draw as many lines as possible that are parallel to $\overleftrightarrow{DE}$, through a point K that is not on $\overleftrightarrow{DE}$.

SOLUTION

The Parallel Postulate indicates that there is only one line that can be drawn through a point not on a line that is parallel to the given line.



One final property of parallel lines is the Transitive Property.

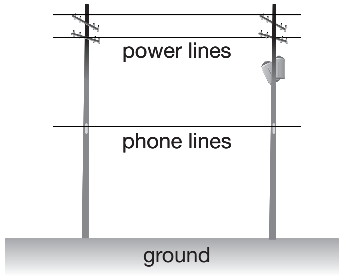
Theorem 5-7: Transitive Property of Parallel Lines
If two lines are parallel to the same line, then they are parallel to one other.

Math Reasoning

Model What measurements should Felix take to be sure that the phone line and the power line are approximately parallel?

Example 4 Application: Power Lines

Felix is repairing power lines and he needs to ensure that the power lines are parallel. After taking some measurements, he determined that the upper power line and the phone line are parallel and the lower power line and the phone line are parallel. Does Felix have enough information to conclude that the upper and lower power lines are parallel?

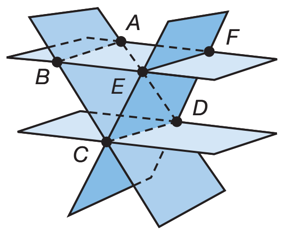


SOLUTION

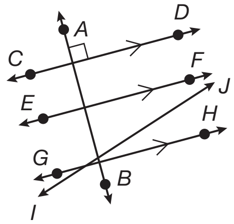
Yes. The Transitive Property of Parallel Lines can be applied. Since each power line is parallel to the phone line, the power lines are parallel to each other.

Lesson Practice

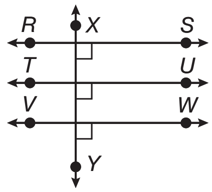
a. *(Ex 1)* If the horizontal planes are parallel, identify the lines that are parallel.



b. *(Ex 2)* In this figure, $\overleftrightarrow{CD} \parallel \overleftrightarrow{GH}$, and $\overleftrightarrow{CD} \perp \overleftrightarrow{AB}$. What is the relationship between $\overleftrightarrow{AB}$ and $\overleftrightarrow{GH}$?

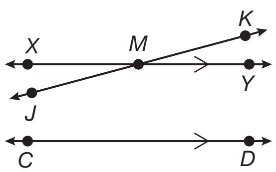


c. *(Ex 2)* If $\overleftrightarrow{RS} \perp \overleftrightarrow{XY}$ and $\overleftrightarrow{VW} \perp \overleftrightarrow{XY}$, what is the relationship between $\overleftrightarrow{RS}$ and $\overleftrightarrow{VW}$?



d. *(Ex 2)* What is the relationship among the angles at the points of intersection of $\overleftrightarrow{RS}$, $\overleftrightarrow{TU}$, and $\overleftrightarrow{VW}$ with $\overleftrightarrow{XY}$?

e. *(Ex 3)* If line $\overleftrightarrow{XY}$ goes through point M, and $\overleftrightarrow{XY} \parallel \overleftrightarrow{CD}$, how do you know that $\overleftrightarrow{JK}$ is not parallel to $\overleftrightarrow{CD}$?



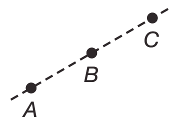
f. *(Ex 4)* Carlotta is assembling the floorboards for her deck. How can she be sure that all of the boards are parallel to one another?

Practice Distributed and Integrated

1. (1) Draw a diagram showing three planes that intersect at a common line.

* **2.** (4) **Error Analysis** Damon states that at least four collinear points are needed to determine a plane. Is Damon correct? Explain.

3. (4) **Write** Why is a plane not defined by the three given points?



4. (SB 3) **Predict** The Pythagorean Theorem, $a^2 + b^2 = c^2$, relates the lengths of the legs of right triangle, a and b, to the length of the hypotenuse, c. There are cases where all three numbers are positive integers. One example is $3^2 + 4^2 = 5^2$. Find one other example. Are there an infinite number of positive integers that satisfy the Pythagorean Theorem?

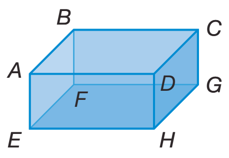
5. (3) What is the measure of the angle between two opposite rays?

* **6.** (5) **Justify** Can two lines be drawn from a point not on a line that will meet the line at a right angle? at a 45° angle? Explain each answer.

7. (SB 9) There are 100 centimeters in 1 meter. How many square centimeters are there in 1 square meter? Explain how you found your answer.

8. (SB 1) **Algebra** Simplify $\frac{2(2+4)}{6} - |-2|$.

* **9.** (5) Consider $\overline{AB}$ in this figure. Name a segment that is skew, one that is parallel, and one that is perpendicular to $\overline{AB}$.



10. (2) Which property is used to state, "If $5 = x$, then $x = 5$"?

11. (SB 13) Plot the points $A(0, 0)$, $B(4, 0)$, $C(4, 4)$ and $D(0, 4)$ on a coordinate grid. Describe the shape that results when the points are connected in alphabetical order.

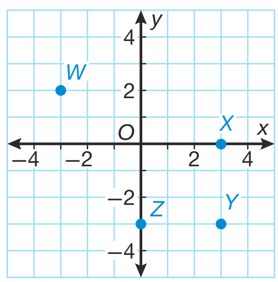
* **12.** (4) **Air Traffic Controller** An air traffic controller requested that a pilot taxi on the runway from point A to point B by traveling in a straight line. How many different options does the pilot have to get from point A to point B? How do you know?

13. (SB 17) For the function $f(x) = 3x^2 - 2x + 5$, find $f(1)$, $f(-2)$ and $f(a)$.

14. (SB 13) **Multiple Choice** Which point has the coordinates $(3, -3)$ on the graph shown?

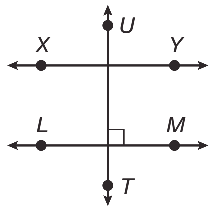
A W **B** Y

C X **D** Z



15. (4) **Photography** It is very important that a camera be held as still as possible when using certain photographic techniques. Explain why professional photographers use tripods to hold their cameras steady instead of a stand with four or more legs. Name a postulate or theorem in your explanation.

*16. If $\overleftrightarrow{XY}$ is parallel to $\overleftrightarrow{LM}$, what is the relationship between $\overleftrightarrow{XY}$ and $\overleftrightarrow{UT}$?
(5)



17. **Food Preparation** The instructions on a container of juice concentrate read, "Seven parts water per 1 part juice concentrate." How much juice can this container make if it holds 250 milliliters of juice concentrate?
(SB 4)

18. Are the points (1, 1), (5, 5), and (7, −4) collinear or coplanar? Explain.
(1)

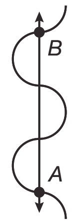
19. Transform this formula to solve for b: $a^3 + b^3 + c^3 = d^3$.
(SB 16)

20. What is the probability, as a fraction, of rolling an even number on a six-sided number cube?
(SB 12)

21. **Justify** How can you use a ruler to determine if a point M is the midpoint of a segment AB?
(2)

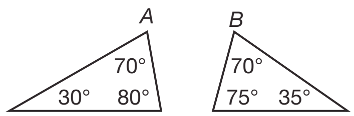
22. **Tipping** In some places, it is customary to give a tip of 15% at restaurants. Does it make a difference whether the 15% is paid on the bill before taxes are added or after taxes are added? Explain.
(SB 4)

23. **Write** How many lines connect points A and B? Explain.
(4)



24. **Generalize** What are the properties of all ordered pairs in Quadrant II of the coordinate plane?
(SB 13)

25. Are angles A and B in the triangles shown congruent? Explain.
(3)



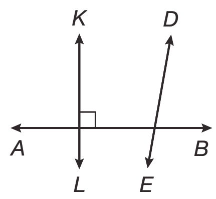
26. **Algebra** Transform the formula for the area of a triangle to solve for the height, h.
(SB 16)

*27. In most rooms, the floor is perpendicular to the wall and the wall is perpendicular to the ceiling. What can you say about the relationship between the floor and ceiling of most rooms if those two statements are true?
(5)

28. **Generalize** Give an example of two acute angles whose measures could be added together to form another acute angle. Give an example of two acute angles whose measures could be added together to form an obtuse angle.
(3)

29. **Music** Musical notes can be identified by their frequencies. The frequency doubles when going up an octave. Middle C is denoted as C_4 and has a frequency of 261.63 hertz. What is the frequency of the note C_5, which is one octave above C_4?
(SB 22)

*30. Does the diagram at right show $\overleftrightarrow{AB}$ perpendicular to $\overleftrightarrow{DE}$? Explain.
(5)



LAB

2

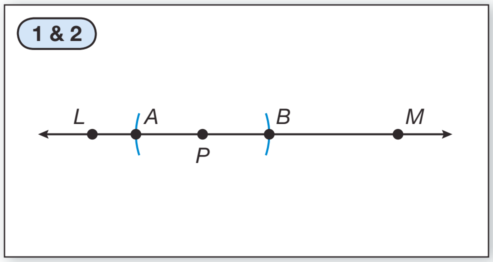
Perpendicular Line through a Point on a Line

Construction Lab 2 *(Use with Lesson 5)*

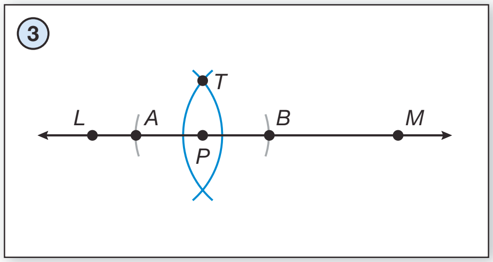
In Lesson 5, you learned about perpendicular lines. This construction lab shows you how to construct a perpendicular line through a given point on a line.

1. Draw $\overleftrightarrow{LM}$ and choose point P on the line as the point at which to construct a perpendicular line.

2. Sweep two arcs centered at P through $\overleftrightarrow{LM}$. Label the two points of intersection with $\overleftrightarrow{LM}$ as A and B, respectively.

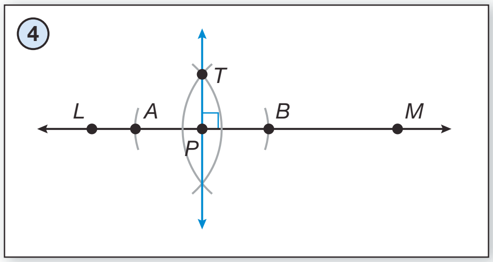


3. Sweep two arcs, one centered at A and one centered at B. Be sure to choose a radius large enough so that the arcs intersect. Label one point of intersection of these two arcs T.



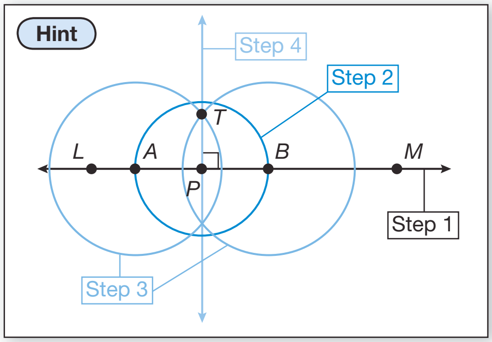
4. Draw $\overleftrightarrow{TP}$.

   Line TP is perpendicular to $\overleftrightarrow{LM}$ at P.



Hint

Instead of sweeping arcs to find points *A* and *B*, you can construct a whole circle centered on point *P*. See the diagram at right.



Lab Practice

a. Draw $\overline{AB}$ that is 3 inches long. Construct a segment perpendicular to $\overline{AB}$ through point P such that $\overline{AP}$ is 1.75 inches.

b. Draw $\overline{MN}$ that is 6 inches long. Construct a segment perpendicular to $\overline{MN}$ through point P such that $\overline{NP}$ is 2 inches.

LESSON 6

Identifying Pairs of Angles

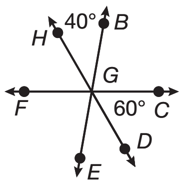
Warm Up

1. (3) **Vocabulary** Two angles that have the same measure are said to be ______________.

2. (SB 15) Solve the equation $x + 135 = 180$.

3. (1) Match each term below with the correct definition.
   - collinear — **A** the point or set of points common to different figures
   - space — **B** on the same line
   - coplanar — **C** the set of all points
   - intersection — **D** in the same plane

4. (3) Identify one acute angle and one obtuse angle in the diagram.

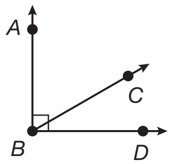
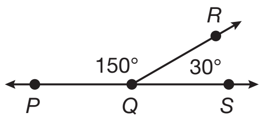


New Concepts

A pair of angles can sometimes be classified by their combined measure. These pairs are known as complementary and supplementary angles.

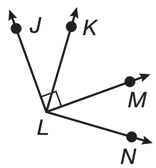
Math Language

The sum of an angle and its **complement** is 90°. The sum of an angle and its **supplement** is 180°.

Complementary Angles	Supplementary Angles
Two angles are **complementary angles** if the sum of their measures is 90°. $m\angle ABC + m\angle CBD = 90°$, so $\angle ABC$ is complementary to $\angle CBD$. 	Two angles are **supplementary angles** if the sum of their measures is 180°. $m\angle PQR + m\angle RQS = 180°$, so $\angle PQR$ is supplementary to $\angle RQS$. 

Example 1 Finding Complements and Supplements

a. Find the angles complementary to $\angle KLM$ if $m\angle KLN = 90°$.



SOLUTION

From the diagram, $m\angle JLK + m\angle KLM = 90°$ and $m\angle KLM + m\angle MLN = 90°$.

So $\angle JLK$ and $\angle MLN$ are complementary to $\angle KLM$.

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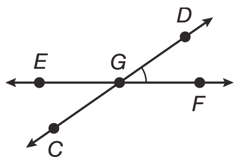
b. Find the angles supplementary to $\angle DGF$.



SOLUTION

From the diagram, $m\angle EGD + m\angle DGF = 180°$ and $m\angle DGF + m\angle FGC = 180°$.

So, $\angle EGD$ and $\angle FGC$ are supplementary to $\angle DGF$.

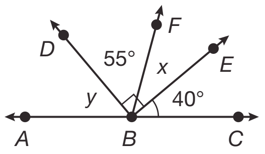
Theorem 6-1: Congruent Complements Theorem
If two angles are complementary to the same angle or to congruent angles, then they are congruent.

Theorem 6-2: Congruent Supplements Theorem
If two angles are supplementary to the same angle or to congruent angles, then they are congruent.

According to Theorem 6-1, if $\angle 1$ is complementary to $\angle 2$ and $\angle 3$ is also complementary to $\angle 2$, then $\angle 1 \cong \angle 3$. Likewise, according to Theorem 6-2, if $\angle 1$ is supplementary to $\angle 2$ and $\angle 3$ is also supplementary to $\angle 2$, then $\angle 1 \cong \angle 3$.

Example 2 Solving with Complements and Supplements

Find the measures of the angles labeled x and y.



SOLUTION

To find x, notice that $\angle DBF$ and $\angle FBE$ are complementary.

> **Hint**
>
> Notice that $m\angle DBC$ is equal to the sum of the measures of three angles: $m\angle DBF + m\angle FBE + m\angle EBC$.

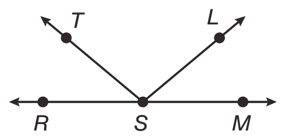
$m\angle DBF + m\angle FBE = 90°$	Definition of complementary angles
$55° + x = 90°$	Substitute.
$55° + x - 55° = 90° - 55°$	Substract 55° from each side.
$x = 35°$	Simplify.

To find y, notice that $\angle ABD$ and $\angle DBC$ are supplementary.

$m\angle ABD + m\angle DBC = 180°$	Definition of supplementary angles
$y + 55° + 35° + 40° = 180°$	Substitute.
$y + 130° = 180°$	Simplify.
$y + 130° - 130° = 180° - 130°$	Subtract 130° from each side.
$y = 50°$	Simplify.

Complementary angles and supplementary angles are related to each other by their angle measures. Angles are also related to each other by their positions relative to each other in the same plane.

Two angles in the same plane that share a vertex and a side, but share no interior points are **adjacent angles**. In the diagram, $\angle TSL$ is adjacent to $\angle LSM$, and $\angle RST$ is adjacent to $\angle TSL$.

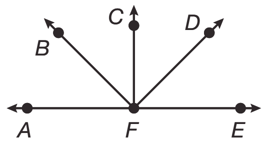


Adjacent angles whose non-common sides are opposite rays are a **linear pair**. In the diagram, $\angle RST$ and $\angle TSM$ are a linear pair. Recall that the measure of a straight line is 180°. Since a linear pair composes a straight line, linear pairs are supplementary.

Theorem 6-3: Linear Pair Theorem
If two angles form a linear pair, then they are supplementary.

Example 3 Identifying Angle Pairs

Identify two sets of adjacent angles and one linear pair.



SOLUTION

There are many adjacent angles in the diagram. Two possible sets are $\angle AFB$ and $\angle BFC$, and $\angle AFC$ and $\angle CFE$.

There are also several linear pairs shown. One is $\angle AFD$ and $\angle DFE$.

Math Reasoning

Generalize Are the statements below true or false?

a. *If two angles are not adjacent, then they are vertical angles.*

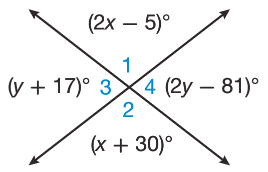
b. *If two angles form a linear pair, then they are adjacent angles.*

Nonadjacent angles formed by two intersecting lines are **vertical angles**. Vertical angles share the same vertex and have no common sides.

Theorem 6-4: Vertical Angle Theorem
If two angles are vertical angles, then they are congruent.

Example 4 Solving with Vertical Angles

Determine the values of x and y.



SOLUTION

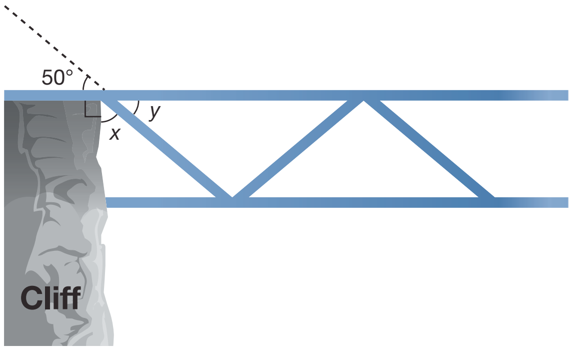
Since $\angle 1$ and $\angle 2$ are vertical angles, they are congruent. The same is true of $\angle 3$ and $\angle 4$. Therefore, $m\angle 1 = m\angle 2$ and $m\angle 3 = m\angle 4$.

$$\begin{aligned} m\angle 1 &= m\angle 2 \\ 2x - 5 &= x + 30 \\ 2x - 5 + 5 &= x + 30 + 5 \\ 2x &= x + 35 \\ 2x - x &= x + 35 - x \\ x &= 35 \end{aligned}$$

$$\begin{aligned} m\angle 3 &= m\angle 4 \\ y + 17 &= 2y - 81 \\ y + 17 - 17 &= 2y - 81 - 17 \\ y &= 2y - 98 \\ y - 2y &= 2y - 98 - 2y \\ -y &= -98 \\ \frac{-y}{-1} &= \frac{-98}{-1} \\ y &= 98 \end{aligned}$$

Example 5 Application: Bridge Supports

The diagram shows the part of a bridge where it contacts a vertical cliff, so that the bridge and the cliff are perpendicular. The angle between the surface of the road and the line extended from the bridge's support measures 50°. It is important that the bridge's support be set at the correct angle to hold the weight of the bridge. What is the angle x that the support makes with the cliff?



Math Reasoning

Formulate How could you have solved this problem by looking at a linear pair of angles instead of a vertical pair of angles?

SOLUTION

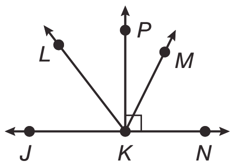
The angle that measures 50° and the angle labeled y are vertical angles. The angles labeled x and y are complementary angles.

$$x + y = 90°$$
$$x + 50° = 90°$$
$$x + 50° - 50° = 90° - 50°$$
$$x = 40°$$

The angle between the support and the cliff measures 40°.

Lesson Practice

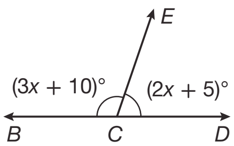
Refer to the diagram to find the complementary and supplementary angles.

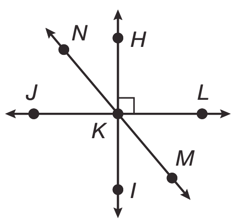


a. Which angle is complementary to $\angle PKM$? *(Ex 1)*

b. Which angle is supplementary to $\angle JKL$? *(Ex 1)*

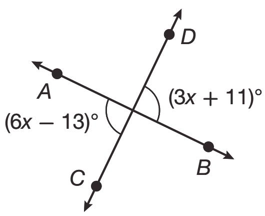
c. Find the value of x. *(Ex 2)*



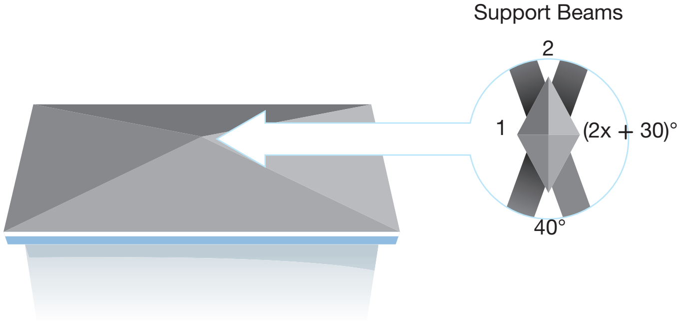


d. Identify three pairs of adjacent angles and two linear pairs. *(Ex 3)*

e. Determine the value of x. *(Ex 4)*



f. *(Ex 5)* A rectangular roof is made of four triangular sections and is supported at its peak by a diamond-shaped support and four wooden beams, as shown in the diagram. What is the value of x? What are the angle measures of $\angle 1$ and $\angle 2$, respectively?



Practice Distributed and Integrated

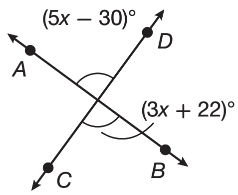
* **1.** *(6)* An angle measuring 32° has a complement that measures $(2x - 16)°$. What is the value of x?

* **2.** *(6)* An angle measures 51°. What is the measure of its supplementary angle?

3. *(SB 1)* **Algebra** Compute $|3 \times 2 - 8| \times 2$.

4. *(4)* Four noncoplanar points define space, but could they be used to form two intersecting planes? Explain why or why not.

* **5.** *(6)* Find the value of x in the diagram at right.



6. *(4)* **Write** Stanley says, "If a point is not on a line, then the line and the point exist on exactly one plane." If the point *is* on the line, how would his statement need to be changed?

7. *(2)* **Mapping** On a grid of their community, Silvio lives at the point (7, 3) and his friend Andy lives at the point (−11, 3). If they decide to meet halfway between their houses, at what point on the grid will they meet?

8. *(SB 14)* Evaluate the expression $3xy + 5x + 2y$ for $x = 2$ and $y = -1$.

* **9.** *(6)* What angle is equal to its complementary angle?

10. *(SB 12)* What is the probability, expressed as a percentage, of choosing a red marble if a bag of marbles contains 20 red, 15 green, 25 orange, and 40 blue marbles?

11. *(1)* **Error Analysis** A student says that two points define a line, three points define a plane, and four points define space. Are the student's definitions correct? Explain.

12. *(2)* Find the midpoint of the values 4 and 8 on the x-axis and the midpoint of the values 1 and 7 on the y-axis.

13. *(3)* If D is in the interior of $\angle ABC$, which measures 74°, and $m\angle ABD = 32°$, what is $m\angle DBC$?

14. (SB 12) **Competition** In 2007, the World Rock-Paper-Scissors Championship was held in Toronto, Ontario, Canada. What is the probability that any two competitors will both have the same combination in a game of rock-paper-scissors, expressed as a fraction?

***15.** (5) Simone has been given the challenge of drawing two coplanar lines that are perpendicular to the same line, but the lines must not be parallel to each other. Will she succeed? Explain why or why not.

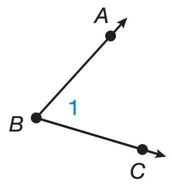
***16.** (5) **Write** Two planes are parallel to each other and are both intersected by a third plane. Describe the property of each intersection and how these intersections are related.

***17.** (SB 14) Evaluate the expression $-x^y + 3xy - \frac{x}{y}$ for $x = 4$ and $y = 2$.

18. (3) **Multiple Choice** Which is *not* a correct way to name the angle in the diagram?

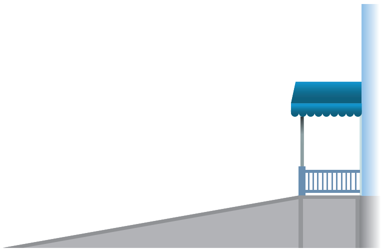
A $\angle 1$ **B** $\angle B$

C $\angle ABC$ **D** $\angle A$



19. (SB 5) **Write** Give an example that requires estimation.

20. (3) **Handicap Access** Use a protractor to determine the angle measure of the wheelchair ramp shown in the diagram.



21. (2) Use the Ruler Postulate to find the distance between the values −7 and 11 on a number line.

***22.** (5) **Analyze** If $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are both parallel to $\overleftrightarrow{EF}$, what conclusion can be made regarding the relationship between $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$?

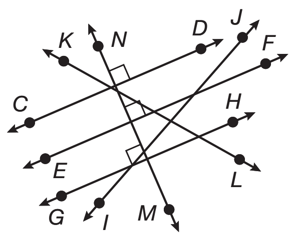
23. (SB 4) Find 20% of 654.

24. (SB 2) **Justify** Does subtraction exhibit the property of closure over the set of real numbers? Is subtraction commutative? If not, give an example to demonstrate.

***25.** (6) **Landscaping** A tree is secured with a support wire that makes a 40° angle with the ground. The angle between the tree and the wire is complementary to the angle between the wire and the ground. What is the measure of the angle between the tree and the wire?

26. (4) If $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect at point K, how many planes are needed to contain the two lines?

***27.** (5) List all of the lines that are parallel to $\overleftrightarrow{CD}$ in the diagram at right.



28. (SB 22) **Comparison Shopping** A price tag reads \$1.59/kg for apples, and \$0.82/lb for oranges. Which fruit is cheaper by weight? Show your steps.

***29.** (6) If $m\angle DEF = (3x + 5)°$ and $m\angle KLM = (x + 31)°$, and both angles are complementary to $\angle PQR$, what are the measures of the angles?

30. (SB 17) **Algebra** Evaluate the function $f(x) = \frac{5}{x + 2}$ for $x = \frac{2}{3}$.

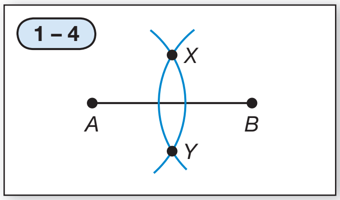
LAB 3

Perpendicular Bisectors and Angle Bisectors

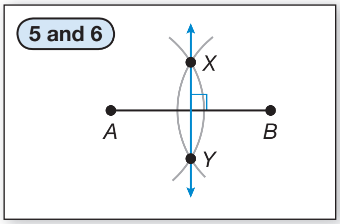
Construction Lab 3 *(Use with Lesson 6)*

> **Math Language**
>
> A **locus** is a set of points that satisfies a list of given conditions.

A **perpendicular bisector** is a line, segment, or ray that intersects a segment at its midpoint, forming 90° angles. Another way to define a perpendicular bisector is as a locus of points that are equidistant from the endpoints of a segment.



1. To construct a perpendicular bisector of a segment, begin with a segment, $\overline{AB}$.
2. Set your compass to a setting wider than half the length of the segment and place one end on *A*.
3. Use your compass to draw an arc through the segment. Be sure to draw the arc both above and below the segment.
4. Repeat this process with the same compass setting, starting at point *B*.



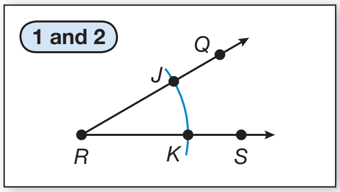
5. Label the points where the two arcs intersect as points *X* and *Y*.
6. Draw the line through points *X* and *Y*. Line *XY* is the perpendicular bisector of $\overline{AB}$.

There are two important theorems about perpendicular bisectors that you will use throughout this program.

Theorem 6-5
If a point lies on the perpendicular bisector of a segment, then the point is equidistant from the endpoints of the segment.

Theorem 6-6
If a point is equidistant from the endpoints of a segment, then the point lies on the perpendicular bisector of the segment.

An **angle bisector** is a line, segment, or ray that divides an angle into two congruent adjacent angles.



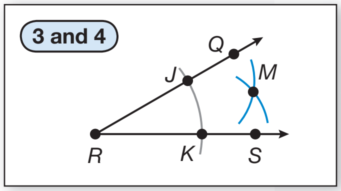
1. To construct the bisector of an angle, begin with an angle, $\angle QRS$.
2. Using a compass, draw an arc centered at *R* that intersects both sides of the angle. Label these points of intersection *J* and *K*.

Math Reasoning

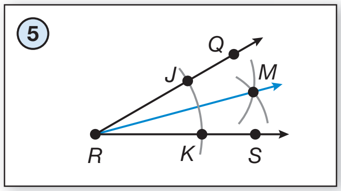
Connect Try using this process to bisect a 180° angle. What do you notice?

3. Using the same compass setting, draw an arc centered at J as shown.

4. Repeat this process with the same compass setting to draw an arc centered at K. Label the intersection of the two arcs M.



5. Draw $\overrightarrow{RM}$, the angle bisector of $\angle QRS$.



There are also two important theorems about angle bisectors that you will use throughout this program.

Theorem 6-7
If a point lies on the bisector of an angle, then the point is equidistant from the sides of the angle.

Theorem 6-8
If a point is equidistant from the sides of an angle, then the point lies on the bisector of the angle.

Lab Practice

a. Use a ruler to draw a 4 inch segment, then use construction techniques to draw a perpendicular bisector.

b. Use a straightedge to draw a segment of any length, then use construction techniques to bisect it. Use a ruler to verify that your bisector divides the segment into two congruent parts.

c. Use a protractor to draw a 108° angle, then use construction techniques to bisect it.

d. Use a straightedge to draw an angle of any measure, then use construction techniques to bisect it. Use a protractor to verify that your bisector divides the angle into two congruent angles.

LESSON 7 Using Inductive Reasoning

Warm Up

1. (6) **Vocabulary** A statement that has been proved true is a __________.

2. (4) Which statement about postulates and/or theorems is false?
   A A postulate is a proven statement.
   B A postulate is assumed to be true.
   C A theorem is a proven statement.
   D Both **B** and **C** are false.

3. (5) Which statement is true?
   A If two lines are perpendicular, they make a 90° angle.
   B If two lines are parallel, they make a 90° angle.
   C Both **A** and **B** are correct.
   D Neither **A** nor **B** is correct.

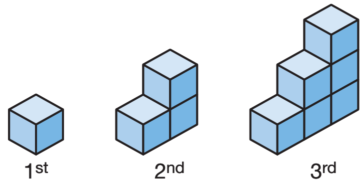
New Concepts

Inductive reasoning is the process of reasoning that a rule or statement is true because several specific cases are true. Inductive reasoning can be used to formulate a conjecture about something. A **conjecture** is a statement that is believed to be true. If a conjecture can be proven, it becomes a theorem.

> **Math Reasoning**
>
> **Analyze** Tad saw a brown bear and made the conjecture, "All bears are brown." Did Tad use inductive reasoning appropriately? Explain.

Example 1 Formulating a Conjecture

Look at the progression of the pattern and formulate a conjecture regarding the number of blocks there will be in the fifth arrangement of this series.



SOLUTION

Each successive step of the series adds a row of blocks to the bottom of the figure that is one block longer than the previous row of blocks. For example, the second arrangement has a bottom row that is 2 blocks long, so to make the third one, a row of 3 blocks is added to the bottom.

To continue, the fourth step would have a row of 4 blocks added for a total of 10, and the fifth would have 5 more blocks added for a total of 15. The statement, "There will be 15 blocks in the fifth step of this pattern," is a conjecture.

Instead of formulating a conjecture by looking at data, it may be necessary to test a conjecture using given data. If even one example can be found that does not support the conjecture, then the conjecture must be incorrect. An example that does not support the conjecture is called a **counterexample**. You will learn more about counterexamples in Lessons 10 and 14.

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> **Caution**
>
> Testing a conjecture is not the same as proving it. Testing a conjecture can disprove it, or it can prove it for certain values, but it cannot prove the conjecture for all possible values.

Example 2 Testing a Conjecture

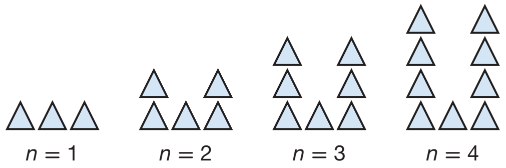
a. Michelle made the conjecture, "The expressions $6n + 1$ and $6n - 1$ will always result in two prime numbers." Show that this conjecture is true for $n = 1$, 2, and 3, but not true for $n = 4$.

SOLUTION

For $n = 1$: $6(1) + 1 = 7$ and $6(1) - 1 = 5$; both are prime.
For $n = 2$: $6(2) + 1 = 13$ and $6(2) - 1 = 11$; both are prime.
For $n = 3$: $6(3) + 1 = 19$ and $6(3) - 1 = 17$; both are prime.
But for $n = 4$: $6(4) + 1 = 25$ and $6(4) - 1 = 23$; 25 is not prime.

Since we have found one case in which the conjecture is incorrect, we can conclude that the conjecture is false.

b. Maria looks at the diagram below and conjectures that the number of triangles in the figure is given by the expression $2n + 1$. Is this conjecture true for the four steps of the pattern shown below?



SOLUTION

Yes, the conjecture is true for all 4 steps.

For $n = 1$, $2(1) + 1 = 3$
For $n = 2$, $2(2) + 1 = 5$
For $n = 3$, $2(3) + 1 = 7$
For $n = 4$, $2(4) + 1 = 9$

Even though conjectures are not proven, scientists often use conjectures to describe real-world phenomena. These conjectures are carefully studied and tested, but it is often difficult to prove them formally.

Example 3 Application: Research

A researcher studying crows for several years made the observation that every crow she studied was black. Her research assistant made this conjecture: "All crows are black." How can this conjecture be tested? Can it be proved?

SOLUTION

The conjecture can be tested by observing as many crows as possible. If even one crow is found that is not black, then the conjecture is disproved. The only way to prove this conjecture is to observe every crow. If every crow can be studied, and they are all black, then the conjecture is true. However, it is impossible to study every crow that exists, so the conjecture cannot be proved.

Lesson Practice

a. (Ex 1) Formulate a conjecture about how the next step in this pattern would be found: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, …

b. (Ex 2) Test the conjecture that every even integer 4 through 14, can be written as the sum of two prime numbers.

c. (Ex 3) How might you disprove the conjecture below?

Apples, pears, lemons, and peaches all grow on trees, therefore all fruits grow on trees.

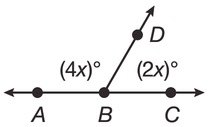
Practice Distributed and Integrated

1. (SB 3) **Justify** In the following description, are rational and irrational numbers defined adequately? Explain.

Rational numbers are defined to be all terminating or repeating decimals, and irrational numbers are defined to be all nonterminating, nonrepeating decimals.

* **2.** (2) **Analyze** Leilani suggests that if the midpoint of a segment connecting 6 and 8 on the x-axis is 7, and the midpoint of a segment connecting 2 and 10 on the y-axis is 6, then the midpoint of a segment connecting the points $(6, 2)$ and $(8, 10)$ must be $(7, 6)$. Is this a valid conclusion? Explain.

3. (6) What is the value of x in the diagram?



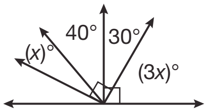
* **4.** (5) **Model** Draw an example of skew lines using a geometric figure. Clearly show the skew lines.

* **5.** (7) **Generalize** Formulate a conjecture about how the next step in this pattern would be found: 4, 16, 36, 64, 100, 144, …

6. (4) **Air Traffic Control** Two airplanes have the same latitudinal and longitudinal coordinates, but the air traffic controller is not concerned. Why would she not be concerned?

7. (SB 7) **Write** A number is rounded to 9000. Is it possible to tell exactly how many significant figures the number had before rounding? Explain.

8. (6) Find the value of x in the diagram.



9. (4) **Justify** What is the fewest number of points that can define a plane? Use a postulate or theorem to justify your answer.

***10.** (7) **Justify** When Raul studied objects that were released from a position of rest, all of the objects fell to the ground. He conjectured that any object released from a position of rest will immediately fall to the ground. Find an example that disproves Raul's conjecture.

***11.** (7) Is the following conjecture true for $x = 5$, $x = 10$, and $x = 15$?

For any x, *the sum of the whole numbers 1 through* x *is equal* to $\frac{1}{2}x(x + 1)$.

12. (SB 10) **Summer Employment** Devondra estimated that she would make $4000 this summer at her part-time job. She actually made $3450. What was her percent error, to the nearest percentage?

13. (1) **Write** Explain what is meant when a term in mathematics is said to be *undefined*.

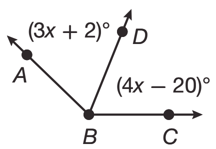
14. (SB 15) **Algebra** Find the value of x in the equation $\frac{2x - 5}{3} = \frac{x + 10}{4}$.

15. (SB 22) **Landscaping** A rectangular field that is 119 square yards in area is to be covered with sod. It is known that the field is 3 yards longer than twice the width. Find the dimensions of the field.

16. (SB 13) **Travel** Marcella lives at (4, 22) on a city grid, and Arnault lives at (−3, −4) on the same grid. One grid line represents one mile. If Arnault's mom drives him to Marcella's house by driving on a road that is parallel to the x-axis, then turns and drives along a road that is parallel to the y-axis, how many miles does Arnault's mom drive?

***17.** (7) **Write** Can a conjecture be proved or disproved by studying examples or cases of the conjecture? Explain.

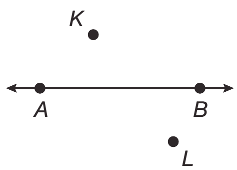
18. (3) **Algebra** Ray BD is the angle bisector of $\angle ABC$. What is the value of x?



***19.** (4) Sketch an example of two lines that are coplanar.

20. (SB 7) How many significant figures does the number 0.0100 have?

***21.** (5) Draw a line parallel to and a line perpendicular to $\overleftrightarrow{AB}$ from each of the points K and L. Discuss the relationship between these drawn lines.



***22.** (7) What type of reasoning is used in the statement below? Provide a counterexample to the conjecture if possible.

Every car I have ever owned ran on gas, therefore all cars run on gas.

23. (SB 16) **Multi-Step** The formula for converting kilometers to miles is $m = 1.609k$, where m is miles and k is kilometers. Transform this formula to find a formula for converting miles to kilometers. Use this formula to determine the number of kilometers in 539 miles and in 7380 miles, rounded to the nearest kilometer.

24. (6) **Justify** Can two acute adjacent angles form a linear pair? Explain why or why not.

25. **Temperature** The conversion for temperature from degrees Celsius to degrees Fahrenheit is given by the formula $F = \frac{9}{5}C + 32$. Transform this formula to find a formula to convert degrees Fahrenheit to degrees Celsius.
(SB 16)

26. **Multiple Choice** Which statement is true?
(4)

A Two distinct coplanar lines can intersect at two points.

B Two distinct coplanar lines can intersect at one point or never intersect if they are parallel.

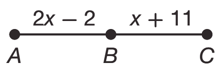
C Two distinct coplanar lines can intersect at one point or intersect everywhere.

D Two distinct coplanar lines intersect at every point if they are parallel.

27. The sum of the measures of three congruent angles is 180°. What would be the complementary angle to each of the angles?
(6)

***28.** **Justify** Is it possible for three planes to intersect at a point? Explain why or why not, and if it is possible, sketch an example showing how.
(1)

29. If B is the midpoint of $\overline{AC}$, find the value of x.
(2)



30. The measure of $\angle XYZ$ is 110°. What would the resulting angles' measures be if the angle bisector to $\angle XYZ$ were drawn?
(3)

LESSON 8

Using Formulas in Geometry

Warm Up

1. **Vocabulary** ________ lines are coplanar lines that do not intersect. *(5)* (***parallel, perpendicular, skew***)

2. If the length of $\overline{AC}$ is 12, and $\overline{AC}$ is bisected at point B, what are the lengths of $\overline{AB}$ and $\overline{BC}$? *(2)*

3. **Multiple Choice** Which of the following is not a theorem? *(6)*
   A Right Angle Congruence Theorem
   B Congruent Complements Theorem
   C Congruent Supplements Theorem
   D Reflexive Property of Equality

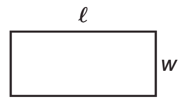
New Concepts

A **formula** is a mathematical relationship expressed with symbols. Some formulas have already been encountered in algebra.

> **Math Reasoning**
>
> **Write** List some other formulas used in other math classes, such as in algebra. How might these formulas be helpful in geometry?

A familiar formula is the formula for **perimeter**. The perimeter is the sum of the side lengths of a closed geometric figure. It is often thought of as the distance around a figure.

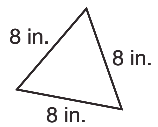
There is a special formula to find the perimeter of a rectangle, where P is the perimeter, ℓ is the length of the rectangular base, and w is the width, or height, of the rectangle.



$$P = 2\ell + 2w$$

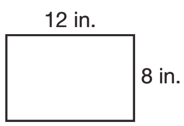
Example 1 Finding Perimeter of a Figure

a. Find the perimeter of the triangle.



SOLUTION Add the lengths of the sides together.
$8 + 8 + 8 = 24$
The perimeter of the triangle is 24 inches.

b. Find the perimeter of the rectangle.



SOLUTION Use the formula for the perimeter of a rectangle.

$P = 2\ell + 2w$	Perimeter formula
$P = 2(12) + 2(8)$	Substitute.
$P = 40$ in.	Simplify.

c. If a regular pentagon has a side length of 8 inches, what is its perimeter?

SOLUTION There are five sides in a pentagon and each side of a regular pentagon has the same measure. Therefore, the perimeter is $5 \times 8 = 40$ inches.

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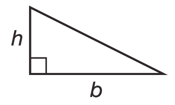
> **Math Reasoning**
>
> **Formulate** Draw a diagonal from one corner of a rectangle to the other. What shapes does the diagonal create? Explain how this relates to the formula for area of a triangle.

The **area** of a figure is the size of the region bounded by the figure.

The area of a rectangle is found by the following formula, where ℓ is the length of the figure's base and w is the length of the figure's height:

$A = \ell w$

The area of a triangle is found by the following formula:



$A = \frac{1}{2} bh$

The area of a figure is always expressed in square units.

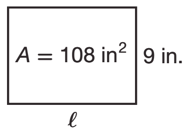
Example 2 Using the Area Formula for a Rectangle

a. Find the area of the rectangle.

SOLUTION

$A = \ell w$	Area formula
$A = (14)(3)$	Substitute.
$A = 42 \text{ cm}^2$	Simplify.

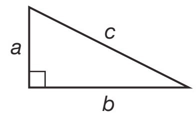
b. Find the length of the rectangle.



SOLUTION

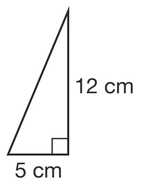
$A = \ell w$	Area formula
$108 = \ell(9)$	Substitute.
$12 \text{ in.} = \ell$	Divide both sides by 9.

> **Theorem 8-1: Pythagorean Theorem**
>
> The sum of the square of the lengths of the legs, a and b, of a a right triangle is equal to the square of the length of the hypotenuse c and is written $a^2 + b^2 = c^2$.



Example 3 Using the Pythagorean Theorem

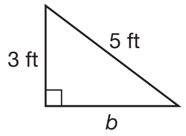
a. Find the length of the hypotenuse.



SOLUTION

$a^2 + b^2 = c^2$	Pythagorean Theorem
$12^2 + 5^2 = c^2$	Substitute.
$144 + 25 = c^2$	Simplify.
$\sqrt{169} = \sqrt{c^2}$	Square root of both sides
$13 \text{ cm} = c$	Simplify.

b. Find the area of the triangle.



SOLUTION

Use the Pythagorean Theorem to find the length of b.

$a^2 + b^2 = c^2$	Pythagorean Theorem
$3^2 + b^2 = 5^2$	Substitute.
$9 + b^2 = 25$	Simplify.
$9 + b^2 - 9 = 25 - 9$	Subtract 9 from both sides.
$b^2 = 16$	Simplify.
$\sqrt{b^2} = \sqrt{16}$	Square root of both sides
$b = 4$ ft	Simplify.

Then calculate the area of the triangle.

$A = \frac{1}{2}bh$	Formula for area of a triangle
$A = \frac{1}{2}(4)(3)$	Substitute.
$A = 6 \text{ ft}^2$	Simplify.

Example 4 Application: Measuring Temperature

Different countries use different units to measure the temperature. Much of the world uses degrees Celsius, but a few countries use degrees Fahrenheit. For scientists and travelers, converting between Celsius and Fahrenheit is an important skill.

To convert to Celsius from Fahrenheit, use the formula: $C = \frac{5}{9}(F - 32)$.

a. If it is 77°F, what is the temperature in degrees Celsius?

SOLUTION

$C = \frac{5}{9}(F - 32)$	Conversion formula
$C = \frac{5}{9}(77 - 32)$	Substitute.
$C = 25$	Simplify.

b. If it is 10°C, what is the temperature in degrees Fahrenheit?

SOLUTION

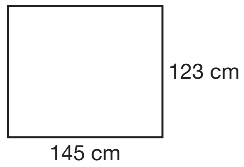
$C = \frac{5}{9}(F - 32)$	Conversion formula
$10 = \frac{5}{9}(F - 32)$	Substitute.
$10 \times \frac{9}{5} = \frac{5}{9}(F - 32) \times \frac{9}{5}$	Multiply by the reciprocal of $\frac{5}{9}$, $\frac{9}{5}$.
$18 = F - 32$	Simplify.
$18 + 32 = F - 32 + 32$	Add 32 to both sides
$50 = F$	Simplify.

> **Caution**
>
> The Pythagorean Theorem only applies to right triangles.
>
> A right angle is denoted with a small square in the corner that has a measure of 90°.

Lesson Practice

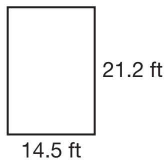
a. Find the perimeter of a triangle with congruent side lengths all equal to 6.5 meters.
(Ex 1)

b. Find the perimeter of the rectangle.
(Ex 1)



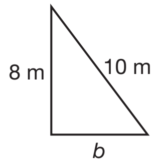
c. Find the perimeter of a six-sided figure with side lengths that are all equal to 16 inches.
(Ex 1)

d. Find the area of the rectangle.
(Ex 2)



e. Find the base of a rectangle with an area of 12 cm^2 and a length of 6 centimeters.
(Ex 2)

f. Use the Pythagorean Theorem to find b.
(Ex 3)



g. Use the Pythagorean Theorem to find the area of a triangle with a hypotenuse of 17 millimeters and a side length of 15 millimeters.
(Ex 3)

h. If it is 0° Fahrenheit, what is the temperature in degrees Celsius? Round to the nearest tenth.
(Ex 4)

i. If it is 100° Celsius, what is the temperature in degrees Fahrenheit?
(Ex 4)

Practice Distributed and Integrated

1. Complete the following conjecture.
(7)
The product of an even and an odd number is _______.

2. **Plants** A bean sprout grows 3 inches in its first week, 2 inches in its second week, and 1.333… inches in its third week. If the sprout's growth follows this pattern, how much will the bean sprout have grown in its fourth week?
(7)

3. What are three undefined terms of geometry?
(1)

4. **Coordinate Geometry** State the quadrant in which each point is located.
(SB 13)
a. $(-2, -2)$ **b.** $(-2, 2)$ **c.** $(2, -2)$

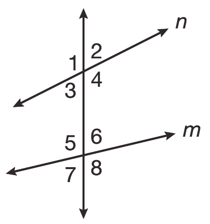
5. If there are twelve turtles in the pet store, what is the probability that a turtle chosen at random will weigh more than the median turtle weight?
(SB 12)

6. **Algebra** The area of a rectangle is 54 cm^2. The side lengths are $2x + 1$ and $x + 2$. What is the measure of each side?
(8)

7. **Analyze** Can a line be intersected at the same point with two different perpendicular lines? Explain why or why not.
(5)

8. How far does a jogger travel to reach the midpoint between a jogging track and a parking lot that are 0.25 miles apart?
(2)

9. Find the next item in the pattern. 2, 4, 8, 16, 32, …
(7)



10. **Multiple Choice** Which pair of angles are congruent?
(6)
A $\angle 1$ and $\angle 8$ **B** $\angle 2$ and $\angle 3$
C $\angle 1$ and $\angle 2$ **D** $\angle 5$ and $\angle 6$

11. **Multiple Choice** Which pair of angles are a linear pair?
(6)
A $\angle 5$ and $\angle 6$ **B** $\angle 6$ and $\angle 7$
C $\angle 1$ and $\angle 4$ **D** $\angle 4$ and $\angle 6$

12. Name all the pairs of supplementary angles formed by line m.
(6)

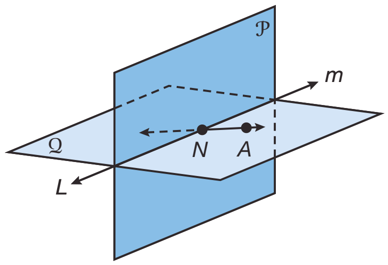
13. **Write** Karl wrote that any three points define a plane. Is his statement true? If not, rewrite his statement so that it is true.
(4)

14. **Verify** A rectangle has side lengths of 30 meters and 36 meters. If the side lengths of the rectangle are doubled, verify that the perimeter also doubles.
(8)

15. **Error Analysis** A student has solved the equation for Pythagorean Theorem in the following way. It is given that $a = 5$ and $b = 12$.
(8)

$$a^2 + b^2 = c^2$$
$$(a + b)^2 = c^2$$
$$(5 + 12)^2 = c^2$$
$$17^2 = c^2$$
$$\sqrt{289} = \sqrt{c^2}$$
$$17 = c$$

Where did the student make an error? What is the actual answer?



16. Identify the intersection between planes $\mathcal{P}$ and $\mathcal{Q}$.
(4)

17. Name the two rays that make $\angle LNA$.
(3)

18. What figure is formed at the intersection of two lines?
(1)

19. **Multiple Choice** Which of the following terms cannot describe lines?
(5)
A perpendicular **B** congruent
C skew **D** parallel

20. Evaluate the expression $\dfrac{x + (3x)^2 + x}{2}$ for $x = \sqrt{2}$. Express the answer in simplified radical form.
(SB 14)

21. (SB 22) **Music** Musical notes are identified by their frequencies, which are recorded in hertz (Hz). Middle C is denoted as C_4 and has a frequency of 261.63 Hz. What is the frequency of the C_3 note, which is one octave below C_4? (Frequency doubles when going up an octave.)

22. (8) Find the perimeter of a triangle if each side length is 136 millimeters.

23. (SB 17) If $f(x) = 3x + x^2$, what does $f(2)$ equal?

24. (SB 11) **Statistics** What is the mode of the following set? {A, B, M, M, H, H, J, P}

25. (3) What is the angle between two opposite rays?

26. (7) **Biology** In a lab experiment, there is a culture that contains 25 bacteria at 2:00. At 2:15, there are 50 bacteria. At 3:15, there are 800 bacteria. Make a conjecture about the rate at which the bacteria increases.

27. (2) By what property is the statement $AB = AB$ justified?

28. (SB 22) **Banking** King is baking a recipe that serves 6 and calls for 1.5 cups of flour. He expects 8 guests and decides to increase the recipe so that it can serve 8. How much flour will he need to make the larger recipe?

29. (8) **Building** A housing developer wants to build a fence around the backyard of a house. If the backyard forms a square and one side is bounded by the house, what formula could the developer use to calculate the amount of fence required?

30. (SB 2) **Verify** Show that the Commutative and Associative Properties of Addition are true when adding the linear functions $f(x) = 3x + 2$, $g(x) = -2x + 3$, and $h(x) = 0$. The first line of each is given below.

Associative Property of Addition	Commutative Property of Addition
$[f(x) + g(x)] + h(x) = f(x) + [g(x) + h(x)]$	$f(x) + g(x) = g(x) + f(x)$

LESSON 9

Finding Length: Distance Formula

Warm Up

1. (SB 1) **Vocabulary** The distance of a number from zero on the number line is the _______________ of that number.

2. (SB 13) In what quadrant is the point (5, −3) on the coordinate plane?

3. (SB 14) Evaluate the expression $\frac{|5x - 8|}{2}$ for $x = 3$.

New Concepts

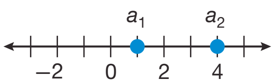
Often, the length of a segment can be measured using a ruler. At other times, it may be necessary to find length by looking at a number line or a coordinate plane.

Hint

An absolute value is used to determine distance and length because it is impossible for something to have a negative length, or for the distance between two points to be negative.

To find the distance between two points on a number line, take the absolute value of the difference between those points' coordinates.

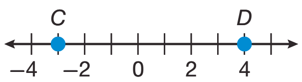
$$d = |a_2 - a_1|$$



The distance between points a_1 and a_2 is $|4 - 1| = 3$.

Example 1 Distance Between Two Points on a Line

Find the distance between the points on the number line.



SOLUTION

Use the formula:

$$\begin{aligned} d &= |a_2 - a_1| \\ &= |4 - (-3)| \\ &= |7| \\ &= 7 \end{aligned}$$

On a coordinate plane, the distance between two points can be found using the distance formula.

Distance Formula
In a coordinate plane, the distance d between two points (x_1, y_1) and (x_2, y_2) is given by the formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

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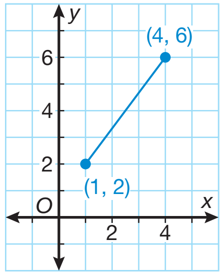
Example 2 Using the Distance Formula

Find the distance between the two points.



SOLUTION

First, choose one point's coordinates to be (x_1, y_1). The other point will be (x_2, y_2). It does not matter which point is chosen.

Let $(1, 2)$ be (x_1, y_1) and $(4, 6)$ be (x_2, y_2).

Substitute into the distance formula.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$d = \sqrt{(4 - 1)^2 + (6 - 2)^2}$$
$$d = \sqrt{3^2 + 4^2}$$
$$d = \sqrt{25}$$
$$d = 5$$

It does not matter which ordered pair is chosen to be (x_1, y_1). It is important, however, that x_1 and y_1 come from the same ordered pair.

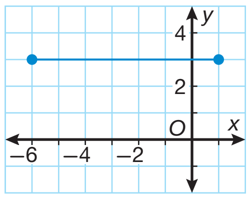
When two points share the same x-value or y-value, the distance formula can be simplified as shown in the next example.

Math Reasoning

Verify Since the diagram shows a horizontal line, an easy way to verify the result of the distance formula is to simply count the number of unit squares the line crosses. Would the result be any different if you flipped the *x* and *y* coordinates of these points?

Example 3 Distance Between Points That Share One Coordinate

Find the distance between the two points.



SOLUTION

Since $y_1 = y_2$, we can substitute and simplify.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_2)^2}$$
$$d = \sqrt{(x_2 - x_1)^2 + 0^2}$$
$$d = \sqrt{(x_2 - x_1)^2}$$

The square root and the square cancel, so with the two identical y-values, the distance formula becomes:

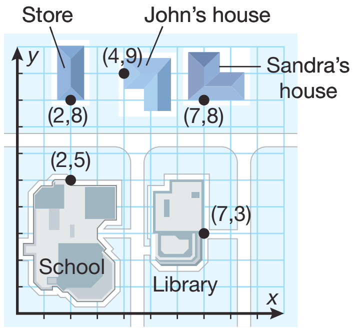
$$d = |x_2 - x_1|$$

An absolute value is used because squaring and then taking the square root of a number always results in a positive number. The resulting formula is identical to the one used to find distance on a number line.

$$d = |x_2 - x_1|$$
$$d = |-6 - 1|$$
$$d = |-7|$$
$$d = 7$$

Example 4 Application: Navigation

Use the following map for each question. The distance is measured from the dot on each building.



a. What is the distance from John's house to the school if each unit on the coordinate plane represents 100 meters? Round to the nearest hundredth.

SOLUTION

John's house is at (4, 9) and the school is at (2, 5).

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d = \sqrt{(4 - 2)^2 + (9 - 5)^2}$

$d = \sqrt{2^2 + 4^2}$

$d = \sqrt{20}$

$d \approx 4.4721$

Since each unit represents 100 meters, multiply the answer by 100.
The distance from John's house to the school is about 447.21 meters.

b. What is the distance from Sandra's house to the store?

SOLUTION

Sandra's house and the store have the same y-coordinate, so the formula for distance on a number line can be used.

$d = |x_2 - x_1|$

$d = |7 - 2|$

$d = 5$

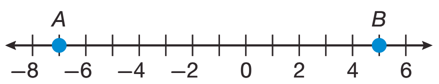
Since each unit represents 100 meters, the distance from Sandra's house to the store is 500 m.

Math Reasoning

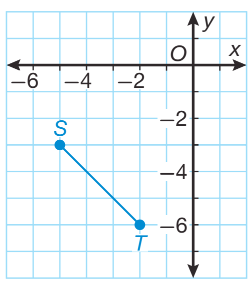
Analyze Can you find a particular route on the map where the distance you would have to walk between two buildings is not accurately expressed by applying the distance formula? Why does the distance formula not work in these situations?

Lesson Practice

a. Find AB.
(Ex 1)



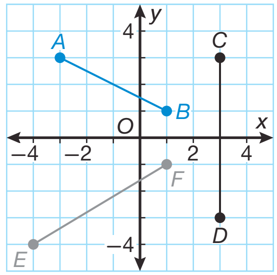
b. What is the distance between points S and T? Round to the nearest hundredth.
(Ex 2)



c. Find the distance between the points (2, 3) and (2, –4).
(Ex 3)

d. The peak of a mountain is located at the coordinate (120, 0). The hiker starts at the bottom of the trail at coordinate (0, 125). If each unit on the coordinate plane represents 10 meters, how far will the hiker walk if he gets to the peak? Round to the nearest tenth.
(Ex 4)

Practice Distributed and Integrated



1. (9) What is the approximate distance between points A and B in the coordinate grid at right?

2. (9) **Verify** What is the approximate distance between the points E and F? Is the answer the same if you find the distance between the points F and E?

3. (SB 18) Factor $(9x^2 - 18x - 7)$.

4. (9) **Coordinate Geometry** Calculate the distance between the points $(9, 14)$ and $(-5, 13)$ to the nearest hundredth.

5. (3) **Clocks** What is the angle between the hands of a clock when it is exactly 4 o'clock?

6. (6) If an angle is obtuse, must its supplement be acute, obtuse, right, or none?

7. (SB 22) **Error Analysis** A student calculated the formula for the perimeter (P) of a regular dodecagon, where n is the side length.

$P = 10n$

Is the student correct? Explain.

8. (SB 1) Evaluate this expression: $\frac{2(2 + 4)}{6} - |-2|$.

9. (4) Is the following statement sometimes, always, or never true?

Two planes intersect at a point.

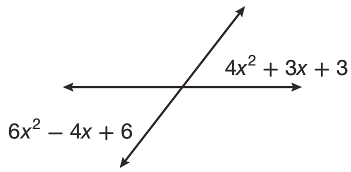
10. (7) Make a conjecture about the pattern at right and find the next line in the pattern.

1
1 1
1 2 1
1 3 3 1

11. (5) **Maps** On a map, the Tropic of Capricorn is parallel to the equator, and the Tropic of Cancer is also parallel to the equator. Do the two tropics ever meet? How do you know?

12. (4) **Write** Describe the difference between a postulate and a theorem.

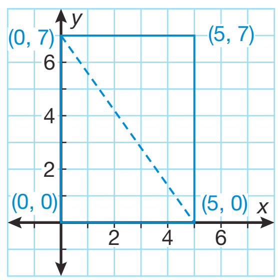
13. (6) **Algebra** Find the value of x in the diagram at right.



14. (4) What two objects intersect at a line? Explain.

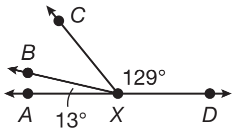
15. (8) **Analyze** Solve the Pythagorean Theorem for c. How is it like the distance formula? Explain your reasoning.

16. (9) **Fencing** If Sven has a rectangular yard with the corners given in the diagram, to the nearest hundredth meter, how much fencing will he need to divide the yard into two triangles? Each unit on the grid represents one meter.



17. (SB 12) There are six bananas, two oranges, and seven apples in a bag. At random, pieces of fruit are chosen from the bag. What is the probability that an apple is chosen first and an orange is chosen second?

18. What is the measure of $\angle BXC$?
(3)



19. **Multiple Choice** Which of the following is not a formula?
(8)
A distance formula
B perimeter formula
C Pythagorean Theorem
D protractor formula

20. Find the product of $(2x - 2)$ and $(x + 1)$.
(SB 18)

21. What property justifies this statement?
(2)
If $\overline{AB} \cong \overline{XY}$ *and* $\overline{CD} \cong \overline{XY}$, *then* $\overline{AB} \cong \overline{CD}$.

22. **Explain** Can two noncoplanar lines intersect? Explain.
(1)

23. **Shopping** If a carton of eggs costs $2.05 with tax included, how many cartons can you buy with $10? Explain how you estimated the answer.
(SB 22)

24. **Multiple Choice** Which of the following is not a pair of angles?
(6)
A supplementary
B adjacent
C right
D vertical

25. **Algebra** If B is the midpoint of $\overline{AC}$, and $AB = \frac{5}{2}x + 12$ and $AC = 12x - 4$, what is the length of $\overline{BC}$?
(2)

26. What is the domain of the function $f(x) = \sqrt{x}$?
(SB 17)

27. How does the distance formula simplify when finding the distance between two points which are directly above each other on a coordinate grid?
(9)

28. Solve $-3w + 2 < 4w + 16$ for w.
(SB 15)

29. A line is perpendicular to another line. Classify the supplementary angles it creates. Are they congruent?
(5)

30. What is the difference between a conjecture and a theorem?
(7)

LESSON **10**

Using Conditional Statements

Warm Up

1. (7) **Vocabulary** ________ reasoning is the process of reasoning in which a rule or statement is considered true because specific cases are true.

2. (4) Melissa notices that all the flowers in her yard bloom during the spring. Based on this observation, she says, "All flowers bloom in the spring." Is this an example of a postulate, a theorem, or a conjecture?

3. (SB 15) Solve the equation $2x + 7 = 4$.

New Concepts

A **conditional statement** is a statement in the form, "If p, then q," where p is the hypothesis and q is the conclusion. For example:

If it is morning, then the sun is in the east.

The **hypothesis** of a conditional statement is the part of the statement that is between the words *if* and *then*. In the statement above, the hypothesis, p, is "it is morning." The **conclusion** of a conditional statement is the part of the statement that follows the word *then*. In the statement above, the conclusion, q, is "the sun is in the east."

Example 1 Identifying the Hypothesis and Conclusion

Identify the hypothesis and conclusion of each conditional statement.

a. *If $2x + 1 = 5$, then $x = 2$.*

SOLUTION

Hypothesis: $2x + 1 = 5$

Conclusion: $x = 2$

b. *If a plant is growing, then it needs water.*

SOLUTION

Hypothesis: *A plant is growing.*

Conclusion: *It needs water.*

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Some conditional statements are true and some are false. This is called the **truth value** of a conditional statement. A statement is only false when the hypothesis is true and the conclusion is false. For example:

If a rectangle has a width of 5 feet and a height of 4 feet, then its area is 30 square feet.

The hypothesis is true, but the conclusion of this statement is false. Since the hypothesis is true but the conclusion is false, the statement's truth value is false.

Math Reasoning

Connect Using the formula you learned in Lesson 9, write the sample statement so it is true.

If a conditional statement's hypothesis is false, then the statement could still be true. For example, consider the statement, "If Ai wins the lottery, he will take a vacation." The hypothesis is false if Ai does not win the lottery, but the statement is still true, because the statement only applies if Ai does win the lottery.

Example 2 Evaluating the Truth Value of a Conditional Statement

Determine whether each statement is true or false. If it is false, explain your reasoning.

a. *If an angle is obtuse, it measures 120°.*

SOLUTION

The hypothesis of this statement is true, but the conclusion is false. An obtuse angle can measure anything greater than 90° and less than 180°. Any obtuse angle that is not 120° could be used to contradict this statement. Therefore, the statement is false.

b. *If two parallel lines intersect, then they form acute angles.*

SOLUTION

The hypothesis of this statement is false because parallel lines are defined as lines that never intersect. When the hypothesis of a conditional statement is false, the conditional statement as a whole has a truth value of "true." The statement cannot be said to be false unless a situation exists where the hypothesis is true.

Math Reasoning

Analyze What is the result of taking the converse of a converse statement?

The **converse** of a statement is the statement formed by exchanging the hypothesis and conclusion. The converse of a statement "if p, then q" has the form "if q, then p." Consider the following conditional statement.

If it is morning, then the sun is in the east.

The converse of this statement is,

If the sun is in the east, then it is morning.

Even if a conditional statement is true, the converse of that statement is not necessarily true. For example:

If an animal is a duck, then it can fly.

The converse of this statement is,

If an animal can fly, then it is a duck.

This statement is not true. There are many animals that can fly that are not ducks.

Example 3 Stating Converses

Write the converse of each statement and determine whether the converse is true.

a. *If an animal is a dog, then it has four legs.*

SOLUTION

Converse: *If an animal has four legs, then it is a dog.*

The converse is not true. We can prove it is not true by finding an untrue example. For example, a cat is also an animal with four legs, but it is not a dog.

b. *If* $x = 4$, *then* $3x + 7 = 19$.

SOLUTION

Converse: *If* $3x + 7 = 19$, *then* $x = 4$.

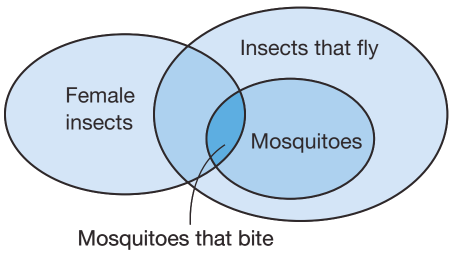
The converse is also true.

Example 4 Application: Biology

Hint

For more about Venn diagrams, see the Skills Bank at the back of this textbook.

Write the converse of each conditional statement. Use the Venn diagram to determine if the converse is true.



a. *If an insect is a mosquito, then it can fly.*

SOLUTION

Converse: *If an insect can fly, then it is a mosquito.*

There are many flying insects besides mosquitoes, so the converse is false.

b. *If a mosquito bites, then it is female.*

SOLUTION

Converse: *If a mosquito is female, then it bites.*

We see from the diagram that the entire region indicating "female mosquitoes" overlaps completely with "mosquitoes that bite," so the converse of this statement is also true.

Lesson Practice

Identify the hypothesis and conclusion of each statement.
(Ex 1)

a. *If* $x = 4$ *and* $y = 2$, *then* $2x + 3y = 14$.

b. *If an apple is a golden delicious apple, then it is yellow in color.*

c. Determine whether the statement is true or false.
(Ex 2)
If two points are collinear, then they are also coplanar.

Find the converse of each statement and determine whether it is true.
(Ex 3)

d. *If* $x^2 = 9$, *then* $x = 3$ *or* -3.

e. *If it is Thanksgiving Day, then it is Thursday.*

f. *If a cardinal is a male, then it is bright red.*

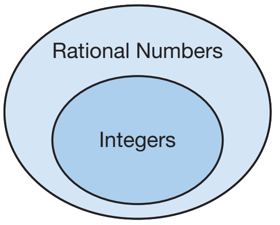
Practice Distributed and Integrated

1. (2) **Driving** Three cities lie on a straight highway. Dawson City is between Orangebourgh and Danteville. If the distance from Dawson City to Danteville is 19 miles and the distance from Orangebourgh to Danteville is 32 miles, what is the distance from Orangebourgh to Dawson City? Name a theorem or postulate that justifies your answer.

* 2. (8) **Signs** What is the perimeter of a regular, triangular yield sign with a side length of 15 inches?

3. (4) **Justify** If two lines are parallel, are they contained in a plane? Why or why not?

* 4. (10) Use the Venn diagram to write a conditional statement.



5. (SB 18) Factor: $x^2 - 4x - 21$.

6. (SB 1) Can the expression $\frac{|x^2 - 4x + 3|}{2}$ ever be a negative number? Explain.

*7. (9) **Multiple Choice** The point (4, 7) is an endpoint of a line segment. The segment is 5 units long. Which point is a possible second endpoint of the line?

A (1, 4) **B** (1, 11)
C (−1, 6) **D** (0, 3)

8. (3) **Write** What is the difference between the Ruler Postulate and the Protractor Postulate?

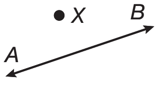
* 9. (10) **Write** A certain conditional statement's conclusion is always true when its hypothesis is true, so the conditional statement is always true. Can you determine for certain that the statement's converse will always be true? Explain and give an example to support your answer.

*10. (10) **Life Span** If an animal is a loggerhead sea turtle, then its expected life span is approximately 70 years. Is this a conditional statement? If it is not, rewrite it as a conditional statement. If so, write its converse.

11. (SB 10) An experiment designed to find the speed of sound shows that it is 768 miles per hour. If the speed of sound is actually 770 miles per hour, what is the experiment's percent error, to the nearest hundredth?

12. (SB 16) Transform the formula $E = mc^2$ to solve for m.

13. (5) **Model** Copy the figure at right and draw a line through the point that is parallel to $\overleftrightarrow{AB}$. Try to draw a different line through the point that is also parallel to $\overleftrightarrow{AB}$. Is it possible to draw a second line? Why or why not?



*14. (9) Find the distance between the two points (2, 3) and (−4, 1) to the nearest hundredth.

***15.** *(9)* The length of a line segment is 7. Its endpoints are (1, 3) and (k, 3). Solve for k. Is there more than one solution? Explain.

16. *(6)* **Model** Is it possible to draw two intersecting lines where the vertical angles are not equal? Why or why not?

17. *(6)* Two streets intersect and are not perpendicular. Two different corners of the streets are at angles that have equal measures. What is another term to describe the angle pair?

18. *(SB 9)* Is it possible to compare two quantities that are recorded in different units of measure? Explain how.

19. *(SB 7)* Give an example of a situation in which scientific notation is commonly used.

20. *(2)* **Justify** What property justifies the statement $x = x$?

***21.** *(8)* **Gardening** A gardener wants to split a rectangular bed of flowers diagonally to make two separate triangular beds. What is the area of one of the right triangles the gardener will form if the diagonal is 13 feet and one side is 12 feet?

22. *(7)* Find the next number in this pattern: 3, 9, 27, 81, …

23. *(SB 7)* Evaluate the expression mc^2 for $m = 1$ and $c = 3 \times 10^8$. Express your answer in scientific notation.

xy^2 ***24.** *(10)* **Algebra** Complete the conditional statement.

If $2x + 7 = 13$, *then* $x =$ ____.

***25.** *(10)* **Error Analysis** Kareem wrote the following statement.

If the measures of three angles combined equal 180°, then the angles are all acute.

Use an example to disprove Kareem's conditional statement, then write a true conditional statement.

26. *(SB 18)* **Error Analysis** A student multiplied the binomial below. Explain where the student made an error. Multiply the binomial correctly.

$(x + 2)(x - 3) = x^2 + x - 6$

27. *(SB 3)* List all the sets of numbers to which -2.8 belongs.

28. *(SB 1)* **Analyze** Find the value of the expression $|3 + (-4) + 6|$. Now find the value of the expression $|3| + |-4| + |6|$. Why are the answers different?

29. *(6)* **Multiple Choice** Which pair of terms is equivalent?

A supplementary angles and vertical angles
B vertical angles and complementary angles
C linear pair and adjacent, supplementary angles
D linear pair and adjacent angles

***30.** *(10)* Draw a Venn diagram to show the relationship in the conditional statement.

If a number is a natural number, then it is a whole number.

INVESTIGATION

1

Transversals and Angle Relationships

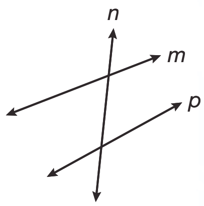
Math Language

Recall that an **angle** is defined as a figure formed by two rays and a common endpoint. It is represented by the symbol ∠.

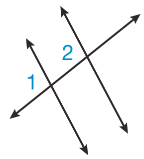
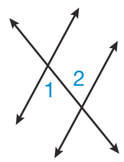
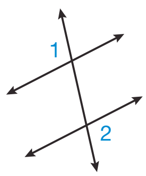
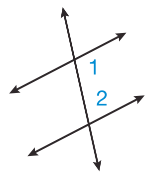
A **transversal** is a line that intersects two or more coplanar lines at different points.

1. Identify the transversal.

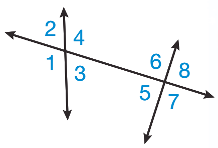
2. How many angles are formed by a transversal crossing two lines?



When two lines are intersected by a transversal, the angles formed are classified according to four types of angle pairs. The example column shows two angles that fit each classification.

Classification	Example
A pair of **corresponding angles** is any pair of angles that lie on the same side of the transversal and on the same sides of the other two lines.	
A pair of **alternate interior angles** is any pair of nonadjacent angles that lie on opposite sides of the transversal and between the other two lines.	
A pair of **alternate exterior angles** is any pair of angles that lie on opposite sides of the transversal and outside the other two lines.	
The **same-side interior angles**, also called the **consecutive interior angles**, are a pair of angles that lie on the same side of the transversal and between the other two lines.	

Give one example of each type of angle pair.



3. corresponding angles

4. alternate interior angles

5. alternate exterior angles

6. same-side interior angles

7. **Generalize** When a transversal intersecting two lines is moved, what happens to the measures of the two angles in a linear pair?

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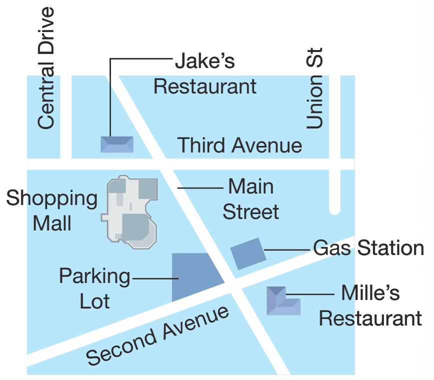
Maps **Identify the following in the town map.**



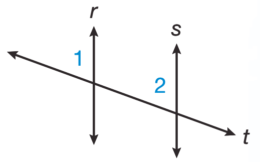
8. A street that is a transversal and the streets that it intersects.

9. Two businesses on street corners that represent an alternate exterior angle pair.

10. The angle pair of the street corners with a shopping mall and parking lot.

A transversal may also intersect two parallel lines.

Multi-Step **Use the diagram to answer the following questions.**



11. What type of angle pair is $\angle 1$ and $\angle 2$?

12. Using a protractor, measure $\angle 1$ and $\angle 2$.

13. What conjecture can you make regarding the measure of a pair of corresponding angles formed when a transversal intersects parallel lines?

Hint

All of the postulates and theorems presented here refer only to transversals that intersect a pair of parallel lines.

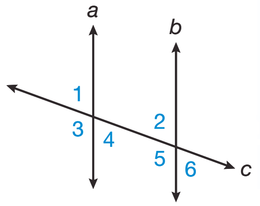
When a transversal intersects parallel lines, the angle pairs that are formed are either supplementary or congruent. Postulate 11 and the theorems below indicate which pairs are congruent and which pairs are supplementary.

Postulate 11: Corresponding Angles Postulate
If two parallel lines are cut by a transversal, then the corresponding angles are congruent.

Theorem 10-1: Alternate Interior Angles Theorem
If two parallel lines are cut by a transversal, then the alternate interior angles are congruent.

Theorem 10-2: Alternate Exterior Angles Theorem
If two parallel lines are cut by a transversal, then the alternate exterior angles are congruent.

Use the diagram to answer these questions.



14. If $m\angle 1 = 50°$, what is $m\angle 2$?

15. **Write** If you know $m\angle 4$, is it possible to know $m\angle 2$? Explain.

Theorem 10-3: Same-Side Interior Angles Theorem
If two parallel lines are cut by a transversal, then the same-side interior angles are supplementary.

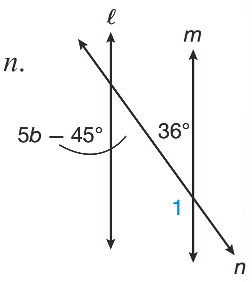
16. If $\angle ABC$ and $\angle DEF$ are a pair of same-side interior angles, what is $m\angle ABC$ when $m\angle DEF = 75°$?

17. **Multi-Step** Lines ℓ and m are intersected by transversal n.

a. What angle pair is represented by the expressions?

b. Find b if $\ell \parallel m$.

c. What is the measure of $\angle 1$?



Investigation Practice

What types of angle pairs are the following?

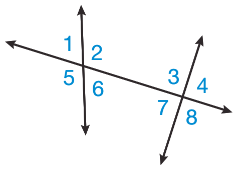
a. $\angle 5$ and $\angle 4$

b. $\angle 6$ and $\angle 7$

c. $\angle 1$ and $\angle 3$

d. $\angle 1$ and $\angle 8$

e. $\angle 3$ and $\angle 6$



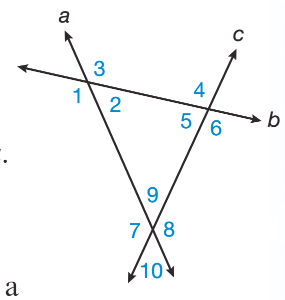
Multi-Step Use the diagram to identify the following.

f. A pair of same-side interior angles with transversal b.

g. A pair of corresponding angles with transversal a.

h. A pair of alternate interior angles with transversal c.

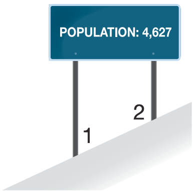
i. Identify the transversal such that $\angle 4$ and $\angle 10$ are a pair of alternate exterior angles.



A sign on a hill has posts that are parallel.

j. Identify the components of the diagram that represent a transversal and the two lines it intersects.

k. What is $m\angle 1$ if $m\angle 2 = 135°$? Justify your answer by naming the theorem used.



l. **Algebra** Determine $m\angle LMP$ and $m\angle ONQ$ when $\overleftrightarrow{MP} \parallel \overleftrightarrow{NQ}$.

m. **Analyze** When a transversal intersects two parallel lines, are all the acute angles congruent? Draw a sketch to demonstrate your answer.