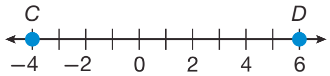
LESSON **11**

Finding Midpoints

Warm Up

1. **Vocabulary** On a coordinate plane, the ordered pair (4, 4) are the __________ of a point. *(SB 13)*

2. What is the distance between points C and D on the graph? *(9)*



3. **Multiple Choice** Which number is halfway between −3 and 5 on the number line? *(2)*

   A 0   **B** 0.5
   C 1   **D** 4

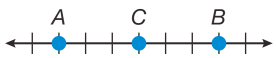
New Concepts

For two points on a number line A and B, the midpoint of $\overline{AB}$ is the point that is **equidistant** from both A and B. For point C to be equidistant from A and B means that the distance from A to C is the same as the distance from B to C.

Midpoint on a Number Line

The midpoint C of $\overline{AB}$ has a coordinate that is the average of the coordinates of A and B:

$$C = \frac{A + B}{2}$$



Math Reasoning

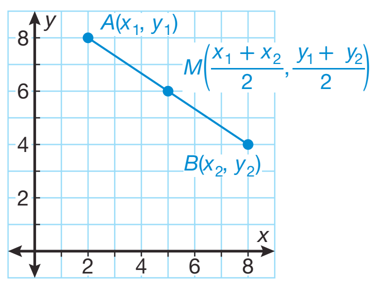
Formulate Describe how the midpoint formula can be inferred from the formula for midpoints on a number line.

The midpoint of $\overline{AB}$ on a coordinate plane is the point M on $\overline{AB}$ that is equidistant from A and B. To find the midpoint of a segment on a coordinate plane, use the midpoint formula given below.

Midpoint on a Coordinate Plane

The midpoint M of $\overline{AB}$ with endpoints $A(x_1, y_1)$ and $B(x_2, y_2)$, has coordinates that are given by the formula:

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$



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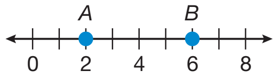
Example 1 Finding the Midpoints

a. What is the coordinate of the midpoint of $\overline{AB}$?

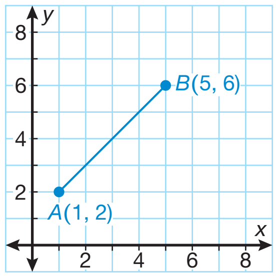


SOLUTION The midpoint is the coordinate on the number line that is the average of the coordinates of the points:

$$C = \frac{2 + 6}{2}$$

$$C = 4$$

b. Determine the midpoint M of $\overline{AB}$ connecting (1, 2) and (5, 6).



SOLUTION

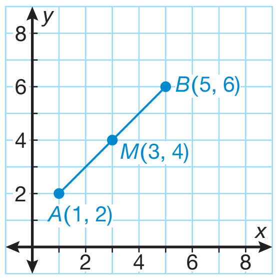
Substitute (1, 2) for (x_1, y_1) and (5, 6) for (x_2, y_2).

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$M\left(\frac{1 + 5}{2}, \frac{2 + 6}{2}\right)$$

$$M(3, 4)$$

To check, plot the point (3, 4). It should lie on $\overline{AB}$.



Also, the distance formula can be used to verify that (3, 4) is equidistant from A and B:

$MA = \sqrt{(3 - 1)^2 + (4 - 2)^2}$	$MB = \sqrt{(3 - 5)^2 + (4 - 6)^2}$
$MA = \sqrt{4 + 4}$	$MB = \sqrt{4 + 4}$
$MA = \sqrt{2(4)}$	$MB = \sqrt{2(4)}$
$MA = 2\sqrt{2}$	$MB = 2\sqrt{2}$ ✓

Math Reasoning

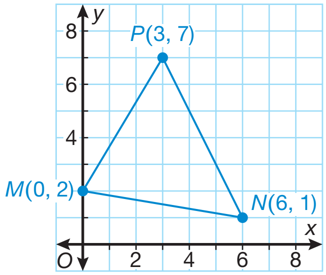
Connect Determine the mean of each pair of numbers: 0 and 4, 1 and 3, and 1.5 and 2.5. How are the concepts of mean and midpoint related?

Math Reasoning

Estimate Before solving Example 2 , look at each side of the triangle and estimate where you think the midpoints might be. This is a useful way to check your answer. How close were your estimates to the actual values?

Example 2 Finding Midpoints of Sides

Determine the midpoint of each side of $\triangle MNP$.



SOLUTION Use the midpoint formula to find A, the midpoint of $\overline{MN}$.

$$A\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$A\left(\frac{0 + 6}{2}, \frac{2 + 1}{2}\right)$$

$A(3, 1.5)$

Similarly, the midpoints B of $\overline{NP}$ and C of $\overline{MP}$ have coordinates:

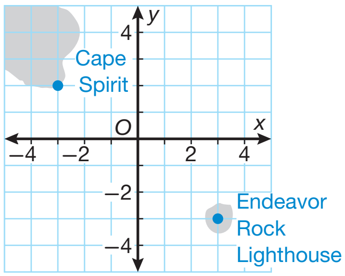
$$B\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \qquad C\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$B\left(\frac{3 + 6}{2}, \frac{7 + 1}{2}\right) \qquad C\left(\frac{0 + 3}{2}, \frac{2 + 7}{2}\right)$$

$B(4.5, 4)$ $\qquad$ $C(1.5, 4.5)$

Example 3 Application: Navigation

A fishing boat dropped its anchor equidistant from Cape Spirit and Endeavor Rock Lighthouse, on the segment joining the two locations. Find the coordinates of the boat.



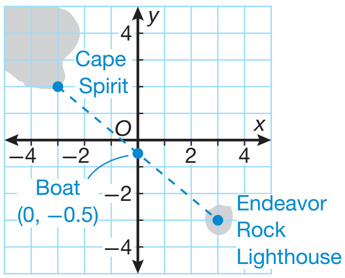
SOLUTION Let point T represent the location of the boat. Point T is the midpoint of the segment with endpoints $(-3, 2)$ and $(3, -3)$.

$$T = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$T = \left(\frac{-3 + 3}{2}, \frac{2 + (-3)}{2}\right)$$

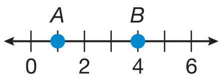
$T = (0, -0.5)$

The diagram shows the location of the boat at $(0, -0.5)$.



Lesson Practice

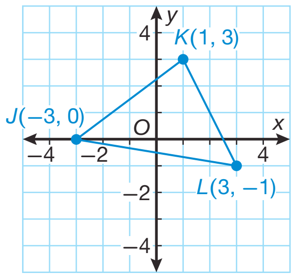
a. On the number line below, what is the midpoint of $\overline{AB}$?
(Ex 1)



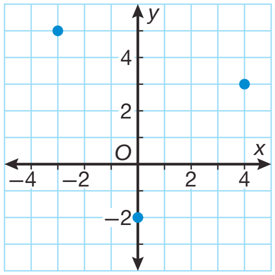
b. Determine the coordinates of the midpoint M for $\overline{AB}$ connecting $A(5, 1)$ and $B(3, 7)$.
(Ex 2)

c. Determine the midpoint of the segment connecting $(-3, 2)$ and $(4, 2)$.
(Ex 2)

d. Determine the coordinates of the midpoint of each side of $\triangle JKL$.
(Ex 3)



e. **Navigation** The map below shows the locations of three buoys. Find the midpoint of the line segments connecting each pair of buoys. State the coordinates of the midpoints.
(Ex 4)



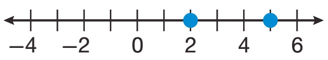
Practice Distributed and Integrated

1. What is the midpoint of the segment connecting 2 and 9 on a number line?
(11)

2. Determine the midpoint M of $\overline{AB}$ connecting $A(3, 2)$ and $B(7, 4)$.
(11)

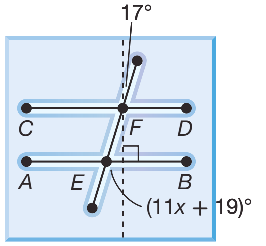
3. Determine the midpoint M of $\overline{PQ}$ connecting $P(-3, 2)$ and $Q(2, -2)$.
(11)

4. **Algebra** Use the number line to determine the value of a, where the midpoint of a and 5 is 2.
(11)



5. **Error Analysis** Fred claims that the midpoint of $\overline{AB}$ connecting $A(-2, 5)$ and $B(2, 0)$ is $(0, 5)$. What mistake did he make?
(11)

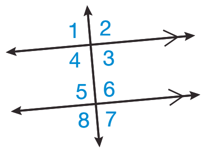
Refer to the figure to answer each question.



6. **Drafting** Hoyt used engineering software to draft a sketch of a metal plate etched with grooves that is needed to complete a highly secure lock. Segments CD and AB are parallel, and $\overline{EF}$ intersects each at 17° from vertical. What is the measure of $\angle AEF$?
(Inv 1)

7. **Drafting** What is the value of x?
(Inv 1)

8. **Verify** What theorem or postulate can be used to justify the statement $\angle 1 \cong \angle 7$?
(Inv 1)



Determine whether each conditional statement is true. If the statement is false, give a counterexample.

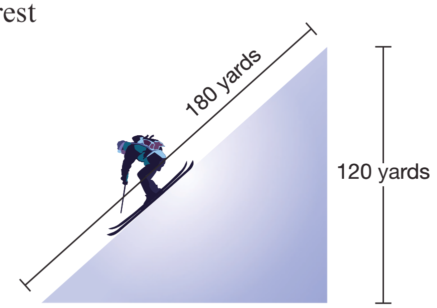
9. If it is winter, then the month is January.
(10)

10. If $2x + 2y = 6$, then $x = 2$ and $y = 1$.
(10)

11. *(9)* Find the distance between points (3, 4) and (−9, 7) to the nearest hundredth.

12. *(9)* Find the distance between points (−1, 4) and (5, −6) to the nearest hundredth.

13. *(8)* **Skiing** A 180-yard ski slope is 120 yards high. What is the horizontal distance of the hill to the nearest yard?



14. *(8)* Find the perimeter of a right triangle with two legs which each measure 6 units. Round to the nearest hundredth.

15. *(8)* Find the side length of a square with a perimeter of 458 centimeters.

16. *(7)* **Generalize** Fill in the next three rows of the following pattern:

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

17. *(7)* Use inductive reasoning to determine the next term: 1, 3, 9, 27, 81, 243, ______

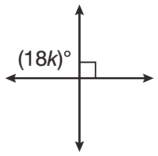
18. *(7)* **Write** Morgan has been following college football for many years and makes the statement that her team will win if she wears her lucky sweater. Comment on the validity of her conjecture.

19. *(6)* Find the complement of a 14° angle.

20. *(6)* Find the supplement of an 85° angle.

21. *(5)* Two coplanar lines are parallel when a third line intersects these two lines at 90°. Can it also be said that two planes are parallel if a third plane intersects these two planes at 90°?

22. *(5)* In this figure, what is the value of k?



23. *(4)* What is the greatest number of planes determined by three points? What is the least?

24. *(4)* Is this statement always, sometimes, or never true? *Two lines intersect at one point*.

25. *(3)* **Multi-Step** If $\overleftrightarrow{SU}$ lies in the right angle $\angle RST$ and $\angle RSU$ is one-fourth the measure of $\angle UST$, what is $m\angle UST$?

26. *(3)* **Clocks** What is the angle of a clock's hands when it is 2:30?

27. *(2)* **Write** Explain the difference between congruence and equality. Use an example.

28. *(2)* Given that P, Q, and R are collinear and Q is between P and R, what postulate can be used to justify this addition? $PQ + QR = PR$

29. *(1)* **Error Analysis** Joy made the statement, "Any three points are noncollinear." Explain where Joy made an error and suggest a new, true statement.

30. *(1)* **Multiple Choice** Which statement is true?

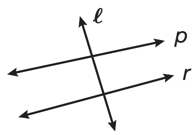
A Any two planes are coplanar.
B Any two points are noncollinear.
C Any two points are coplanar.
D Any two lines are coplanar.

LESSON 12

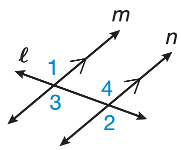
Proving Lines Parallel

Warm Up

1. *(Inv 1)* **Vocabulary** In this diagram, line ℓ is a(n) __________ to lines p and r.



2. *(Inv 1)* What type of angles are ∠1 and ∠2?



3. *(Inv 1)* **Multiple Choice** What type of angles are ∠3 and ∠4?

A Alternate interior angles
B Alternate exterior angles
C Corresponding angles
D Same-side interior angles

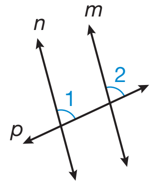
New Concepts

In Investigation 1, you learned Postulate 11: if two parallel lines are cut by a transversal, the corresponding angles formed are congruent. The converse of Postulate 11 is also true, and can be used to show that two lines are parallel.

Postulate 12: Converse of the Corresponding Angles Postulate

If two lines are cut by a transversal and the corresponding angles are congruent, then the lines are parallel.

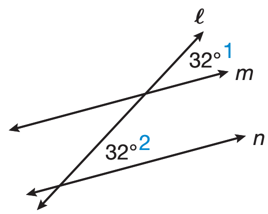
If $\angle 1 \cong \angle 2$, then $m \parallel n$.



> **Hint**
>
> The postulate and theorems in Investigation 1 refer to two parallel lines cut by a transversal to prove that certain angles are congruent or supplementary. The postulate and theorems in this lesson work conversely. That is, they use known angle relationships to prove that lines are parallel.

Example 1 Proving Parallelism: Corresponding Angles

Prove that lines m and n in this diagram are parallel.



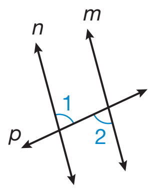
SOLUTION

Angles 1 and 2 both measure 32°, so by the definition of congruent angles, ∠1 and ∠2 are congruent. Since ∠1 and ∠2 are corresponding congruent angles, lines m and n are parallel by Postulate 12.

Theorem 12-1: Converse of the Alternate Interior Angles Theorem

If two lines are cut by a transversal and the alternate interior angles are congruent, then the lines are parallel.

If $\angle 1 \cong \angle 2$, then $m \parallel n$.



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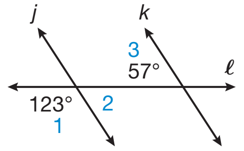
Example 2 Proving Parallelism: Alternate Interior Angles

Prove that lines j and k in this figure are parallel.



SOLUTION

Angles 1 and 2 form a linear pair, which means they are supplementary.

$m\angle 1 + m\angle 2 = 180°$

$123° + m\angle 2 = 180°$

$m\angle 2 = 57°$

Since $m\angle 2 = m\angle 3$, $\angle 2 \cong \angle 3$. Angles 2 and 3 are congruent alternate interior angles, so by Theorem 12-1, lines j and k are parallel.

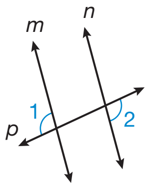
Math Language

Two angles are **supplementary** if the sum of their measures equals 180°.

Theorem 12-2: Converse of the Alternate Exterior Angles Theorem

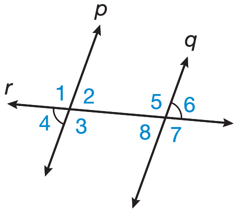
If two lines are cut by a transversal and the alternate exterior angles are congruent, then the lines are parallel.

If $\angle 1 \cong \angle 2$, then $m \parallel n$.



Example 3 Proving Parallelism: Alternate Exterior Angles

a. Identify both pairs of alternate exterior angles in this figure.



SOLUTION

$\angle 1$ and $\angle 7$ are alternate exterior angles.

$\angle 4$ and $\angle 6$ are also alternate exterior angles.

b. Prove that lines p and q are parallel.

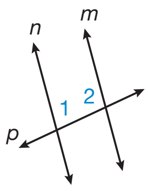
SOLUTION

The angle congruency marks show that the alternate exterior angles, $\angle 4$ and $\angle 6$, are congruent. Therefore lines p and q are parallel by the Converse of the Alternate Exterior Angles Theorem (Theorem 12-2).

Theorem 12-3: Converse of the Same-Side Interior Angles Theorem

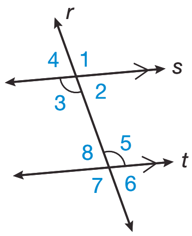
If two lines are cut by a transversal and the same-side interior angles are supplementary, then the lines are parallel.

If $m\angle 1 + m\angle 2 = 180°$, then $m \parallel n$.



Example 4 Proving Parallelism: Same-Side Interior Angles

a. Identify both pairs of same-side interior angles in this figure.



SOLUTION

Angle 2 and ∠5 are same-side interior angles. Angle 3 and ∠8 are also same-side interior angles.

b. Use the Converse of the Same-Side Interior Angles Theorem (Theorem 12-3) to prove that lines *s* and *t* are parallel.

SOLUTION

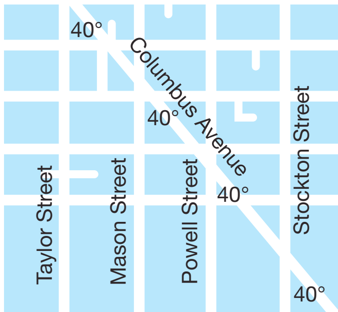
It is shown in the drawing that ∠3 ≅ ∠5. Angle 5 and ∠8 are supplementary since they form a straight line. Therefore, by substitution, ∠3 and ∠8 are supplementary. Since ∠3 and ∠8 are also same-side interior angles, Theorem 12-3 proves that lines *s* and *t* are parallel.

Math Reasoning

Analyze Could another theorem in this lesson have been used to prove that lines *s* and *t* are parallel?

Example 5 Application: City Planning

In San Francisco, California, Columbus Avenue crosses Stockton, Powell, Mason, and Taylor Streets as shown on the map. Columbus Avenue makes a 40° angle with each of these four streets.



a. What geometric term best describes Columbus Avenue?

SOLUTION

Columbus Avenue is a transversal.

b. Prove that Powell, Mason, and Taylor streets are all parallel to each other.

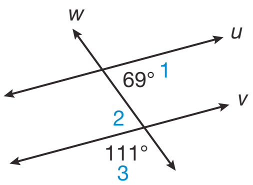
SOLUTION

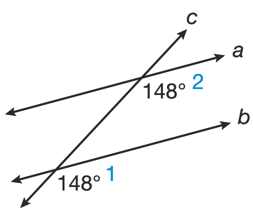
The two 40° angles at the intersections of Columbus and Mason, and Columbus and Powell are congruent by definition. They are also corresponding angles. By Postulate 12, Mason and Powell are parallel. Taylor is parallel to Mason and Powell for the same reason. Since two lines that are parallel to the same line are also parallel to each other (Theorem 5-7), all three streets are parallel to one another.

Lesson Practice

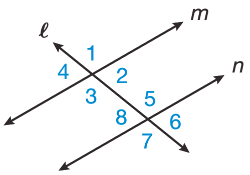
a. Prove that lines *a* and *b* in this figure are parallel.
(Ex 1)

b. Prove that lines *u* and *v* in this figure are parallel.
(Ex 2)





Use the diagram to answer problems c through f.
(Ex 3, 4)



c. Identify both pairs of alternate exterior angles in this figure.

d. Given that $\angle 1 \cong \angle 7$, prove that lines m and n are parallel.

e. Identify both pairs of same-side interior angles in this figure.

f. Given that $\angle 2 \cong \angle 6$, use Theorem 12-3 to prove that lines m and n are parallel.

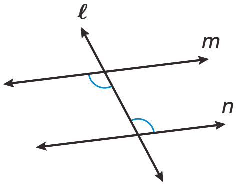
Use the diagram to answer problems g and h.
(Ex 5)

g. **City Planning** Speer Boulevard crosses Fox Street, Elati Street, and Delaware Street. Give the geometric term for Speer Boulevard.

h. **City Planning** Prove that Fox, Elati, and Delaware streets are all parallel to one another.

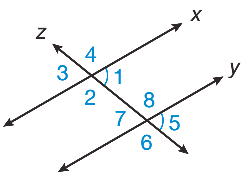
Practice Distributed and Integrated

* **1.** Prove that lines m and n are parallel.
(12)



2. **Justify** What type of reasoning is being used in forming the following statement?
(7)
The numbers 5, 10, 15, 20, and 25 all end in either a 5 or a 0, and can all be divided by 5. Therefore, all numbers ending in 5 or 0 can be divided by 5.

* **3.** **a.** Use the Converse of the Corresponding Angles Postulate to prove that lines x and y are parallel.
(12)
b. Identify the same-side interior angles in the figure.



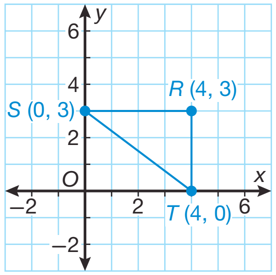
* **4.** **Write** A pair of same-side interior angles formed by a transversal across two lines measure $(3x + 10)°$ and $(2x)°$. Explain how to find a value of x that would indicate that two lines are parallel.
(12)

5. What is the midpoint of the segment connecting 4 and 8 on the number line?
(11)

6. **Carpentry** Karen drilled holes in a 12-inch wooden ruler at 1 inch, 6 inches, and 11 inches along its length. She wants to drill two more holes at the midpoints between the holes she already has. Where should the new holes go?
(11)

7. (9) **Write** One endpoint of a segment lies on the origin. If the second endpoint is known to lie on the line $x = 4$, and the segment is a units long, explain how you could find the coordinates of the other endpoint.

Refer to the figure to find the requested points.



8. (11) What is the midpoint of $\overline{SR}$? of $\overline{RT}$?

9. (11) What is the midpoint of $\overline{ST}$?

Determine whether each conditional statement is true. If the statement is false, give an example why it is false.

10. (10) If you have two points, then they can be connected by exactly one line.

11. (10) **Track and Field** If a runner wins the gold medal, then that runner was the fastest in the race.

12. (6) **Algebra** Two angles are supplementary. One measures $(4x + 2)°$ and the other $(6x + 8)°$. What is the value of x?

13. (8) **Building** A builder wants to build a diagonal beam across a decorative window. If the window is 3 feet by 5 feet, then to the nearest hundredth, how long is the diagonal?

14. (11) What is the midpoint of the segment connecting the points $(-3, -4)$ and $(4, 3)$?

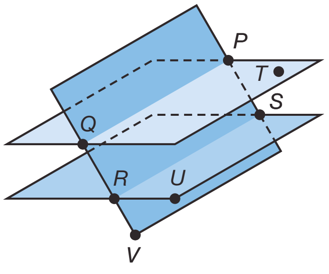
15. (4) Is this statement always, sometimes, or never true?

Two planes intersect in exactly one line.

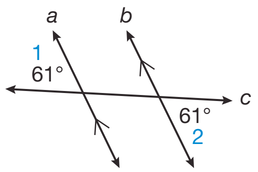
16. (8) Find the side length of a square with an area of 182.25 m^2.

17. (8) **Tiling** A square ceramic tile is 18 inches in length. If a room is 12 feet by 18 feet, how many tiles are needed?

18. (5) The parallel planes *PQT* and *SRU* are cut by the plane *PQR*. Name one pair of parallel lines in the figure.



***19.** (12) Prove that lines a and b are parallel.



20. (6) **Landscaping** A support wire to a newly planted tree makes a 32° angle with the ground. Determine the angle made by the wire at the point of contact to the tree by finding the complement of 32°.

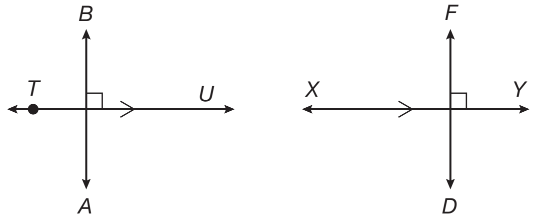
21. (10) **Multiple Choice** Which statement disproves the following conditional statement?

If $y = 2x^2 - 5$, then y is positive.

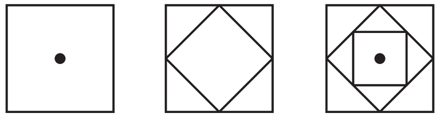
A $x = 1$ **B** $x = 2$

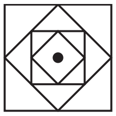
C $x = 3$ **D** $x = 100$

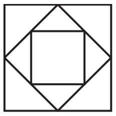
22. *(5)* Is $\overline{AB}$ parallel to $\overline{DF}$? Explain.

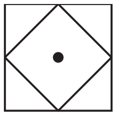


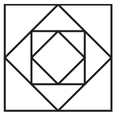
23. *(7)* **Multiple Choice** Use inductive reasoning to determine the next item in the sequence.



A 

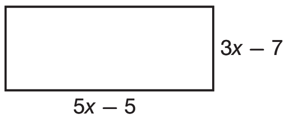
B 

C 

D 

24. *(5)* **Write** How many different ways can coplanar lines intersect? Explain each situation.

25. *(8)* **Multi-Step** Find the approximate area of the rectangle if the perimeter is 84 inches.



26. *(4)* **Error Analysis** Sarita said that if there are two lines, they are both contained in the same plane. Explain a situation where she could be wrong.

27. *(3)* **Clocks** What is the angle made by the hands of a clock when it is 8 o'clock?

28. *(2)* **Multiple Choice** Which of the following is not a property of congruence?

A Mirror Property
B Transitive Property
C Symmetric Property
D Reflexive Property

29. *(2)* Point C lies between A and B on $\overline{AB}$. If $CB = 18$ and $AB = 42$, what is AC?

30. *(1)* How many possible points of intersection can three different coplanar lines have?

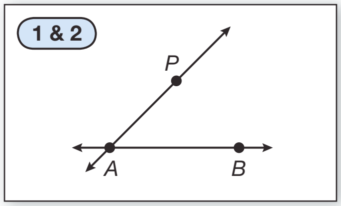
LAB 4

Parallel Line through a Point

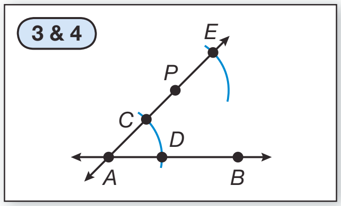
Construction Lab 4 *(Use with Lesson 12)*

In Lesson 12, you practiced proving lines parallel by showing that certain angle pairs are congruent. This lab will show you how to construct a line parallel to a given line through a point using corresponding angles.

1. Begin by drawing a line and a point not on the line, labeled P. Label any two points on the line A and B.
2. Draw a line through P such that it intersects $\overleftrightarrow{AB}$ at A.



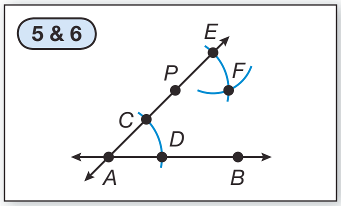
3. With the compass centered at A, draw an arc across both rays of $\angle PAB$. Label the intersection points C and D as shown.
4. Using the same compass setting, center the compass at P. Draw an arc across $\overleftrightarrow{AP}$ such that it intersects $\overleftrightarrow{AP}$ at point E, as shown. Be sure the arc extends downward below P. Point P should lie between points A and E.



Hint

This is the same procedure you learned in Construction Lab 2 for constructing an angle congruent to a given angle.

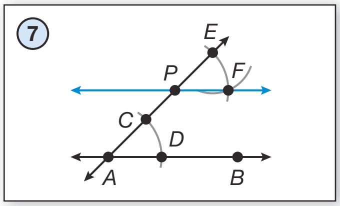
5. Set the compass width to the distance between C and D.
6. Center the compass at E and draw an arc that intersects the arc drawn in step 4. Label the intersection of the two arcs as point F.



7. Draw a line through points P and F.

   Line PF is parallel to $\overleftrightarrow{AB}$.

   $$\overleftrightarrow{PF} \parallel \overleftrightarrow{AB}$$



This construction is a direct application of the Converse of the Corresponding Angles Postulate.

Lab Practice

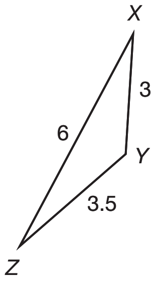
Use a straightedge to draw a line and a point not on the line on a blank sheet of paper. Trade with a partner and construct a line parallel to the given line through the given point. Use a protractor to verify that corresponding angles are equal.

LESSON 13

Introduction to Triangles

Warm Up

Use the diagram to answer problems 1 to 3.



1. **Vocabulary** In $\triangle XYZ$, $\angle Y$ is a(n) ________ angle. *(3)*
   (***obtuse, acute, right, straight***)

2. Determine the perimeter of $\triangle XYZ$. *(8)*

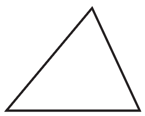
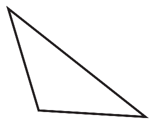
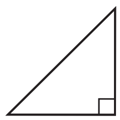
3. **Multiple Choice** Classify the angles X and Z in $\triangle XYZ$. *(3)*
   A acute, right **B** acute, acute
   C obtuse, obtuse **D** congruent

New Concepts

A **triangle** is a three-sided polygon. A triangle can be classified by its angles or by its sides. The table below shows three ways to classify a triangle according to its angles.

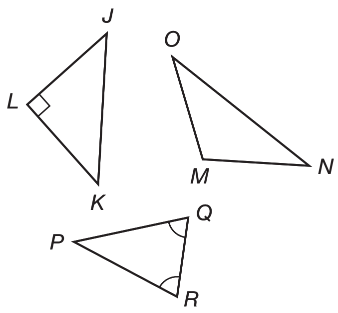
Math Reasoning

Model An obtuse triangle has exactly one obtuse angle. Try to draw a triangle with two obtuse angles. What do you notice?

Acute Triangle	Obtuse Triangle	Right Triangle
Any triangle that has three acute angles is an **acute triangle**. 	Any triangle that has one obtuse angle is an **obtuse triangle**. 	Any triangle that has one right angle is a **right triangle**. 

A special kind of acute triangle is an **equiangular triangle**, which has three congruent angles.

Example 1 Classifying Triangles by Angles



a. In the diagram, which triangle is obtuse?

SOLUTION

$\triangle MNO$ is obtuse because it has one obtuse angle M.

b. Which triangle is a right triangle?

SOLUTION

$\triangle JLK$ is a right triangle because $\angle L$ is a right angle.

c. Are any of the triangles equiangular?

SOLUTION

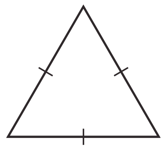
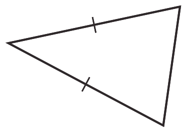
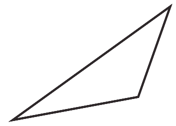
No. $\triangle JLK$ and $\triangle MNO$ are not acute, so they cannot be equiangular. $\triangle PQR$ is acute (because all its angles are acute), but is not equiangular because $\angle P$ is not congruent to the other two angles.

Online Connection
www.SaxonMathResources.com

Triangles may also be classified by the lengths of their sides. The table below summarizes three ways to classify a triangle by its sides.

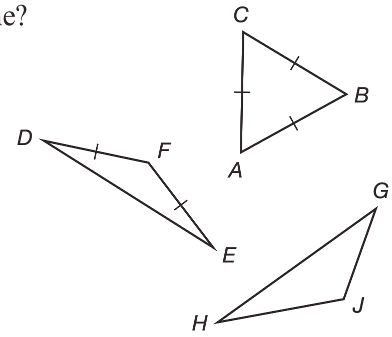
Math Reasoning

Write Explain why an equilateral triangle is always isosceles, but not vice versa.

Equilateral Triangle	Isosceles Triangle	Scalene Triangle
Any triangle that has three congruent sides is an **equilateral triangle.** 	Any triangle with at least two congruent sides is an **isosceles triangle.** 	Any triangle that does not have any congruent sides is a **scalene triangle.** 

Example 2 Classifying Triangles by Sides

a. In the diagram, which triangle is scalene?



SOLUTION

$\triangle GHJ$ is scalene, because none of its sides are congruent.

b. Which triangle is equilateral?

SOLUTION

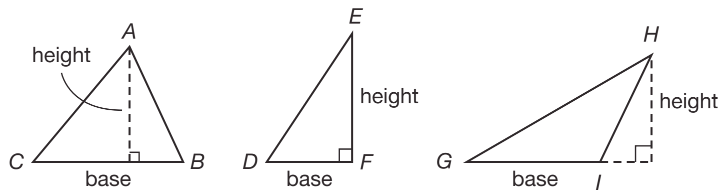
$\triangle ABC$ is equilateral, because all three sides are congruent.

c. Are any of the triangles isosceles but not equilateral?

SOLUTION

Yes. $\triangle ABC$ and $\triangle DEF$ are both isosceles, because at least two sides are congruent. $\triangle DEF$ is not equilateral because its third side is not congruent to the other two.

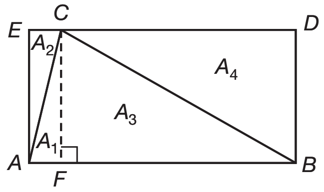
A **vertex of a triangle** is one of the points where two sides of the triangle intersect. A **base of a triangle** can be any one of the triangle's sides. The **height of a triangle** is the perpendicular segment from a vertex to the line containing the opposite side. The length of that segment is also called the height.



In $\triangle GHI$, the perpendicular segment from H does not intersect the base. The base is extended so a perpendicular segment can be drawn to show the height. To find the area of a triangle, both the base and the height must be known.

Area of a Triangle
The area of a triangle is given by the formula below, where b is the length of the triangle's base and h is the height. $$A = \frac{1}{2}b \times h$$

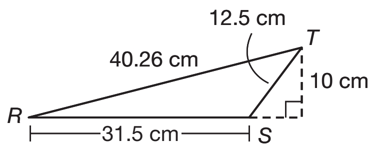
The diagram shows $\triangle ABC$ enclosed in rectangle $ABDE$. Notice that $\triangle AFC$ and $\triangle CEA$ have the same base and height, so areas A_1 and A_2 are equal. Similarly, $A_3 = A_4$. The area of rectangle $ABDE$ is $b \times h$. Therefore,



$$\begin{aligned}\text{Area of } ABDE &= A_1 + A_2 + A_3 + A_4 \\ b \times h &= A_1 + A_1 + A_3 + A_3 \\ b \times h &= 2(A_1 + A_3) \\ \frac{1}{2}b \times h &= A_1 + A_3 \\ \frac{1}{2}b \times h &= \text{Area of } \triangle ABC\end{aligned}$$

Example 3 Finding Perimeter and Area of a Triangle

a. Determine the perimeter of $\triangle RST$.



SOLUTION

$$\begin{aligned}P &= TR + RS + ST \\ &= 40.26 + 31.5 + 12.5 \\ &= 84.26\end{aligned}$$

The perimeter is 84.26 cm.

b. Determine the area of $\triangle RST$.

SOLUTION

$$\begin{aligned}A &= \frac{1}{2}bh \\ &= \frac{1}{2}(31.5)(10) \\ &= 157.5\end{aligned}$$

The area is 157.5 square centimeters.

Math Language

The **perimeter** of a closed figure is the sum of its side lengths. So, to find the perimeter of a triangle, add the lengths of its three sides together.

Example 4 Application: Farming

A triangular plot of land has a northwestern boundary measuring 64.6 yards, a southern boundary measuring 138.0 yards, and a north-eastern boundary measuring 114.1 yards. The perpendicular distance from the southern boundary to the northern corner of the plot is 53.0 yards.

Hint

To aid in solving this example, draw a picture of the plot of land being described.

a. How much fencing is required to surround the plot?

SOLUTION The perimeter is

$$\begin{aligned}P &= 64.6 + 138.0 + 114.1 \\ &= 316.7\end{aligned}$$

316.7 yards of fencing are required.

b. It takes 100 pounds of barley seed to seed 2400 square yards of land. How much seed is needed for the whole plot, to the nearest pound?

SOLUTION The area of the plot is

$$A = \frac{1}{2}b \times h$$

$$= \frac{1}{2}(138.0)(53.0)$$

$$= 3657 \text{ yd}^2$$

100 pound of barley covers 2400 square yards

Use a proportion:

$$\frac{100}{2400} = \frac{x}{3657}$$

$$(3657)(100) = (2400)(x)$$

$$365{,}700 = 2400x$$

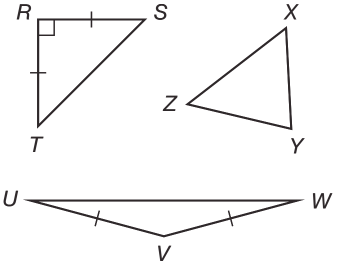
$$\frac{365{,}700}{2400} = x$$

$$152.375 = x$$

To the nearest pound, 152 pounds of seed is needed for the whole plot.

Lesson Practice

Use the diagram to answer problems a through d.



a. Which triangle is obtuse?
(Ex 1)

b. Which triangle is acute?
(Ex 1)

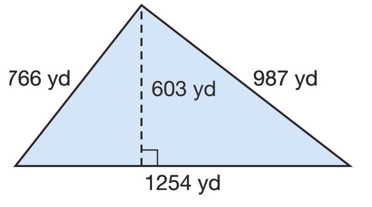
c. Which triangle is a right isosceles triangle?
(Ex 2)

d. Are any of the triangles scalene?
(Ex 2)

e. A right isosceles triangle has legs measuring 13.2 centimeters and a hypotenuse measuring 18.7 centimeters. What is its perimeter?
(Ex 3)

f. What is the area of the triangle in part e?
(Ex 3)

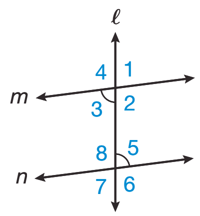
g. **Surveying** A golf course is planned for the plot of land shown on the right. How much boundary fencing is required to surround the plot?
(Ex 4)



h. **Surveying** Grass sod is needed for 95% of the area of the golf course shown above. What area of sod is needed, rounded to the nearest square yard?
(Ex 4)

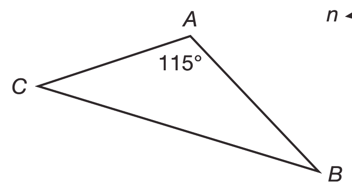
Practice Distributed and Integrated

1. (12) Identify a pair of angles in the diagram that can be used to prove lines parallel using the Converse of the Same-Side Interior Angles Theorem.



2. (10) **Write** Write a conditional statement that has a false converse.

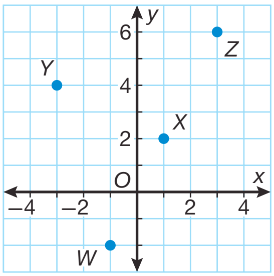
3. (13) Classify $\triangle ABC$ as acute, right, or obtuse.



4. (4) **Analyze** Is it possible to draw three points that are noncoplanar? What about drawing four points?

5. (7) **Algebra** Test the conjecture that the sum of the first n odd whole numbers is given by n^2 using $n = 3$, $n = 5$, and $n = 7$.

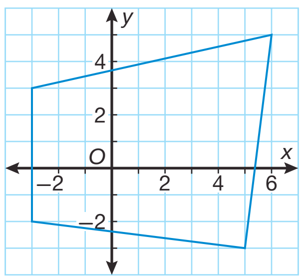
Use the diagram for the next two questions. Round to the nearest hundredth.



6. (9) What is the length of $\overline{YZ}$?

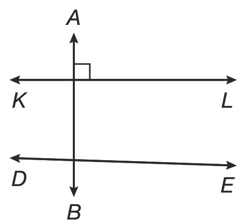
7. (9) What is the length of $\overline{WX}$?

8. (11) **Surveying** A hedge surrounds the field shown on the grid. There are openings at the midpoints of the northern and southern sides of the hedge for a hiking trail. Determine the coordinates of the openings in the hedge.

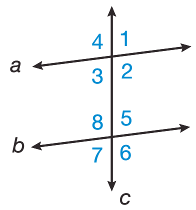


9. (7) **Write** Shanice hears that the weather forecast predicts rain tomorrow. She thinks, "Since they have been wrong every day this week, it will not rain." Is Shanice's conjecture valid? Explain.

10. (5) Is $\overleftrightarrow{AB}$ perpendicular to $\overleftrightarrow{DE}$? Explain.



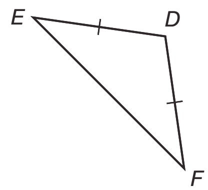
11. (12) Suppose ∠3 and ∠8 are supplementary. Which postulate or theorem should be used to prove that lines a and b are parallel?



12. (1) **Multiple Choice** What is the greatest number of intersection points that four coplanar lines can have?

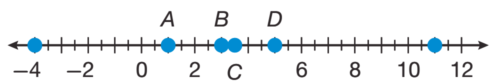
A 0 **B** 2
C 4 **D** 6

13. (13) Classify $\triangle DEF$ by its sides. Is $\triangle DEF$ also equilateral?



14. (7) **Error Analysis** Alejandro and Jessica were asked to find the next number in the sequence 2, 4, ____. Alejandro makes the conjecture that the next number will be 6, while Jessica makes the conjecture that the number will be 8. Who is correct?

15. (11) **Multiple Choice** Which point is the midpoint of the segment connecting –4 and 11?



A 1 **B** 3
C 3.5 **D** 5

16. **Write** Explain why an obtuse angle cannot be congruent to an acute angle.
(3)

17. **Passports** Write a conditional statement from the following statement.
(10)
People born in the United States can have an American passport.

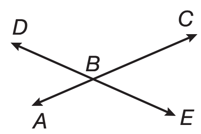
18. **Agriculture** A farmer is building a rectangular pen for his chickens with an area of 40 square feet. If one side measures 8 feet, what is the length of the other side?
(8)

19. Draw a pair of parallel lines and label them n and m. Then draw a transversal that is not perpendicular to either line and label it k. Mark all congruent acute angles on the figure.
(Inv 1)

20. **Architecture** A certain tower has eight triangular faces. Four of these faces have approximate base lengths of 200 feet and heights of 1280 feet. Estimate the number of panes of glass needed for one of these faces, if each pane of glass has an area of approximately 45 ft^2.
(13)

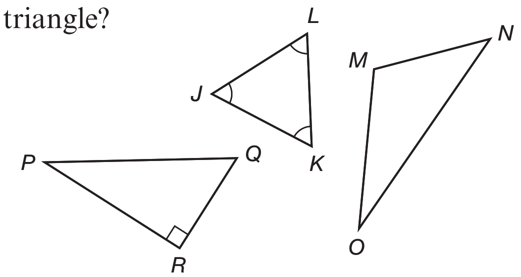
21. Write the converse of the following conditional statement.
(10)
If a bird is a flamingo, then it is pink.

22. **Justify** Is $\angle ABD$ congruent to $\angle CBE$? Justify your answer.
(6)



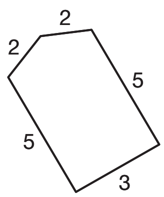
23. Is the following statement always, sometimes, or never true?
(4)
Three noncoplanar points are contained in a single space.

24. **a.** Of the three triangles shown, which triangle is a right triangle?
(13)
b. Which triangle is obtuse?
c. Which triangle is equiangular?

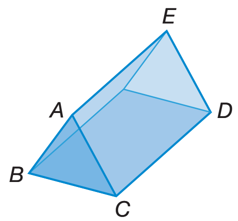


25. **a.** Find the midpoint of the segment connecting -5 and 0.
(11)
b. Draw a number line containing -5 and 0. Label the midpoint.

26. What is the perimeter of the pentagon shown?
(8)

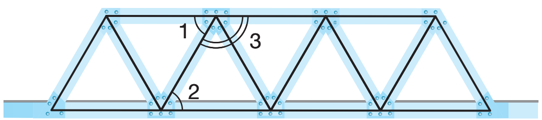


27. **Algebra** Suppose E is the midpoint of $\overline{CD}$, $CD = 2x + 7$, and $DE = 4x - 13$. What is CE?
(2)



28. **Justify** Three planes, BCD, ABE, and ACD are arranged so that they intersect in pairs, forming the plane arrangement shown at right. Is the information given sufficient to determine that $\overline{AE} \parallel \overline{CD}$? Explain.
(5)

29. **Civil Engineering** Each diagonal girder on this bridge forms a transversal with the upper and lower horizontal girders.
(12)



a. Suppose the angles 1 and 2 are congruent. Prove that the upper and lower girders are parallel.
b. Describe another way you could prove that the upper and lower girders are parallel.

30. If the measure of an acute angle is $(12x + 30)°$, what is the range of values for x?
(3)

LESSON 14

Disproving Conjectures with Counterexamples

Warm Up

1. (7) **Vocabulary** A statement that is believed to be true but has not been proved is a __________.

2. (10) **Multiple Choice** What is the hypothesis of the conditional statement below?
If it rains today, the road will be slick.

 A It rains today.
 B The road will be slick.
 C It does not rain today.
 D The road will not be slick.

3. (10) **Multiple Choice** What is the conclusion of the conditional statement below?
If a triangle has one acute angle and one obtuse angle, then the remaining angle is acute.

 A A triangle has one acute angle and one obtuse angle.
 B The remaining angle is acute.
 C A triangle has one obtuse angle.
 D A triangle has one acute angle.

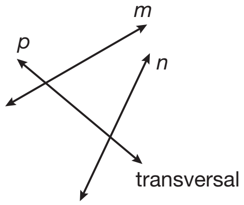
New Concepts

Consider the simple conjecture given below.

If two lines are both intersected by a transversal, then they are parallel.

This conjecture is false: two lines do not have to be parallel to be intersected by a transversal. A simple way to prove that this statement is not true is to use a counterexample.

A **counterexample** is an example that proves a conjecture or statement false. For example, the diagram shows a pair of lines that are not parallel, but they are intersected by a transversal. It disproves the statement given above because it gives a specific example where the statement is *not* true. To construct a counterexample, find a situation where the hypothesis of the statement is true but the conclusion is false.



Math Reasoning

Write Give a counterexample to prove the following conjecture false: *if a fruit is an apple, then it is green*. Write a false conjecture about an object you see or use every day, and give a counterexample to show it is false.

Example 1 Finding a Counterexample to a Geometric Conjecture

Use the conjecture to answer **a** and **b**.

If a triangle is isosceles, then it is acute.

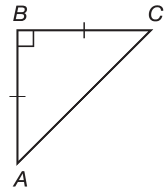
a. What is the hypothesis of the conjecture? What is its conclusion?

SOLUTION

Hypothesis: *The triangle is isosceles.*
Conclusion: *The triangle is acute.*

b. Find a counterexample to the conjecture.

SOLUTION



A counterexample would be an example of a triangle for which the hypothesis is true, but the conclusion is false; that is, a triangle that is isosceles but not acute. Consider this right triangle, ABC.

Since $\overline{BC}$ and $\overline{AB}$ are congruent, $\triangle ABC$ is isosceles.

Since $\angle B$ is a right angle, $\triangle ABC$ is not an acute triangle.

Therefore, $\triangle ABC$ is a counterexample to the conjecture.

> **Math Reasoning**
>
> **Model** Is there another kind of triangle that could be a counter-example to this statement?

Not all conjectures are geometric. Counterexamples can be used to disprove algebraic conjectures or any other kind of conjecture.

Example 2 Finding a Counterexample to an Algebraic Conjecture

a. Find a counterexample to the conjecture.

Every quadratic equation has either no solution or two solutions.

SOLUTION

You probably remember that sometimes quadratic equations can have only one solution. Such an equation would have to have only one x-intercept. A simple example is the parent function for quadratic equations.

$0 = x^2$

This equation can be solved by graphing, using the quadratic formula, or factoring. The answer is $x = 0$. Since this is a quadratic equation that has only one solution, the statement is proven false by this counterexample.

b. Find a counterexample to the conjecture.

If $5x - 10 = 15$, then $2x + y > 9$.

SOLUTION

First, solve the hypothesis of this statement. We find that for the hypothesis to be true, $x = 5$. Then substitute $x = 5$ into the conclusion to solve for y.

$$\begin{aligned} 2x + y &> 9 \\ 2(5) + y &> 9 \\ y &> -1 \end{aligned}$$

So for the conclusion to be true, y must be greater than -1. A counterexample to the statement is any value of y that is less than -1. Only one counterexample is needed, so a possible answer is $y = -2$.

Math Reasoning

Analyze Do you need to calculate the proportion for Saturn as well as for Earth and Mars? Why or why not?

Example 3 Application: Astronomy

Use the data in the table to prove the conjecture false.

If a planet orbits our Sun, its orbital period (year) is proportional to its distance from the Sun.

Planet	Orbital Period (days)	Distance from Sun (million miles)
Earth	365	93
Mars	687	142
Saturn	10,760	888

SOLUTION The hypothesis of the conjecture, *the planet orbits our Sun*, is true for all three planets in the table. If the conclusion were true, the ratio $\frac{\text{orbital period}}{\text{distance from Sun}}$ should be the same for all three planets. Extend the table by calculating this proportion for each planet:

Planet	Orbital Period (days)	Distance from Sun (million miles)	Proportion
Earth	365	93.0	$\frac{365}{93.0} \approx 3.92$
Mars	687	142	$\frac{687}{142} \approx 4.84$
Saturn	10,760	888	$\frac{10,760}{888} \approx 12.12$

By looking at the fourth column of the table, it is clear that the proportion is not always the same. Any two of these planets provide a counterexample that proves the statement false.

Lesson Practice

Use the conjecture below to answer a and b.
(Ex 1)
If line a is perpendicular to line b and to line c, then lines b and c are perpendicular.

a. What is the hypothesis of the conjecture? What is its conclusion?

b. Find a counterexample to the conjecture.

Use the conjecture below to answer c and d.
(Ex 2)
If $x^2 = 9$, then $x = 3$.

c. What is the hypothesis of the conjecture? What is its conclusion?

d. Find a counterexample to the conjecture.

e. (Ex 3) The masses of two sedimentary rocks are 327 grams and 568 grams, respectively. Their volumes are 275 cm^3 and 501 cm^3, respectively. Explain how this data disproves the conjecture below.
If a rock is sedimentary, then its mass is proportional to its volume.

Practice Distributed and Integrated

Base Length (cm)	Area (cm^2)
5.2	15.3
2.9	9.7
1.8	5.6
10.3	20.6
0.5	2.4

1. (14) This table shows the base lengths and areas of some triangles.

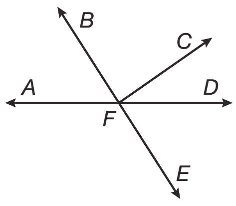
a. (12) Does the table give a counterexample to this conjecture? Explain. *If the base length of a triangle is less than 3 centimeters, then its area is less than 10 cm^2.*

b. Draw a triangle, not listed in the table, that is a counterexample to the conjecture.

2. (1) Classify the following statement as sometimes true, always true, or never true. *Three points can be noncoplanar.*

3. (8) **Analyze** How could you find the perimeter of an isosceles right triangle if you already know the area? Is it possible to find the leg lengths?

4. (6) List two adjacent angles and a linear pair from the diagram.



5. (8) **Algebra** If a regular triangle has a perimeter of $12x + 3$, what is an expression for the length of one of its sides?

6. (Inv 1) Line p is a transversal that intersects parallel lines m and n. If two of the alternate exterior angles measure $(2x - 10)°$ and $(-3x + 20)°$, what is the measure of each exterior angle?

7. (14) Consider this conjecture. *If a shape is a pentagon, then all its interior angles are obtuse.*

a. What is the hypothesis of the conjecture? What is its conclusion?

b. Find a counterexample to the conjecture.

8. (11) **Multiple Choice** Which point is the midpoint of -13 and 5?

A -3 **B** -4

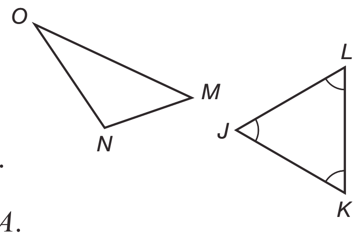
C -6 **D** -8

9. (5) An angle in a triangle is labeled as $\angle ABC$. Explain why $\overleftrightarrow{AB}$ and $\overleftrightarrow{BC}$ can be used to define a plane.

10. (13) **a.** In this figure, which triangle is acute?

b. Which triangle is obtuse?

c. Which triangle is equiangular?



11. (6) **Write** Do complementary angles need to be adjacent? Explain.

12. (4) Draw and label two opposite rays with a common endpoint, A.

13. (10) Identify the hypothesis and conclusion of this statement. *If one dozen eggs cost $2.49, then two dozen eggs cost $4.98.*

14. (9) **Error Analysis** A student calculated the distance between G (12, 4) and H (12, 2) as approximately 12.81. Where did the student make the error and what should the answer be?

15. (13) **Multi-Step** A star-shaped logo for a shop consists of three isosceles triangles, each with a base length of 12.5 millimeters and a height of 24.5 millimeters, fitted around an equilateral triangle with a base length of 12.5 millimeters and a height of 10.8 millimeters. Find the approximate sum of the four triangles' areas.

16. (Inv 1) State the converse of the statement below.
If two lines are cut by a transversal and the alternate interior angles are congruent, then the lines are parallel.

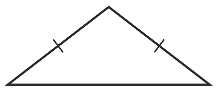
Write a conditional statement from each sentence.

17. (10) **Football** The team scores three points if the kicker makes a field goal.

18. (10) A number squared is a positive number.

19. (14) Find a counterexample to this conjecture. *If* $(1 - a)(1 - b) = 0$, *then* $a = b = 1$.

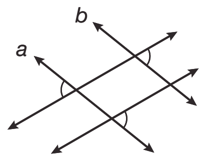
20. (13) **Packaging** This figure shows the triangular shaped label for a brand of cheese wedges. Classify the triangle by its sides and angles.



21. (Inv 1) If a pair of parallel lines are cut by a transversal, which pairs of angles will be supplementary?

22. (9) **Analyze** If you are given the first endpoint of a segment and the length of the segment, how many possible locations are there for the second endpoint of the segment? Explain.

23. (12) **Visual Arts** Seth is creating an abstract design with parallel lines, shown here. He adds angle markings as a reminder that the angles are congruent. How do you know that $a \parallel b$?



24. (7) **Write** When can a conjecture be considered a theorem?

25. (11) Determine the midpoint of each side of $\triangle KLM$ with coordinates $K(-2, -5)$, $L(-7, 4)$, and $M(0, -1)$.

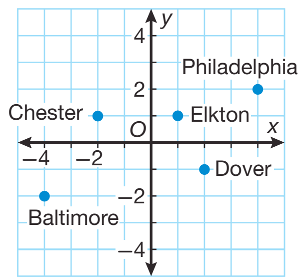
26. (5) Describe all of the possible types of intersections of three planes.

27. (7) Use inductive reasoning to find the next term: $-4, -1, 2,$ _____

28. (14) Find a counterexample to this conjecture. *If a triangle is isosceles, then it is equilateral.*

29. (8) **Painting** A man wants to paint the walls of a room. If the floor is a rectangle that is 10 feet by 12 feet and the ceiling is 10 feet high, what is the area he will have to paint?

30. (11) **Geography** Jacob lives in Philadelphia and his friend Antonio lives in Baltimore. The two plan to meet in the town that is closest to the midpoint of these two cities.



a. What are the coordinates of the midpoint of Philadelphia and Baltimore on the grid?

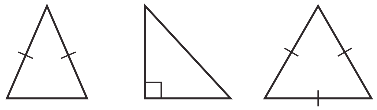
b. What is the name and the coordinates of the town where Jacob and Antonio will meet?

LESSON 15

Introduction to Polygons

Warm Up

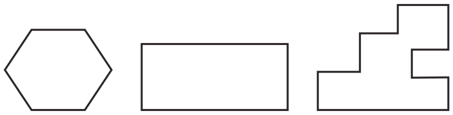
1. (2) **Vocabulary** A and B are the __________ of $\overline{AB}$.
2. (3) How many endpoints does a ray have?
3. (13) Classify each of the triangles in the diagram by both sides and angles.



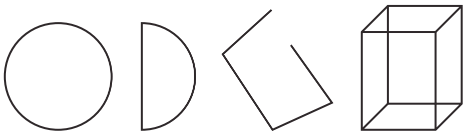
New Concepts

A **polygon** is a closed plane figure formed by three or more segments. Each segment intersects exactly two other segments only at their endpoints. No two segments with a common endpoint are collinear.

The segments that form a polygon are called its **sides**. A **vertex of a polygon** is the intersection of two of its sides.



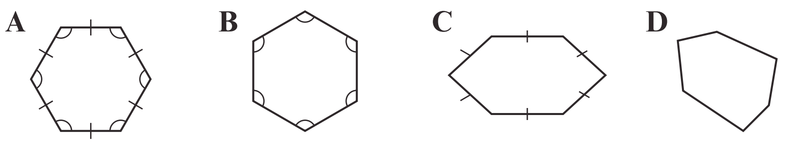
Polygons



Not Polygons

> **Hint**
>
> Equilateral and equiangular polygons have the same traits as equilateral and equiangular triangles, as introduced in lesson 13.

An **equiangular polygon** is a polygon in which all angles are congruent. An **equilateral polygon** is a polygon in which all sides are congruent. If a polygon is both equiangular and equilateral, then it is called a **regular polygon**. If a polygon is not equiangular and equilateral, then it is called an **irregular polygon**.



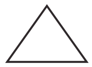
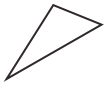
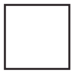
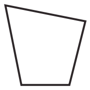
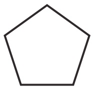
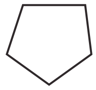
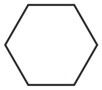
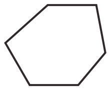
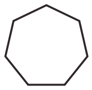
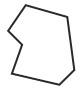
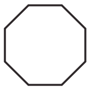
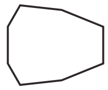
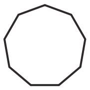
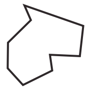
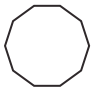
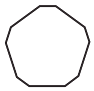
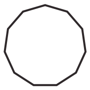
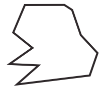
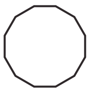
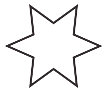
In the diagram, polygons A and B are equiangular. Polygons A and C are equilateral. Since polygon A is both equiangular and equilateral, it is a regular polygon. Polygons B, C and D are all irregular.

Online Connection
www.SaxonMathResources.com

Polygons are named by the number of sides they have. The chart below shows some common polygons and their names.

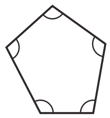
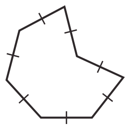
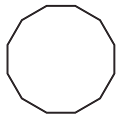
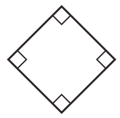
Math Language

An ***n*-gon** is a polygon with *n* sides. Problems may refer to *n*-gons when the number of sides of a polygon is not known, or when a solution is desired for all possible polygons.

Name	Sides	Regular Polygon	Irregular Polygon
Triangle	3		
Quadrilateral	4		
Pentagon	5		
Hexagon	6		
Heptagon	7		
Octagon	8		
Nonagon	9		
Decagon	10		
Hendecagon	11		
Dodecagon	12		

Example 1 Classifying Polygons

Classify each polygon. Determine whether it is equiangular, equilateral, regular, irregular, or more than one of these.

A

B

C

D


SOLUTION

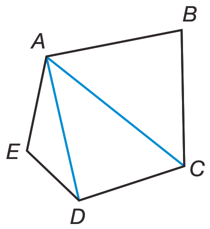
Polygon **A** has 5 sides, so it is a pentagon. It is equiangular but not equilateral, so it is irregular.

Polygon **B** has 7 sides, so it is a heptagon. It is equilateral and irregular.

Polygon **C** is a dodecagon. It is irregular.

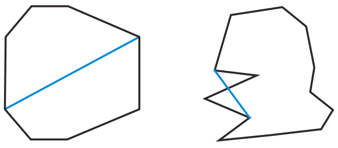
Polygon **D** is a quadrilateral. It is equilateral and equiangular, so it is regular.

A **diagonal of a polygon** is a segment that connects two nonconsecutive vertices of a polygon. For example, pentagon *ABCDE* has two diagonals, $\overline{AC}$ and $\overline{AD}$, from vertex *A*. Three other diagonals could be drawn: $\overline{BD}$, $\overline{BE}$, and $\overline{CE}$.



> **Math Language**
>
> The **exterior of a polygon** is the set of all points outside a polygon. The **interior of a polygon** is the set of all points inside a polygon.

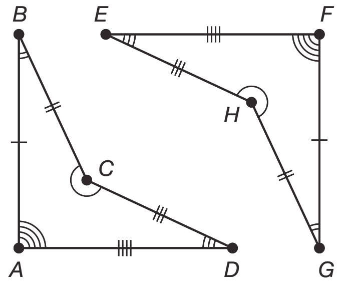
Diagonals can help determine whether a polygon is concave or convex. In a **convex polygon**, every diagonal of the polygon lies inside it, except for the endpoints. In a **concave polygon**, at least one diagonal can be drawn so that part of the diagonal contains points in the exterior of the polygon.



Convex polygon Concave polygon

If two polygons have the same size and shape, they are **congruent polygons**.

Example 2 Identifying Polygon Properties



a. Find a diagonal that contains points in the exterior of polygon *ABCD*.

SOLUTION

Diagonal $\overline{BD}$ lies outside polygon *ABCD*, except for its endpoints.

b. Determine whether polygon *EFGH* is convex or concave. Explain.

SOLUTION

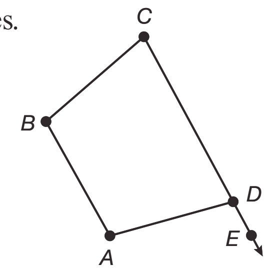
Diagonal $\overline{EG}$ contains points in the exterior of polygon *EFGH*. Therefore, polygon *EFGH* is concave.

c. Are polygons *ABCD* and *FGHE* congruent? Justify your answer.

SOLUTION

Write a congruency statement for all corresponding sides and angles. Angle pairs $\angle A \cong \angle F$, $\angle B \cong \angle G$, $\angle C \cong \angle H$, and $\angle D \cong \angle E$. Sides $\overline{AB} \cong \overline{FG}$, $\overline{BC} \cong \overline{GH}$, $\overline{CD} \cong \overline{HE}$, and $\overline{DA} \cong \overline{EF}$. Therefore, $ABCD \cong FGHE$.

At each vertex of a polygon, there are two special angles. An **interior angle of a polygon** is an angle formed by two sides of a polygon with a common vertex. An **exterior angle of a polygon** is an angle formed by one side of a polygon and the extension of an adjacent side. In the diagram, $\angle CDA$ is an interior angle and $\angle ADE$ is an exterior angle.

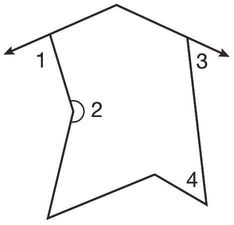


Math Reasoning

Generalize How many interior angles does a convex *n*-gon have? How many exterior angles does a convex *n*-gon have?

Example 3 Identifying Interior and Exterior Angles of Polygons

For each numbered angle in the polygon, determine whether it is an interior angle or an exterior angle.

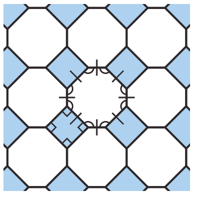


SOLUTION

Angles 2 and 4 are interior. Angles 1 and 3 are exterior.

Example 4 Application: Tile Patterns

This floor tile pattern uses polygonal tiles that fit together exactly.



a. Name the two types of polygons used in the pattern. Are they regular or irregular? Explain.

SOLUTION

Square and octagon; both types are regular, because they have all sides and all angles congruent, respectively.

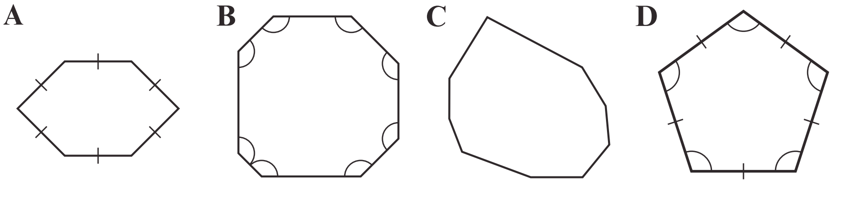
b. Pick any pair of unshaded polygons. Are they congruent? Are they convex or concave? Explain.

SOLUTION

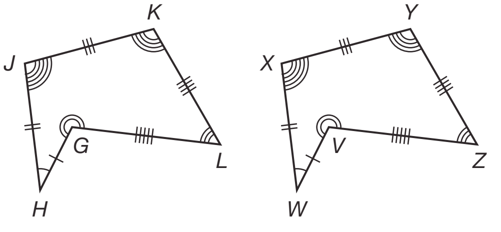
All pairs of unshaded polygons are congruent, because corresponding sides and angles are congruent. Each unshaded polygon is convex, because none of the polygon's diagonals contain points in its exterior.

Lesson Practice

a. Name each polygon. Determine whether it is equiangular, equilateral, regular, irregular, or more than one of these. *(Ex 1)*



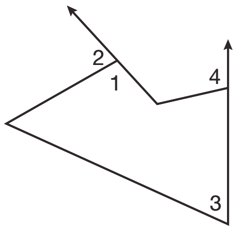
b. Find a diagonal in polygon *GHJKL* that contains points in the exterior of the polygon. *(Ex 2)*



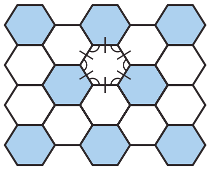
c. Determine whether polygon *VWXYZ* is convex or concave. Explain. *(Ex 2)*

d. Are polygons *GHJKL* and *VWXYZ* congruent? Justify your answer. *(Ex 2)*

e. *(Ex 3)* For each numbered angle in the polygon, determine whether it is an interior angle or an exterior angle.



f. *(Ex 4)* Name the type of polygon used in this pattern. Are the polygons regular or irregular? Explain.



g. *(Ex 4)* Pick any pair of polygons in this pattern. Are they congruent? Are they convex or concave? Explain.

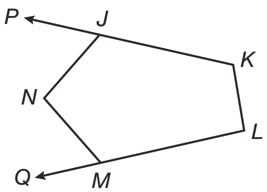
Practice Distributed and Integrated

1. *(14)* **Write** Explain why the pair of numbers 3 and 3 is a counterexample to this statement. *If the sum of two numbers is even, then both numbers are even.*

2. *(5)* **Agriculture** A farm is being divided so that each section of land has equal access to the canal running through the property for watering crops. If the road on the opposite side of the property runs parallel to the canal, explain how this can be done.

3. *(8)* **Wallpaper** A family wants to install wallpaper around the bottom half of a room. If the room has 12-foot tall ceilings and each wall is 14 feet long, calculate the area the wallpaper will cover.

4. *(15)* In polygon $JKLMN$, name each angle and identify it as an interior or exterior angle.



Write a conditional statement from each sentence.

5. *(10)* The absolute value of a number is a nonnegative number.

6. *(10)* A bilingual person speaks two languages.

7. *(11)* Determine the midpoint M of $\overline{XY}$ connecting $X(0, 2)$ and $Y(6, 1)$.

8. *(14)* **Model** Find a counterexample to this conjecture.
If two lines intersect, then any third coplanar line intersects both of them.

9. *(7)* Use inductive reasoning to determine the pattern in the following sequence:

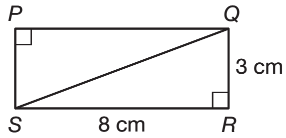
2, 3, 5, 9, 17, 33, 65

10. *(2)* Identify the property that justifies this statement:
$a = b$ *and* $b = c$, *so* $a = c$.

11. Name all the pairs of angles that are congruent when a transversal cuts a pair of parallel lines.
(Inv 1)

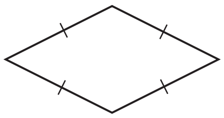
12. Find the length of the segment connecting (1.3, 4.1) and (2.8, 6.1).
(9)

13. **Verify** Rectangle $PQRS$ is divided into two triangles by diagonal $\overline{QS}$.
(13)



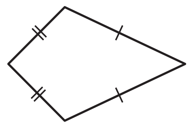
a. Determine the area of rectangle $PQRS$.
b. Given that $\triangle PQS$ and $\triangle QRS$ have equal areas, use your answer to part **a** to determine the area of $\triangle PQS$.
c. Verify that the formula for the area of a triangle gives the same answer as part **b** for the area of $\triangle PQS$.

14. Classify this polygon. Is it equiangular? Is it equilateral? Is it regular?
(15)



15. **Home Renovation** Tasha is using two wooden rails to construct a pair of stair rails along the walls of a staircase. To make sure that the rails are parallel, she measures the acute angle each rail makes with the vertical edge of the wall at the base of the stairs.
(12)
a. What type of angles are these?
b. Tasha measures each angle to be 42°. Explain how she can make sure that the rails are parallel.

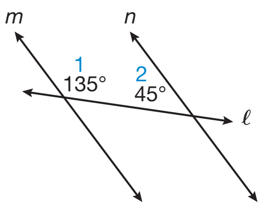
16. **Verify** Confirm that this quadrilateral is a counterexample to the conjecture.
(14)



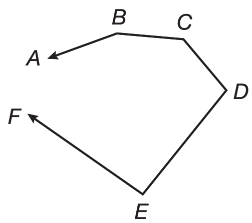
If a quadrilateral has two pairs of congruent sides, then both pairs of opposite sides are parallel.

17. If two parallel lines are cut by a transversal and one pair of same-side interior angles has angles that measure $(10x + 90)°$ and $(4x + 6)°$, what is the measure of each angle?
(Inv 1)

18. **Write** In this figure, the transversal line ℓ intersects lines m and n. Write a paragraph explaining how you know that m and n are parallel.
(12)

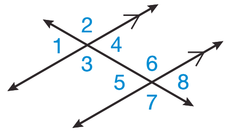


19. This figure shows a polygon with one vertex and two sides missing.
(15)



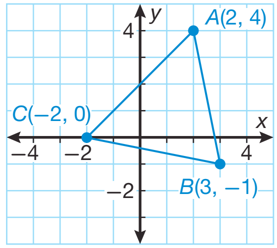
a. Copy the figure and add a point G that makes $ABCDEFG$ concave.
b. Make a second copy of the figure and add a point H that makes $ABCDEFH$ convex.

20. Name every pair of corresponding angles in the diagram.
(Inv 1)



21. **Building** A builder wants to add a diagonal beam to add support to a structure. The beam needs to extend across a height of 15 feet and a distance of 28 feet. What will the length of the beam be to the nearest hundredth of a foot?
(8)

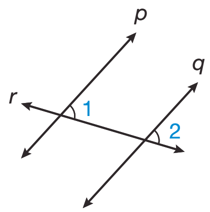
22. Determine the midpoint of each side of $\triangle ABC$.
(11)



23. **Algebra** The height of a triangle is 12.7 centimeters and its area is 31.75 square centimeters. Use the Triangle Area Formula to determine its base length.
(13)

24. **Multiple Choice** Which statement is always true?
(4)
A Two planes intersect in a straight line.
B Two lines are contained in exactly one plane.
C Two lines can intersect at two points.
D Any four points can be contained in exactly one plane.

25. **Multiple Choice** If $\angle 1$ and $\angle 2$ are congruent, which of these should be used to prove that lines p and q are parallel?
(12)
A Converse of the Alternate Exterior Angles Theorem
B Converse of the Alternate Interior Angles Theorem
C Converse of the Corresponding Angles Postulate
D Converse of the Same-Side Interior Angles Theorem



26. **Predict** Use inductive reasoning to find the next term in this sequence. Explain the rule for the pattern.
(7)

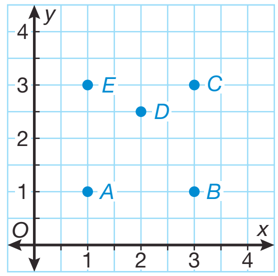
2, 3, 5, 9, 17, …

27. **Multi-Step** Find the perimeter of a square if its area is 289 square centimeters.
(8)

28. If two parallel lines are intersected by a transversal, what is the sum of the measures of all four interior angles that are formed?
(Inv. 1)

29. Find the perimeter of a regular hexagon with side lengths of 6.8 inches
(8)

30. If the points A, B, C, D, and E are connected, is polygon $ABCDE$ convex or concave?
(15)



LESSON 16

Finding Slopes and Equations of Lines

Warm Up

1. (SB 13) **Vocabulary** The pair of numbers (3, −1) is referred to as a(n) ____________. ***(linear pair, ordered pair)***

2. (SB 14) Given the equation $x + 2y = 8$, find the value of y when $x = 3$.

3. (SB 15) Which of these equations has $x = -2$ as its solution?

   A $x - 2 = 0$ **B** $-2 + x = 0$

   C $2 + x = 0$ **D** $2 - x = 0$

New Concepts

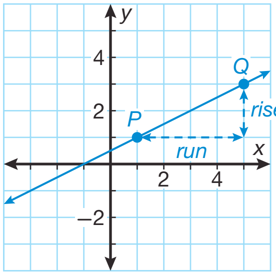
A **linear equation** is an equation whose graph is a line. Some examples are:

$y = 3x - 1$ $\quad 2x + 5y = 7$

$10 = 2x$ $\quad \frac{x}{4} + \frac{y}{13} = 1$

The variables in linear equations never have exponents other than 1. Linear equations connect algebra (equations in x and y) to geometry (lines in a coordinate plane).

Suppose (x_1, y_1) and (x_2, y_2) are points P and Q on a line in a coordinate plane. The **rise** from P to Q is the *vertical* change between P and Q, and equals $y_2 - y_1$. The **run** from P to Q is the *horizontal* change between P and Q, and equals $x_2 - x_1$.

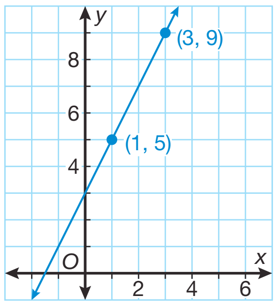


Math Reasoning

Analyze If one point on a line is in Quadrant I and another point is in Quadrant III, what can be determined about the slope of the line?

Example 1 Finding the Slope of a Line

Determine the slope of this line.



SOLUTION

Use the points (1, 5) and (3, 9) to calculate the slope. The rise is 4 units and the run is 2 units.

The slope is $\frac{\text{rise}}{\text{run}} = \frac{4}{2} = 2$.

Math Language

Slope is a **rate of change**, which is a ratio that compares the amount of change in a dependent variable to the amount of change in an independent variable.

The **slope** of a line is defined as the ratio of the vertical change (rise) between two points on a line to the horizontal change (run).

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

There are two special cases. For a horizontal line, the rise is always zero, so the slope is $\frac{0}{\text{run}} = 0$. For a vertical line, the run is zero, so the slope is undefined because division by zero is undefined.

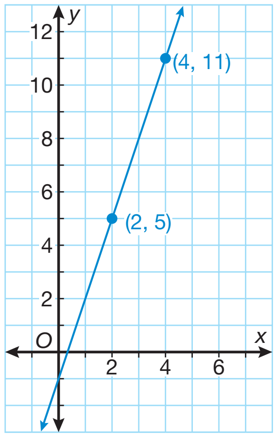
Reading Math

Slope is often represented by the letters *m* or *a*.

The **slope-intercept form** of a linear equation is a way of writing a linear equation using the slope (m) and the y-intercept (b) of the line. This way of writing the equation has the form $y = mx + b$.

Example 2 Writing the Equation of a Line

a. Use this graph of a line to write its equation.



SOLUTION

First, determine the slope m using the points (2, 5) and (4, 11).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{11 - 5}{4 - 2}$$
$$= 3$$

Read the y-intercept b directly from the graph.

$b = -1$

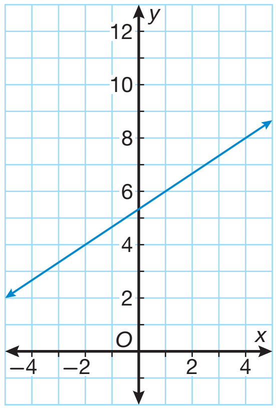
Substite for m and b in the slope-intercept form.

$$y = mx + b$$
$$y = 3x - 1$$

Math Reasoning

Analyze How can the slope-intercept form be used to find the equation of this line, instead of the slope formula?

b. Write the equation of the line that has slope $\frac{2}{3}$ and passes through $(-2, 4)$.



SOLUTION

Since $m = \frac{2}{3}$, substitute it into the slope formula using the given point $(-2, 4)$ and a general point.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$\frac{2}{3} = \frac{y - 4}{x - (-2)}$$

Rearrange this equation to remove the denominators.

$$\frac{2}{3} = \frac{y - 4}{x - (-2)}$$

$2(x - (-2)) = 3(y - 4)$	Cross-multiply.
$2x + 4 = 3y - 12$	Simplify.
$2x = 3y - 16$	Subtract 4 from each side.

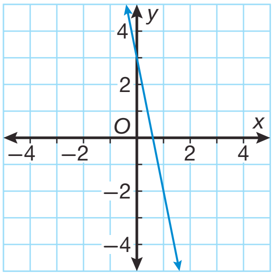
Finally, solve for y to put this equation into slope-intercept form.

$2x = 3y - 16$	
$2x - 3y = -16$	Subtract $3y$ from each side.
$-3y = -16 - 2x$	Subtract $2x$ from each side.
$y = \frac{2}{3}x + \frac{16}{3}$	Divide by -3.

Online Connection
www.SaxonMathResources.com

Example 3 Graphing a Linear Equation

a. Graph the line that has the equation $y = -5x + 3$.



SOLUTION

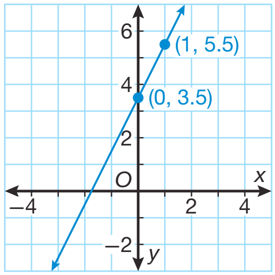
The equation is in slope-intercept form. Since the y-intercept is 3, the line passes through 3 on the y-axis. The slope is -5. For each run of 1 unit, the line has a rise of -5, so it drops 5 units. Use this fact to plot points starting at (0, 3).

b. Graph the line that has the equation $2y - 4x = 7$.

SOLUTION

The equation is not in slope-intercept form. Convert it to slope-intercept form.

$2y - 4x = 7$	Add $4x$ to each side.
$2y = 7 + 4x$	Divide each side by 2.
$y = 2x + 3.5$	Simplify.



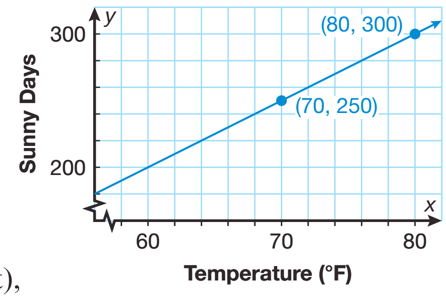
The slope of the line is 2 and the y-intercept is 3.5. The graph is shown in the diagram.

Math Reasoning

Write How can the x- and y-intercepts of $2y - 4x = 7$ be used to graph the equation?

Example 4 Application: Meteorology

Kim believes that there is a linear relationship between the average July temperature in the city of Brightdale in a particular year, and the number of days of sunshine Brightdale enjoys that year. She defines x to be the average July temperature (daily high, in Fahrenheit), and y to be the number of days of sunshine. Kim's model is shown on this graph.



a. Determine the slope of the graph. What does the slope represent?

SOLUTION

Use the formula to find the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{300 - 250}{80 - 70}$$

$$m = 5$$

A slope of 5 means that for every 1°F increase, the number of days of sunshine increases by 5. The slope represents the rate at which the number of days of sunshine increases as the average temperature increases.

b. Write an equation for Kim's model.

SOLUTION

Find the y-intercept using the slope and a point on the line.

$y = mx + b$	
$250 = 5(70) + b$	Substitute.
$250 = 350 + b$	Simplify.
$250 - 350 = 350 + b - 350$	Subtract 350 from each side.
$b = -100$	Simplify.

Kim's model has the equation $y = 5x - 100$.

c. Use the equation to predict the average July temperature if there are 280 days of sunshine.

SOLUTION

Substitute 280 for y in your equation from part **b**:

$280 = 5x - 100$	Substitute.
$280 + 100 = 5x - 100 + 100$	Add 100 to each side.
$380 = 5x$	Simplify.
$\frac{380}{5} = \frac{5x}{5}$	Divide each side by 5.
$76 = x$	Simplify.

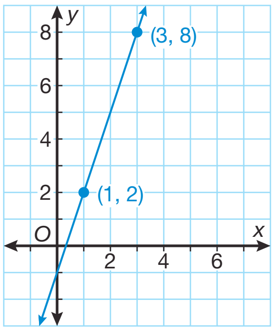
The average July temperature should be 76°F.

Math Reasoning

Analyze Through what quadrants could Kim's graph reasonably cross? Why?

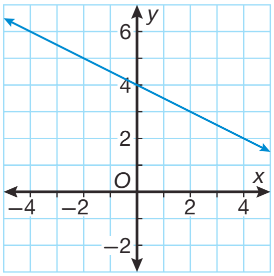
Lesson Practice

a. Determine the slope of this line.
(Ex 1)



b. Use the slope formula to determine the slope of the line passing through (1, 2) and (3, 4).
(Ex 1)

c. Use the graph to write an equation of the line.
(Ex 2)



d. Write the equation of the line that has slope $\frac{3}{2}$ and passes through $(4, -1)$.
(Ex 2)

e. Write the equation of the line that passes through $(0, -2)$ and $(5, 2)$. *(Ex 2)*

f. Graph the line with the equation $y = 3 - \frac{1}{3}x$. *(Ex 3)*

g. Graph the line with the equation $3x + y = 6$. *(Ex 3)*

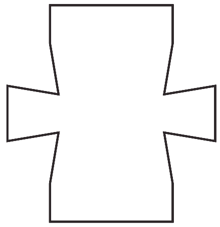
Practice Distributed and Integrated

1. *(15)* **Packaging** This figure shows the cardboard which will be folded into a package.

a. Determine whether the shape is equiangular and whether it is equilateral.

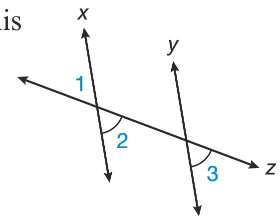
b. Determine whether the shape is regular or irregular.

c. Determine whether the shape is concave or convex, and explain how you know.



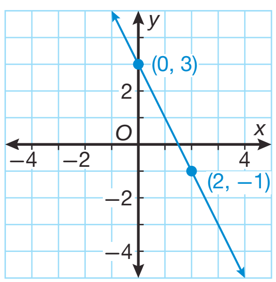
2. *(16)* Determine the slope of the line passing through (3, 1) and (5, 5).

3. *(12)* **Verify** Which postulate or theorem can you use to determine that x and y in this figure are parallel? Explain.



4. *(16)* **a.** Determine the slope of this line.

b. Write an equation for the line.



5. *(10)* **Write** Write a conditional statement that has a false converse.

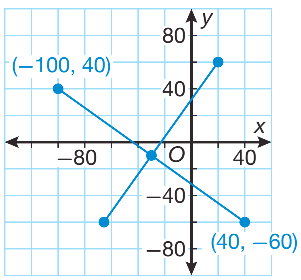
6. *(8)* **Tiling** The area of a single tile is exactly 1 square foot. If a developer wants to put tile in a room that measures 17 feet by 14 feet and each tile costs \$1.29, how much will it cost to tile the room?

7. *(16)* Graph the line with equation $y + 2x = 0$.

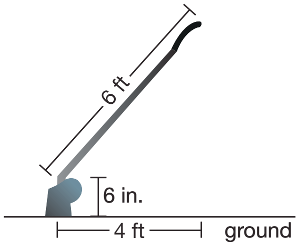
8. *(Inv 1)* Two parallel lines cut by a transversal form corresponding angles that measure $(2x)°$ and $(4x - 60)°$. What is the measure of each angle?

9. *(11)* **Multi-Step** If the midpoint of the segment connecting $(-8, x)$ and $(x, 4x)$ is (y, y), what is the value of y?

10. *(11)* **Space Exploration** NASA plans to launch Terrestrial Planet Finder 1, a telescope array that uses five spacecraft flying in formation to search for Earth-like planets in other solar systems. Suppose the locations of the top-left and bottom-right telescopes in the picture are $(-100, 40)$ and $(40, -60)$. Given that the central combiner telescope lies exactly halfway between these two, what are the coordinates of the central combiner telescope?



11. (8) **Farming** A plow has a length of 6 feet. If the plow has dimensions as shown, how far off the ground are the plow handles, to the nearest hundredth of a foot?

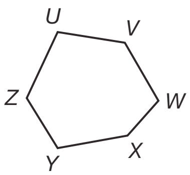


12. (3) **Multi-Step** If $\overrightarrow{BD}$ intersects straight angle $\angle ABC$ and $\angle ABD$ is one-eighth the size of $\angle DBC$, what is $m\angle DBC$?

13. (13) **Algebra** The base of a triangle measures 4.5 inches and its area is 24.75 in^2. Use the Triangle Area Formula to determine its height.

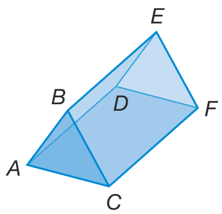
14. (5) **Error Analysis** Mohammed reads a theorem that says there is exactly one line through a point not on a given line that is perpendicular to that line, and extends this statement to read that a point on the line would also have one perpendicular line through it. Comment on the validity of his conclusion in two dimensions and in three dimensions.

15. (15) Copy polygon *UVWXYZ* and draw in all diagonals. Classify polygon *UVWXYZ* as convex or concave.



16. (8) Find the area of a right triangle with a hypotenuse of 36 inches and a leg of 15 inches. Round to the nearest hundredth.

17. (5) In the right triangular prism shown, list pairs of lines that appear to be parallel, perpendicular, and skew.



18. (4) **Verify** Points *A*, *B*, *C*, and *D* all lie on the same line. Are the points also coplanar? Which postulate or theorem can be used to justify your response?

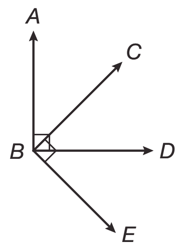
19. (7) **Formulate** Show that the conjecture "$2n - 1$ is a prime number" is true for $n = 3$ and $n = 4$.

20. (6) **Multiple Choice** Which angle measure is supplementary to 80°?

A 100° **B** 10°

C 80° **D** 110°

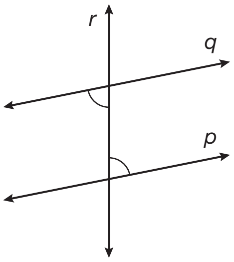
21. (6) $\angle ABD$ and $\angle CBE$ are right angles. Are $\angle ABC$ and $\angle DBE$ congruent? Explain.



22. (16) Does the line $y = -1.2x + 3$ slope up or down when you move left to right? How do you know?

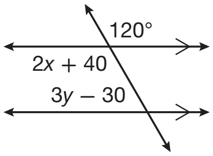
23. (4) **Analyze** Is it possible to draw two planes that intersect at a point? Explain.

24. (12) **Write** In this figure, the transversal line *r* intersects lines *q* and *p*. Write a paragraph explaining why *q* and *p* are parallel.



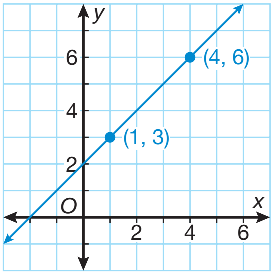
25. (7) Use inductive reasoning to determine the next term in the series: 11, 16, 15, 20, 19, 24, 23, ______

26. (Inv 1) What are the values of *x* and *y* in the diagram, given that $m \parallel n$?



27. (14) **Write** State the hypothesis and the conclusion of this conjecture. *If the product of two numbers is at least 4, then both numbers are at least 2.* Is the conjecture true? Explain how you know.

28. (16) Determine the slope of this line.



29. (13) **Model** Use a straightedge and a protractor to draw a scalene, an isosceles, and an equilateral triangle. Mark each congruent angle with arc marks and each congruent side with tick marks.

30. (14) **Multiple Choice** Which is a counterexample to the following conjecture? *If two numbers are not positive, then their product is positive.*

A 2 and −3
B −1 and 0
C −4 and −1
D 2 and 7

LESSON 17

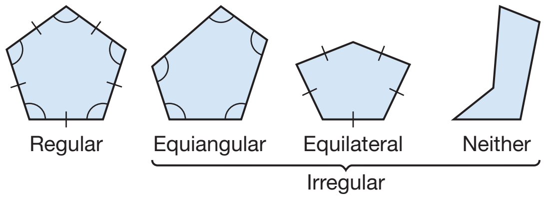
More Conditional Statements

Warm Up

1. (10) **Vocabulary** In the statement below, "A triangle is equilateral," is the _________ of the statement.
   If a triangle is equilateral, then it is isosceles.

2. (10) Give the converse of the conditional statement.
   If I drink enough water, I am not dehydrated.

3. (15) For a polygon, which statement is true?
   A If a polygon is irregular, then it is not equilateral.
   B If a polygon is irregular, then it is not equiangular.
   C If a polygon is not equilateral, then it is irregular.
   D If a polygon is equiangular, then it is equilateral.



New Concepts

In Lesson 10, you learned that a conditional statement has the form, "If p, then q." It is formed from two other statements: the hypothesis, p, and the conclusion, q. If you switch the statements, the result is the converse of the conditional statement, "If q, then p."

Math Language

The **converse** of a statement is formed by switching the conclusion and the hypothesis. The converse of a statement is sometimes true but can also be false.

Example 1 Analyzing the Truth Value and Converse of Conditional Statements

Consider the conditional statement, "If Sylvester walks to work, then it is a Wednesday."

a. State the hypothesis and conclusion of this statement, and write its converse.

SOLUTION

Hypothesis: *Sylvester walks to work.*
Conclusion: *It is a Wednesday.*
Converse: *If it is a Wednesday, then Sylvester walks to work.*

b. If the original statement is true, is the converse true?

SOLUTION

The converse is not necessarily true. Though we know that Sylvester only walks to work on Wednesdays, we do not know that he walks to work every Wednesday. He might drive to work on some Wednesdays. Therefore, the converse is false.

The **negation** of a statement is the opposite of that statement. The negation of a statement p is "not p," and is written as $\sim p$. For example, the negation of "a pentagon is regular" is "a pentagon is not regular."

Example 2 Examining the Negation of Conditional Statements

Identify the hypothesis and the conclusion in the statement below. Then, write the negation of each.

If a pentagon is regular, then it is equiangular.

SOLUTION

The hypothesis is: *A pentagon is regular.*

The conclusion is: *It is equiangular.*

The negation of the hypothesis is: *A pentagon is not regular.*

The negation of the conclusion is: *It is not equiangular.*

> **Math Reasoning**
>
> **Verify** Find the inverse of the statement in Example 1. Is the inverse true or false? Do the inverse and the converse have the same truth value?

The **inverse** of a conditional statement is formed when its hypothesis and conclusion are both negated. The inverse of "If p, then q" is "If $\sim p$, then $\sim q$." The converse of a conditional statement and the inverse of the same conditional statement always have the same truth value: either both are true or both are false. When two related conditional statements have the same truth value, they are called **logically equivalent statements.**

Example 3 Examining the Inverse of Conditional Statements

Write the inverse of the statement below. Is the statement true? Is the inverse of the statement true?

For two lines that are cut by a transversal, if alternate interior angles are congruent, then the lines are parallel.

SOLUTION

The inverse of the statement is, "For two lines that are cut by a transversal, if alternate interior angles are not congruent, then the lines are not parallel."

Since the converse and the inverse of a statement have the same truth value, the converse can be used to determine the truth value of the inverse.

The original statement is the converse of Theorem 10-1. Since the converse of the statement is known to be true, the inverse of the statement is also true.

The **contrapositive** of a conditional statement is formed by both exchanging and negating its hypothesis and conclusion. The contrapositive of, "If p, then q," is, "if $\sim q$, then $\sim p$." A conditional statement and its contrapositive are logically equivalent statements: either both are true or both are false.

We summarize the different types of conditional statements in this table.

	Form
Statement	If p, then q
Converse	If q, then p
Inverse	If $\sim p$, then $\sim q$
Contrapositive	If $\sim q$, then $\sim p$

Example 4 Examining the Contrapositive of Conditional Statements

a. Determine the contrapositive of the statement.

If Mai finishes school at 1 p.m., then it is a Thursday.

SOLUTION

In this statement, p is "Mai finishes school at 1 p.m." and q is "it is a Thursday." Therefore, the contrapositive statement "If $\sim q$, then $\sim p$" is "If it is not a Thursday, then Mai does not finish school at 1 p.m."

b. Determine the contrapositive of the solution to part **a**. What do you notice?

SOLUTION

The new p and q are "it is not a Thursday" and "Mai does not finish school at 1 p.m." Therefore, its contrapositive is "If $\sim$(Mai does not finish school at 1 p.m.), then $\sim$(it is not a Thursday)," which is the same as "If Mai finishes school at 1 p.m., then it is a Thursday." This is the original statement.

Math Reasoning

Verify A conditional statement and its contrapositive always have the same truth value. If the conditional statement in this example is true, is the contrapositive true? If the statement is false, is the contrapositive false?

Lesson Practice

a. (Ex 1) State the hypothesis and conclusion of this statement and its converse.

"*If a polygon is regular, then it is convex*"

b. (Ex 1) If the statement in problem **a** is true, is the converse true?

c. (Ex 2) Identify the hypothesis and the conclusion in the statement below. Then write the negation of each.

If Durrell buys juice, then he buys pretzels.

d. (Ex 3) Write the inverse of the statement below. Is the statement true? Is the inverse of the statement true?

For two lines that are cut by a transversal, if the lines are parallel, then the same-side interior angles are congruent.

e. (Ex 4) Determine the contrapositive of the statement.

If two angles are complementary, then the sum of their measures is 90°.

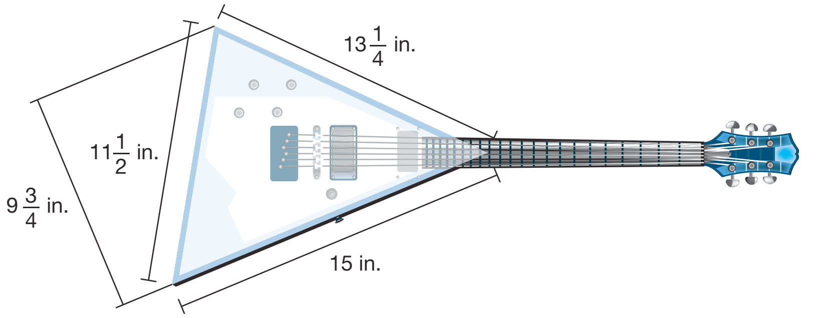
f. (Ex 4) Determine the *converse* of the solution to problem **e**. What do you notice?

Practice Distributed and Integrated

1. (13) **Music** Chris is planning to make an electric guitar in this shape.

a. What area of material will he need for the face of the triangular body, to the nearest square inch?

b. Chris wants to edge the body in luminescent material. If the body is $\frac{3}{4}$ inch. thick, what area of material will he need for the edging? (Round to the nearest square inch.)

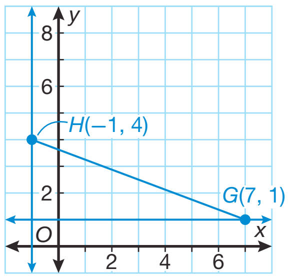


2. (14) **Economics** A cost function relates the total cost (C) of production to the number of items (x) produced per day. Acme Industries has a cost function $C_1 = 300x + 500$, while Amalgamated Widgets' cost function is $C_2 = 420x + 250$. Using these facts, find a counterexample to this conjecture.

For any value of x, Acme Industries has lower costs than Amalgamated Widgets.

3. (8) If a right triangle has a base of 6 inches and a height of 12 inches, what is the length of the hypotenuse to the nearest inch?

4. (11) **Formulate** Points $G(7, 1)$ and $H(-1, 4)$ lie on the coordinate plane.



a. Think of the x-coordinates 7 and -1 as points on a number line. What is the midpoint of the segment connecting them? What is the midpoint of the segment connecting y-coordinates 1 and 4? What is the midpoint of $\overline{GH}$?

b. Use part **a** to write a statement of the form: $\left(\frac{\_+\_}{2}\right), \left(\frac{\_+\_}{2}\right) = (\_, \_)$

5. (17) Write the inverse of this statement. *If Noah has pasta for lunch, then it is Tuesday.*

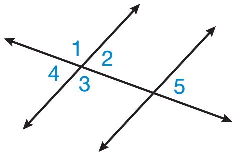
6. (1) Classify the statement as sometimes true, always true, or never true. Explain why.

If two lines are noncoplanar, then they are skew.

7. (10) This conditional statement is not in if-then form. Rewrite it in if-then form, then identify the hypothesis and the conclusion.

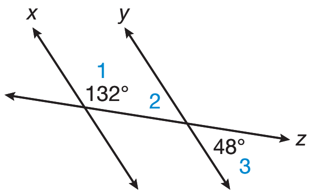
All PE students wear blue shorts to class.

8. *(12)* **Multiple Choice** In this figure, ∠5 and which other angle can prove the lines parallel using the Alternate Exterior Angles Converse?



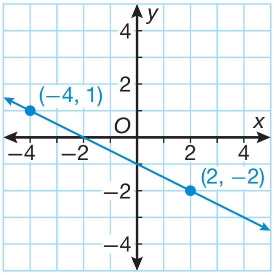
A 1 **B** 2
C 3 **D** 4

9. *(12)* **Write** In this figure, the transversal line z intersects lines x and y. Write a paragraph explaining how to show that x and y are parallel.



10. *(16)* **Multiple Choice** The slope of this graph is:

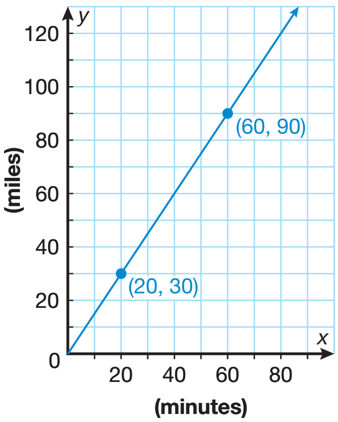
A 0.5 **B** 2
C −0.5 **D** −2



11. *(17)* **Algebra** Examine the statement, "If $x^2 > 4$, then $x > 2$." Is the statement true or false? Write its converse. Is the converse true or false?

12. *(11)* **Write** Explain why the midpoint of a and $-a$ on a number line is 0, for any value of a.

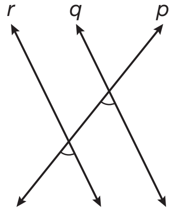
13. *(16)* **Physics** This graph shows the distance a train must travel in miles against the length of time of its journey in minutes. Determine the slope of the line. What does it represent?



14. *(2)* Identify the property that justifies the statement:
$WX = YZ$, so $YZ = WX$.

15. *(7)* Draw an acute, a right, and an obtuse triangle to test the conjecture made by Marc that the side which is opposite the largest angle in a triangle is the largest side in the triangle.

16. *(12)* Prove that lines r and q in this figure are parallel.



17. *(13)* Copy and complete this table showing the side lengths of several triangles.

Side Lengths	Isosceles or Scalene?	Equilateral?
3, 4, 5		
7, 13, 7		
39, 39, 39		

18. State the contrapositive of this statement.
(17)

If a bird is black, then it is not a swan.

Does an individual white swan prove the contrapositive statement? If all the swans you have ever seen are white, does that prove the statement? Does it provide evidence to support the statement? Explain your reasoning.

19. **Verify** Confirm that a triangle with side lengths of 5, 4, and 3 units is a counterexample to this statement.
(14)

If a triangle has no congruent sides, then it is obtuse or acute.

20. **Multiple Choice** Many rooms have a baseboard that lines the room where the walls and floor intersect. Which of the following best describes this situation?
(4)

A Two unique points make a line.
B Two noncollinear points define a plane.
C Two noncoplanar lines make a plane.
D Two distinct planes intersect in a line.

21. Identify the hypothesis and conclusion of this statement.
(10)

A drink is a soda if it has bubbles.

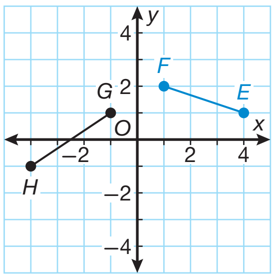
22. **Civics** An opinion poll shows support for Sally Smith at 60% in her congressional race. Assuming the poll is accurate, plot a line to represent how many votes Smith would get against different voter turnouts. Use scales from 0 to 300,000 in steps of 50,000.
(16)

23. Find a counterexample to this conjecture. *If three lines in a plane are all non-parallel, then they divide the plane into seven regions.*
(14)

24. Write the converse of this statement. *If a polygon is a triangle, then the sum of its angle measures is 180°.*
(17)

xy^2 **25.** **Algebra** Find a counterexample to this conjecture. *If* $xy < 1$, *then* $x < 1$ *and* $y < 1$.
(14)

26. **Multi-Step** Find the length of the given segments shown and determine if they are congruent.
(9)



xy^2 **27.** **Analyze** A heptagon has sides with positive integer lengths. If the heptagon is regular, what can you conclude about its perimeter? Explain.
(15)

28. Find the area of a right triangle with the hypotenuse of 17 centimeters and a leg of 10 centimeters. Round to the nearest hundredth.
(8)

29. **Multi-Step** Find the perimeter of a right triangle with both legs equal, if the area is 32 square meters. Round to the nearest hundredth.
(8)

30. Draw an irregular convex hexagon. Draw and label an exterior angle and the associated interior angle. What is the relationship between these two angles?
(15)

LESSON 18

Triangle Theorems

Warm Up

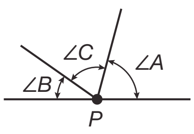
1. (3) **Vocabulary** An angle that measures less than 90° is a(n) ______ angle.
2. (13) Classify all three angles of a right triangle.
3. (15) What kind of triangle is formed by two sides and one diagonal of a square?

New Concepts

Exploration Developing the Triangle Angle Sum Theorem

In this exploration, you will use patty paper to discover the relationship between the measures of the interior angles of a triangle.

1. On a sheet of notebook paper, trace and label $\triangle ABC$ in Theorem 18-1.
2. On a piece of patty paper, draw a line and label a point on the line P.
3. Place the patty paper on top of $\triangle ABC$. Align the papers so that $\overline{AB}$ is on the line you drew and P and B coincide. Trace $\angle B$. Rotate the triangle and trace $\angle C$ adjacent to $\angle B$. Rotate the triangle once more and trace $\angle A$ adjacent to $\angle C$. The diagram shows your final step.



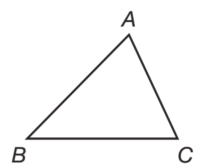
4. What do you notice about the three angles of the triangle you traced?
5. Draw a new triangle and repeat the activity using the new triangle. What is the result?
6. Write an equation describing the relationship you found between the three angles of $\triangle ABC$.

The angles of a triangle have special relationships. The most basic relationship is given by the Triangle Angle Sum Theorem.

Theorem 18-1: Triangle Angle Sum Theorem

The sum of the measures of the angles of a triangle is equal to 180°.

$$m\angle A + m\angle B + m\angle C = 180°$$



Online Connection
www.SaxonMathResources.com

Example 1 Using the Triangle Angle Sum Theorem

In the right triangle $\triangle ABC$, $m\angle B = 35°$ and the right angle is at vertex A. Find the measure of $\angle C$.

SOLUTION

From the Triangle Angle Sum Theorem:

$m\angle A + m\angle B + m\angle C = 180°$

$90° + 35° + m\angle C = 180°$ Substitute.

$m\angle C = 55°$ Solve.

Math Reasoning

Justify Explain in words how Corollary 18-1-2 follows from Theorem 18-1.

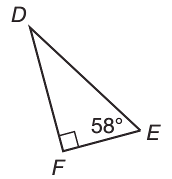
A **corollary** to a theorem is a statement that follows directly from that theorem. The Triangle Angle Sum Theorem has several useful corollaries.

Triangle Angle Sum Theorem Corollaries
Corollary 18-1-1: If two angles of one triangle are congruent to two angles of another triangle, then the third angles are congruent.
Corollary 18-1-2: The acute angles of a right triangle are complementary.
Corollary 18-1-3: The measure of each angle of an equiangular triangle is 60°.
Corollary 18-1-4: A triangle can have at most one right or one obtuse angle.

Each of these corollaries is a direct result of the Triangle Angle Sum Theorem. For example, Corollary 18-1-3 is true because every triangle has three angles and by Theorem 18-1, they add up to 180°. For all three angles to be congruent, they must each measure $\frac{180°}{3} = 60°$.

Example 2 Finding Angle Measures in Right Triangles

a. Find the measure of $\angle D$ in $\triangle DEF$.



SOLUTION

By Corollary 18-1-2, $\angle D$ and $\angle E$ are complementary.

$m\angle D + m\angle E = 90°$

$m\angle D + 58° = 90°$ Substitute for $m\angle E$.

$m\angle D = 32°$ Subtract 58° from each side.

b. In right $\triangle KLM$, $\angle K \cong \angle L$. Determine $m\angle K$.

SOLUTION

By Corollary 18-1-4, $\angle K$ and $\angle L$ cannot both be right angles. Therefore they are the acute angles of $\triangle KLM$. Since they are congruent, $\angle K$ can be substituted for $\angle L$.

$m\angle K + m\angle L = 90°$

$m\angle K + m\angle K = 90°$ Substitute for $m\angle L$.

$2m\angle K = 90°$ Simplify.

$m\angle K = 45°$ Divide both sides by 2.

Math Reasoning

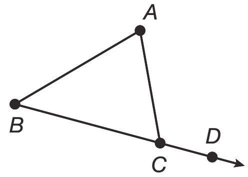
Analyze Explain why an obtuse triangle must have at least one acute exterior angle.

In any polygon, a **remote interior angle** is an interior angle that is not adjacent to a given exterior angle. In triangles, every exterior angle has a special relationship to its two remote interior angles.

Theorem 18-2: Exterior Angle Theorem

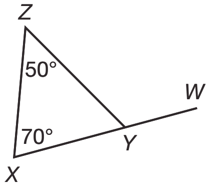
The measure of each exterior angle of a triangle is equal to the sum of the measures of its two remote interior angles.

$$m\angle DCA = m\angle A + m\angle B$$



Example 3 Using the Exterior Angle Theorem

a. For $\triangle XYZ$, determine the measure of $\angle WYZ$.



SOLUTION

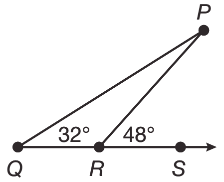
The remote interior angles are at X and Z. Therefore,

$m\angle X + m\angle Z = m\angle WYZ$

$70° + 50° = m\angle WYZ$

$120° = m\angle WYZ$

b. Determine the measure of $\angle P$ in $\triangle PQR$.



SOLUTION

Apply the Exterior Angle Theorem with the exterior angle at vertex R:

$m\angle P + m\angle Q = m\angle PRS$

$m\angle P + 32° = 48°$ Substitute.

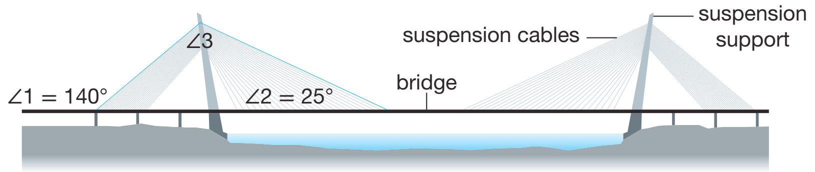
$m\angle P = 16°$ Subtract 32° from each side.

Hint

The *apex* of something is the highest part of it. In this case, the apex of the triangle is the topmost angle of the triangle.

Example 4 Application: Civil Engineering

A bridge uses cables to support its 2000 foot span. Use the data in the image to determine the measure of the angle at the apex of the marked cable structure.



SOLUTION

Apply the Exterior Angle Theorem.

$m\angle 2 + m\angle 3 = m\angle 1$

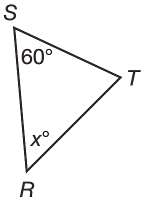
$25° + m\angle 3 = 140°$ Substitute.

$m\angle 3 = 115°$ Subtract 25° from each side.

The angle at the apex measures 115°.

Lesson Practice

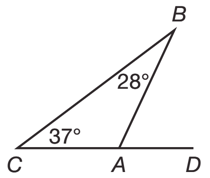
Use this figure to answer a and b.



a. If $x = 50$, determine the measure of $\angle T$. *(Ex 1)*

b. Determine $m\angle T$ if $x = 60$. *(Ex 1)*

c. In right triangle $\triangle PQR$, the measure of one acute angle is 20°. What is the measure of the other acute angle in $\triangle PQR$?



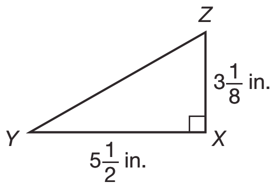
d. In $\triangle ABC$, determine the measure of $\angle DAB$. *(Ex 3)*

e. In $\triangle JKL$, $\angle K$ measures 60° and the exterior angle at vertex L measures 100°. Make a sketch of $\triangle JKL$ showing the given interior and exterior angle measures. *(Ex 3)*

f. Determine the measure of $\angle J$ in $\triangle JKL$ in problem **e**. *(Ex 3)*

g. **Civil Engineering** A planned glass pyramid structure has four triangular faces. The angles at the base of each face are congruent. Each of the angles at the apex of the pyramid measures 68°. What are the measures of the congruent base angles? *(Ex 3)*

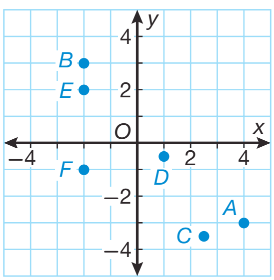
Practice Distributed and Integrated



1. Use $\triangle XYZ$ to answer problems **a** and **b**. *(8)*
   a. In $\triangle XYZ$, what is the length of side $\overline{YZ}$? Round to the nearest eighth of an inch.
   b. Determine the perimeter of $\triangle XYZ$.

xy^2 **2.** **Algebra** Two congruent angles are complementary to a 22° angle. These angles are defined by the expressions $3x + 5$ and $7y - 2$, respectively. What are the values of x and y? *(6)*

3. **Geology** Consider this statement. *If a rock is metamorphic, then it is crystalline.* *(17)*
   a. Write the inverse of this statement.
   b. Assuming that the original statement is true, can you conclude that its inverse is true?



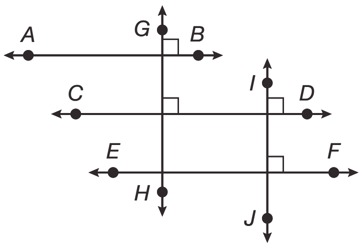
4. **Verify** Two of the labeled points on this coordinate grid have a third labeled point as their midpoint. Identify the two points and their midpoint, and verify your answer. *(11)*

5. Find an example to disprove the converse of the following statement. *(10)*
   If $x = -4$, then $x^2 = 16$.

6. **Multiple Choice** If a line has the length of 5 units and it has one endpoint of $(3, -2)$, what is a possible second endpoint? *(9)*

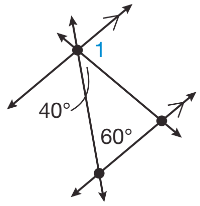
A $(0, 1)$ **B** $(-1, 2)$
C $(-1, 1)$ **D** $(6, -6)$

7. Is $\overleftrightarrow{EF}$ parallel to $\overleftrightarrow{AB}$? Explain.
(5)

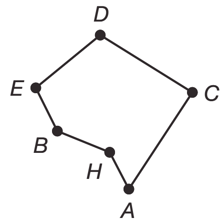


8. **Algebra** A rectangle has a perimeter of 40 inches. If the longer side is $(4x + 8)$ inches and the shorter side is $(3x - 2)$ inches, what are the dimensions of the rectangle?
(8)

9. What is the measure of angle 1 in the diagram?
(Inv 1)

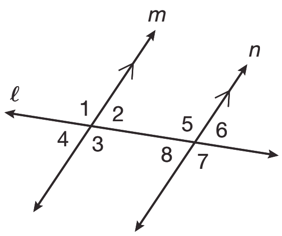


10. **Geography** Mia, a traveling sales representative, has to stop at five customers' locations in one day: *A*, *B*, *C*, *D*, and *E*. She begins and ends at her hotel, *H*. Mia figures that she will drive the shortest total distance if she makes the stops in the sequence *A*, *C*, *D*, *E*, *B*. Classify the polygon that her route, *HACDEB* forms, including any features which are congruent, and whether it is convex or concave.
(15)

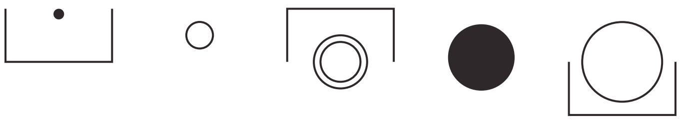


11. If two parallel lines are cut by a transversal that is perpendicular to both lines, which pairs of angles will be supplementary?
(Inv 1)

12. **Analyze** Use the Alternate Interior Angles Theorem to explain why corresponding angles are congruent.
(Inv 1)



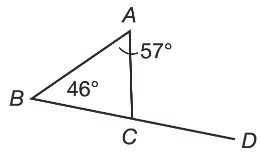
13. **Justify** Use inductive reasoning to create the next item in the sequence, and explain your reasoning.
(7)



14. Two angles of a triangle measure 30° and 45°. What is the measure of the third angle?
(18)

15. **Algebra** Transform the equation $x - 3y = 6$ to put it in slope-intercept form.
(16)

16. Determine the measure of the exterior angle at *C* in $\triangle ABC$.
(18)



17. **Verify** Consider this statement. "If a polygon is irregular, then it is concave."
(17)

a. Write the converse of the statement.

b. Write the inverse of the original statement.

c. Verify that the inverse statement and converse statement have the same truth value.

18. All exterior angles of a certain triangle are obtuse. Classify the triangle by its angles and explain how you know.
(18)

19. **Computer Science** An "OR" logic gate has two inputs and one output, related by the table shown.
(14)

In		Out
A	***B***	
0	0	0
0	1	1
1	0	1
1	1	1

a. Find a counterexample to this conjecture.
If the output of an OR logic gate is 1, input B is 1.

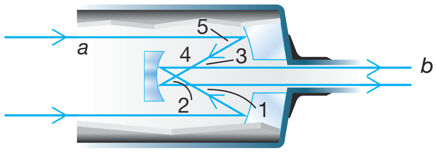
b. Violet made the conjecture: *If neither input of an OR gate is 1, the output is 0.*
Does the table contain a counterexample to her conjecture?

20. Write the inverse of this statement, "If a triangle has all three sides congruent, then it is obtuse." Which is true: the statement, its inverse, both, or neither?
(17)

21. **Write** Write a conditional statement that has a true converse.
(10)

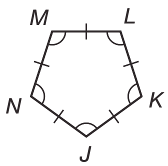
22. Find an expression for the perimeter of a regular hexagon if the sides are $2x - 6$.
(8)

23. **Optics** This optical diagram shows the paths of light rays in a reflecting telescope. Identify the two angles you need to prove congruent in order to prove that rays *a* and *b* are parallel. Which postulate or theorem would you use, and why?
(12)



24. **Multiple Choice** Polygon *JKLMN* is a(n):
(15)

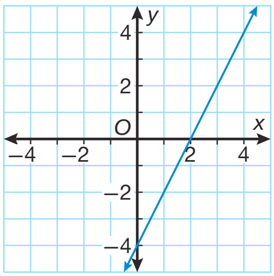
A irregular pentagon **B** equiangular nonagon
C irregular hexagon **D** regular pentagon



25. **Multi-Step** How many true conditional statements can you write using the following three statements?
(10)

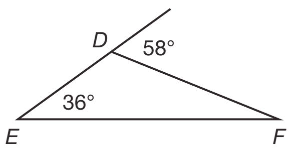
p: ∠1 and ∠2 are complementary
q: $m\angle 1 + m\angle 2 = 90°$
r: ∠1 and ∠2 are acute

26. **Error Analysis** Ricardo has drawn this graph for the line with equation $x - 2y = -4$. Identify two errors Ricardo has made.
(16)



27. **Write** Explain why the distance formula is not needed to determine if three points lie on the same line.
(9)

28. In △*DEF*, the exterior angle at *D* measures 58° and $m\angle E = 36°$. Determine $m\angle F$.
(18)



29. Classify △*DEF* by its angles.
(13)

30. Two distinct lines which are not contained in plane *H* intersect plane *H*. At how many points could the lines intersect the plane?
(4)

LESSON 19

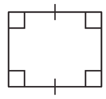
Introduction to Quadrilaterals

Warm Up

1. **Vocabulary** A polygon with all angles and all sides congruent is _________.
(15)

2. **Vocabulary** A four-sided polygon is called a _____________.
(15)

3. Identify the figure at left as a square, a rectangle, or neither.
(15)

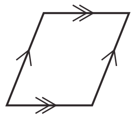
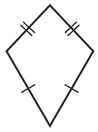
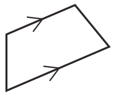
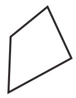


4. The interior angles of a square are:
(15)

A Right angles
B Congruent to each other
C Congruent to the corresponding exterior angles
D All of the above

New Concepts

A **quadrilateral** is a polygon with four sides. Quadrilaterals are classified according to the number of congruent and parallel sides they have.

Quadrilateral	Properties	Example
Parallelogram	Both pairs of opposite sides are parallel.	
Kite	Exactly two pairs of consecutive sides are congruent.	
Trapezoid	Exactly one pair of opposite sides are parallel.	
Trapezium	No sides are parallel.	

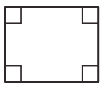
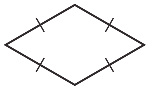
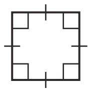
Reading Math

Sometimes symbols are used to name quadrilaterals. For example, ▭*PQRS* means "rectangle *PQRS*" and ▱*WXYZ* means "parallelogram *WXYZ*."

In addition to the quadrilaterals listed above, there are three types of parallelograms. Parallelograms are classified based on whether or not their sides are congruent and whether or not they have right angles.

Hint

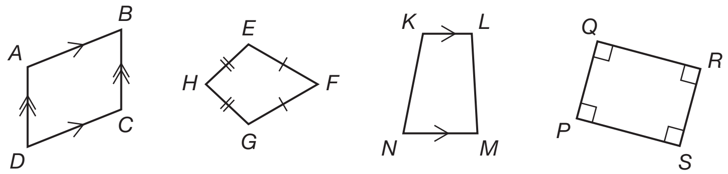
Though parallelograms can often be given several names, always try to find the most specific name. For example, a quadrilateral with four right angles could be called a parallelogram, but it is more specific to call it a rectangle.

Parallelogram	Properties	Example
Rectangle	A parallelogram with four right angles	
Rhombus	A parallelogram with four congruent sides	
Square	A parallelogram with four right angles and four congruent sides	

Some quadrilaterals can be named in several ways. For example, a square is also a rectangle, a rhombus, and a parallelogram; a kite is also a trapezium.

Example 1 Classifying Quadrilaterals

Classify each quadrilateral. Give multiple names if possible.



SOLUTION

In quadrilateral *ABCD*, sides $\overline{AB}$ and $\overline{CD}$ are parallel. Also, $\overline{AD} \parallel \overline{BC}$. Therefore, *ABCD* is a parallelogram.

In quadrilateral *EFGH*, $\overline{EF} \cong \overline{FG}$ and $\overline{EH} \cong \overline{GH}$. Since both of these pairs of sides are consecutive, *EFGH* is a kite. Since no sides are parallel, *EFGH* is also a trapezium.

In quadrilateral *KLMN*, $\overline{KL} \parallel \overline{MN}$. *KLMN* is a trapezoid.

Quadrilateral *PQRS* has four right angles, so it is a rectangle.

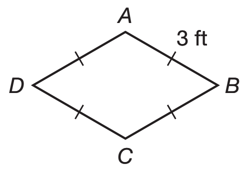
Math Reasoning

Model Could a kite or a trapezoid ever be a parallelogram? If so, draw an example.

Example 2 Sketching Quadrilaterals

Sketch each quadrilateral based on its description.

a. In quadrilateral *ABCD*, each side measures 3 feet.



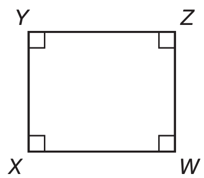
SOLUTION

1. Draw four sides of equal length. (The sides need not be perpendicular.)
2. Label the vertices *A*, *B*, *C*, and *D* in order.
3. Mark one side "3 ft".
4. Use tick marks on all four sides to show that they are congruent.

b. In quadrilateral *WXYZ*, each angle measures 90°.

SOLUTION

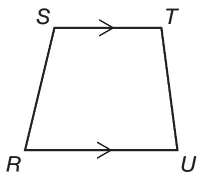
1. All four angles are right angles. Draw a rectangle.
2. Label the vertices *W*, *X*, *Y*, and *Z* in order.
3. Mark each angle with the symbol for a right angle.



c. In quadrilateral *RSTU*, $\overline{ST} \parallel \overline{RU}$.

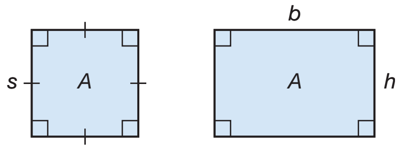
SOLUTION

1. Draw one pair of parallel sides.
2. Mark these two sides with arrows to show they are parallel.
3. Draw the other two sides connecting the first two sides.
4. Label the vertices so that the parallel sides are $\overline{ST}$ and $\overline{RU}$.



Online Connection
www.SaxonMathResources.com

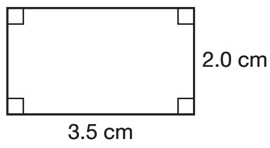
If a rectangle has a base of length b and height h, then its area is $A = bh$. Since all four sides of a square are congruent, its base and height are identical. Therefore, a square's area is given by squaring the length of any side, s, of the square: $A = s^2$.



Example 3 Finding Perimeters and Areas of Rectangles and Squares

> **Math Reasoning**
>
> **Model** If one base of a rectangle is slid over, so it no longer had right angles, what quadrilateral results? Does this new quadrilateral have the same area or a different area than the rectangle?

a. Determine the perimeter and area of this rectangle.



SOLUTION

The length of the rectangle is 3.5 centimeters and its width is 2.0 centimeters.

The perimeter is the sum of the side lengths:

$P = 3.5 + 2.0 + 3.5 + 2.0$

$P = 2(3.5) + 2(2.0)$

$P = 11.0$

The area is:

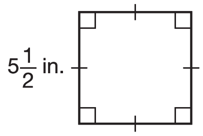
$A = bh$

$A = (3.5)(2.0)$

$A = 7.0$

The rectangle's perimeter is 11.0 cm and its area equals 7.0 cm^2.

b. Determine the perimeter and area of this square.



SOLUTION

The square has side lengths of $5\frac{1}{2}$ inches.

$P = 5\frac{1}{2} + 5\frac{1}{2} + 5\frac{1}{2} + 5\frac{1}{2}$

$P = 4(5.5)$

$P = 22$

$A = s^2$

$A = (5.5)^2$

$A = 30.25$

The perimeter of the square is 22 inches and its area equals 30.25 in^2.

Example 4 Sports

Each side of a baseball diamond measures 30 yards. Each of its corners is a right angle.

a. What kind of quadrilateral is a baseball diamond? Give as many different names for it as possible.

SOLUTION

Since the sides are congruent, each measuring 30 yards, and each corner forms a right angle, the most specific name for the diamond is a square. So, it follows that it can also be called a rhombus, rectangle, and parallelogram.

b. What distance must a batter run for a homerun?

SOLUTION

The distance is the perimeter of the diamond:

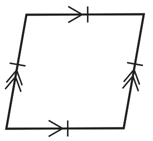
$P = 30 + 30 + 30 + 30$

$P = 120$

The batter must run 120 yards.

Lesson Practice

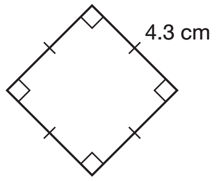
a. Classify this quadrilateral. Give multiple names if possible.
(Ex 1)



b. In quadrilateral $PQRS$, $\overline{PQ} \parallel \overline{RS}$ and $\overline{PS} \parallel \overline{QR}$. Also, $\overline{PQ}$ is approximately twice as long as $\overline{QR}$. Sketch $PQRS$.
(Ex 2)

c. **Sports** An Olympic swimming pool is a rectangle measuring 25 meters by 50 meters. Find its perimeter and area.
(Ex 3, Ex 4)

d. Determine the perimeter and area of this square.
(Ex 3)

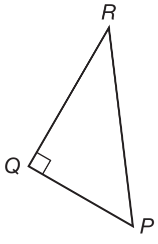


Practice Distributed and Integrated

* **1.** In quadrilateral $STUV$, $\overline{ST}$ and $\overline{TU}$ both measure 2.5 centimeters, and $SV = UV = 5.0$ centimeters. Sketch $STUV$.
(19)

2. **Multiple Choice** Choose the best classification of $\triangle PQR$ by its angles.
(13)

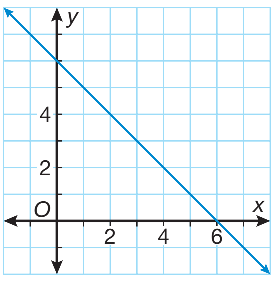
A right B acute and equiangular
C acute D obtuse



* **3.** **Write** Write the negation of the Converse of Alternate Exterior Angles Theorem.
(17)

4. **Algebra** A triangle has a perimeter measuring 133.1 centimeters, and two of its sides measure 23.8 centimeters and 49.3 centimeters, respectively. The height from the third side to the opposite vertex is 19.0 centimeters. Determine the triangle's area.
(13)

5. Determine the slope of this line.
(16)

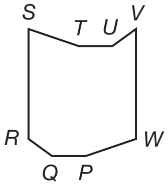


6. (7) **Generalize** Use inductive reasoning to determine the missing value: 11, 13, 17, 19, 23, ____, 31, 37

7. (Inv 1) A pair of lines is cut by a transversal, forming a pair of alternate interior angles that measure 85° and $(3x - 17)°$. For what value of x are the lines parallel?

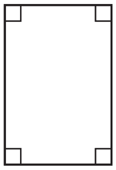
8. (4) What is the intersection of a line, a plane containing the line, and a point on the line?

9. (15) Find a diagonal in polygon *PQRSTUVW* that lies partly in the exterior of the polygon. Classify polygon *PQRSTUVW* as convex or concave.



***10.** (18) Two angles of a triangle measure 68° and 43°, respectively. What is the measure of the third angle?

***11.** (19) Classify this quadrilateral.



12. (14) **Verify** Confirm that a triangle with side lengths 5, 5, and 3 units is a counterexample to the statement, "If a triangle has two congruent sides, then it is obtuse."

13. (10) Identify the hypothesis and conclusion of this conditional statement.
If you have the blood type O, then you are the universal donor.

14. (5) **Air Traffic Control** On her screen, Rita notices that two planes are flying paths that are parallel to each other. She knows that if they continue flying straight, their paths will never cross What theorem or postulate can be used to justify Rita's conclusion?

15. (11) Determine the midpoint of the points $P(-3, 0)$ and $Q(-7, -3)$.

***16.** (17) **Write** Write the inverse of the statement.
If an animal is warm-blooded, then it is a mammal.

17. (11) **Verify** Let M be the midpoint of $A(3, 7)$ and $B(5, 1)$ on the coordinate plane. Verify that M lies on $\overline{AB}$ and that $AM = BM$.

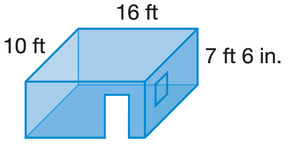
18. (8) Find an expression for the area of a rectangle if one side is twice as long as the other side.

***19.** (19) Larissa is redecorating her room. The ceiling will be white and she plans to paint the walls blue. The diagram shows the room's dimensions.

a. What is the area to be painted white?

b. To the nearest square foot, what is the area to be painted blue?

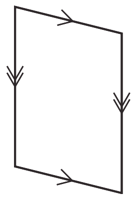
c. The actual area to be painted blue does not include the 6-by-2 foot door and does not include one 3-by-4 foot window. Find the actual area to be painted.



*20. (14) **Engineering** This table shows the gas mileage of a car at various cruising speeds. Do the data give a counterexample to the conjecture, "The product of cruising speed and gas mileage is constant"? Explain.

Cruising Speed (mi/h)	Gas Mileage (mi/gal)
37	35
56	30
75	22

*21. (18) In triangles JKL and MNO, $\angle J$ is congruent to $\angle M$, and $\angle K$ is congruent to $\angle N$. What can you say about $\angle L$ and $\angle O$?

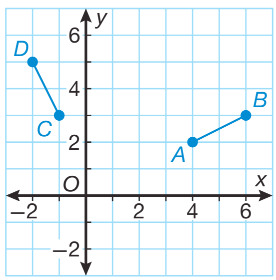


*22. (19) Classify this quadrilateral.

23. (11) What is the midpoint of $(-4, 1)$ and $(4, -1)$?

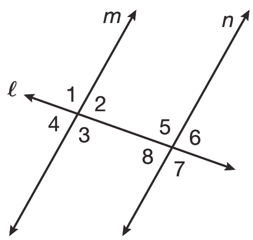
24. (7) **Write** In a biological experiment looking at the migratory patterns of geese, all of the geese that were observed actually flew to a warmer climate during the cold winter months. Is the conjecture, "All geese fly to warmer climates for the winter," valid? Why or why not?

25. (9) **Multi-Step** Find the length of the given segments and determine if they are congruent.



26. (2) Identify the property that justifies the statement.
If $\overline{AB} \cong \overline{CD}$, *then* $\overline{CD} \cong \overline{AB}$.

27. (Inv 1) **Analyze** Use the Same-Side Interior Angles Theorem to explain why corresponding angles are congruent. Assume that m is parallel to n.



28. (16) **Write** an equation for the line passing through $(3, 0)$ and $(5, -1)$.

*29. (18) **Geography** Three streets in Chicago, Illinois—N. Wabash St., N. Rush St., and E. Chestnut St.—make a city block in the shape of a right triangle. The acute angle at the north end of the block measures 28°. What is the exterior angle measure at N. Rush St. and E. Chestnut St.?

30. (6) **Error Analysis** Victoria states that angles that measure 50° and 130°, respectively, are complementary, but her friend Shen disagrees, stating that they are not complementary, but instead are supplementary. Who is correct?

LESSON 20

Interpreting Truth Tables

Warm Up

1. (10) **Vocabulary** The statement "If q, then p," is the _________ of the statement "If p, then q."

Consider this statement to answer problems 2 and 3. (10)
If two numbers are both positive, then their product is positive.

2. Write the converse of the statement.

3. Which is true: the statement, its converse, both, or neither?

New Concepts

A conditional statement has the form, "If p, then q." Recall that several other statements can be constructed from the conditional statement.

- the converse statement, "If q, then p"
- the inverse statement, "If $\sim p$, then $\sim q$"
- the contrapositive statement, "If $\sim q$, then $\sim p$"

Reading Math

If p is the statement, "An angle is acute," then the **negation** $\sim p$ ("not p") is the statement, "An angle is not acute."

The combination of a conditional statement and its converse is called a **biconditional statement**. A biconditional statement is true only when both the original statement and its converse are true. The biconditional of "If p, then q" and "if q, then p" can be written as "p if and only if q."

Example 1 Analyzing Conditional Statements

a. State the converse of this statement: *If* $x^2 \leq 4$, *then* $x \leq 2$.

SOLUTION Switch the hypothesis and conclusion: *If* $x \leq 2$, *then* $x^2 \leq 4$.

b. Determine if the statement and converse from part **a** are true.

SOLUTION Suppose the hypothesis is true: $x^2 \leq 4$. Solving for x shows that x must be 2 or -2. Values greater than -2 and less than 2 also satisfy this equation. In fact, all the possible solutions are less than or equal to 2. Therefore, the statement is true.

For the converse, you can find a counterexample. For example, if $x = -3$, the conclusion is not true.

$$x^2 \leq 4$$
$$(-3)^2 \leq 4$$
$$9 \leq 4$$

So, the converse is not true.

c. Write the biconditional of the statement. "If $x^2 \leq 4$, then $x \leq 2$." Is it true? Explain why or why not.

SOLUTION The biconditional is "$x^2 \leq 4$ if and only if $x \leq 2$."

For the biconditional to be true, both the statement and its converse must be true. In this case, the converse is false, so the biconditional is not true.

A **truth table** is a table that lists all possible combinations of truth values for a hypothesis, a conclusion, and the conditional statement or statements they form. Truth tables are useful tools because they show all the true/false possibilities at a glance:

Hypothesis: p	Conclusion: q	Statement: If p, then q
T	T	T
T	F	F
F	T	T
F	F	T

Math Reasoning

Analyze Why is a conditional statement true even when both its hypothesis and conclusion is false?

The highlighted row is the only combination for which the statement is *not* true. For example, to prove that the statement, "If a quadrilateral is equiangular, then it is a rhombus," is false, you need an example of an equiangular quadrilateral (p is true) that is not a rhombus (q is false).

Example 2 Using a Truth Table

a. Complete a truth table for the statement in Example 1.

SOLUTION There are four possibilities for the hypothesis and conclusion: both are true, only the hypothesis is true, only the conclusion is true, or both are false. Use these to create the rows of the truth table.

Hypothesis: $x^2 \leq 4$	Conclusion: $x \leq 2$	Statement: If $x^2 \leq 4$, then $x \leq 2$
T	T	T
T	F	F
F	T	T
F	F	T

Notice that the statement is only false when the hypothesis is true but the conclusion is false (the highlighted row of the table). For the statement "If $x^2 \leq 4$, then $x \leq 2$," this is impossible. Therefore, the statement is always true.

b. Add columns to your truth table for the statement's converse and its biconditional. Complete the table for these two statements.

SOLUTION

Hypothesis: $x^2 \leq 4$	Conclusion: $x \leq 2$	Statement: If $x^2 \leq 4$, then $x \leq 2$	Converse: If $x \leq 2$, then $x^2 \leq 4$	Biconditional: $x^2 \leq 4$ if and only if $x \leq 2$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

A **compound statement** combines two statements using *and* or *or*. It is similar to a conditional statement, except that p and q are related by "and" or "or" rather than by "if" and "then".

> **Math Reasoning**
>
> **Analyze** Is a biconditional statement a conjunction or disjunction? Explain.

A compound statement that uses *and* is called a **conjunction**. Conjunctions have the form "p and q." For example, statement p stands for "I had bacon for breakfast," and q stands for "I had eggs for breakfast." If you have bacon and eggs for your breakfast, the conjunction, "I had bacon and eggs for breakfast," is true because p and q are both true. But if you have bacon and toast, the conjunction is false, because p is true but q is not.

A compound statement that uses *or* is called a **disjunction**. Disjunctions have the form "p or q." For example, suppose a lunch menu offers the choice of "soup or salad" as an appetizer. This is considered a disjunction because you can choose one or the other.

Example 3 Analyzing Compound Statements

A clothing store accepts cash or credit cards but not personal checks. It gives discounts on all cash purchases. Consider the statements, "a customer makes a credit-card purchase," and, "a customer gets a discount."

a. What is the conjunction of these statements? Use a truth table to assess its truth value.

SOLUTION

The conjunction is: "a customer makes a credit-card purchase *and* gets a discount."

Statement: p	Statement: q	Conjunction: p and q
T	T	T
T	F	F
F	T	F
F	F	F

For the conjunction to be true, the store would have to give discounts (q is true) on credit-card purchases (p is true). Discounts are only given on cash purchases though, so the conjunction is false.

b. What is the disjunction of these statements? Is it true or false?

> **Math Reasoning**
>
> **Justify** Why is a disjunction true in more cases than a conjunction?

SOLUTION

The disjunction is, "A customer makes a credit-card purchase *or* gets a discount." Extend your truth table:

Statement: p	Statement: q	Conjunction: p and q	Disjunction: p or q
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	F

For the disjunction to be false, the sale would have be a cash purchase (p is false) with no discount (q is true). Since this does not happen, the disjunction is true.

Example 4 Application: Astronomy

When stars run out of fuel, they either become black holes or degenerate stars. Consider the statements, "A star will become a degenerate star," and "A star will become a black hole." Form the conjunction and disjunction of these statements. Is the conjunction true? Is the disjunction true? Explain.

SOLUTION

Conjunction: *A star will become a degenerate star and a black hole*.

Disjunction: *A star will become a degenerate star or a black hole.*

The conjunction is false because a star cannot be both a degenerate star and a black hole. The disjunction is true, because all stars eventually become either degenerate stars or black holes.

Lesson Practice

For a–c, consider the statement, "If a quadrilateral is equiangular, then it is a rhombus."

a. State the converse of the statement. *(Ex 1)*

b. Determine whether the statement is true. Also determine whether its converse is true. *(Ex 1)*

c. Write the biconditional of the statement. Is it true? Explain. *(Ex 1)*

Use the description of the restaurant to answer d–g.

Restaurants The chef's special at a five-star restaurant offers its customers a complimentary appetizer based on their choice of entrée. If customers order a filet mignon entrée, they receive leek soup for the appetizer. If customers order grilled salmon for the entrée, they receive baby spinach salad for the appetizer.

d. Use truth tables to represent the given statements. Interpret the tables for the statements. *(Ex 2)*

e. Add columns to your truth tables to address the statements converses and biconditionals. Interpret the tables for these statements. *(Ex 2)*

For f and g, consider the statements, "A customer orders a filet mignon entrée," and, "A customer receives a baby spinach salad appetizer."

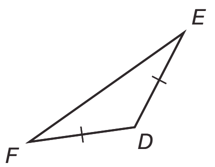
f. What is the conjunction of these statements? Use a truth table to assess its truth value. *(Ex 3, Ex 4)*

g. What is the disjunction of these statements? Is it true or false? *(Ex 3, Ex 4)*

Practice Distributed and Integrated

1. Graph the line with equation $y = 0.5x - 1.5$. *(16)*

2. Classify $\triangle DEF$ by its sides. *(13)*



* 3. (20) Find the converse of the statement, "If two angles are congruent, then their measures are equal," and write a biconditional statement equivalent to them.

* 4. (19) **Architecture** Each floor of an office building has 30 rectangular windows measuring 57 inches by 38 inches. The building has 23 floors.

a. Determine the total area of glass in all the windows, in square inches.

b. Express this area in square feet.

5. (10) Draw a Venn diagram to represent the following conditional statement.
A number is an integer if it is a whole number.

6. (4) **Aim** Julio is playing a game where he rolls a ball from one corner of the room to the opposite corner, without touching the walls. In how many different directions could Julio roll the ball to reach its target? How do you know?

7. (10) Find the converse of the following statement.
If you are an only child, then you do not have a brother.

8. (14, 19) **Multiple Choice** Which is a counterexample to the conjecture, "If a quadrilateral is equiangular, then it is equilateral"?

A A rectangle that is not a rhombus **B** A rhombus that is not rectangular
C Any square **D** Any parallelogram

9. (17) **Geography** Write the converse of the following statement.
If a state lies to the east of longitude 125°W, then it is one of the lower 48 states.

10. (3) **Write** Can an obtuse and an acute angle make a straight angle? Can an obtuse and acute angle make a right angle? Explain.

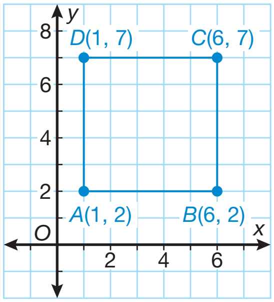
11. (7) **Write** Explain why testing a conjecture is not a generally accepted method of proof.

12. (12) **Analyze** Given two lines m and n with a transversal ℓ, consider these statements:
p: *Alternate interior angles formed by* ℓ *are congruent.*
q: *Alternate exterior angles formed by* ℓ *are congruent.*
r: *Lines* m *and* n *are parallel.*

a. Prove that if statement p is true, then statement q is true.

b. Use part **a** and the Converse of the Alternate Exterior Angles Theorem to prove the Converse of the Alternate Interior Angles Theorem.

13. (11) Determine the midpoint of each side of square $ABCD$.



14. (14) **Write** What can be done to disprove a conjecture? Explain.

*15. (19) **Error Analysis** A student calculates the area and perimeter of a rectangle with length 7.5 cm and width 3 cm as follows:

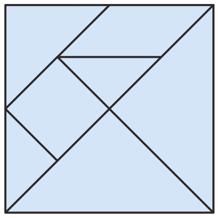
$$A = 3(7.5) \qquad \text{and} \qquad P = 3 + 7.5$$
$$= 22.5 \text{ cm}^2 \qquad\qquad\qquad = 10.5 \text{ cm}$$

Explain the student's error.

*16. (20) **a.** State the converse of the statement, "If a quadrilateral is a square, then it has four congruent sides." Write the statement and its converse as a biconditional statement.

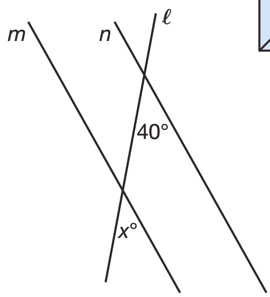
b. Determine whether the biconditional is true. Use a truth table to explain why or why not.

*17. (19) **Games and Puzzles** This tangram set includes two quadrilaterals. Classify them.



*18. (18) An exterior angle of a triangle is a right angle. What can you say about the two remote interior angles? Explain.

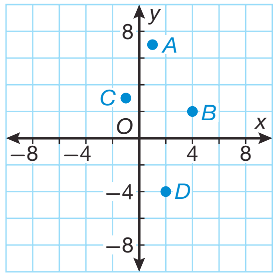
19. (12) **Algebra** What value of x allows you to apply the Converse of the Corresponding Angles Theorem to this figure? If x has this value, how are lines m and n related?



20. (16) Determine the slope of the line passing through (3, 6) and (6, 0).

21. (6) **Write** Can the sum of the angles in a triangle be considered a group of supplementary angles? Explain.

Use the diagram at right for the next two questions. Round to the nearest hundredth.



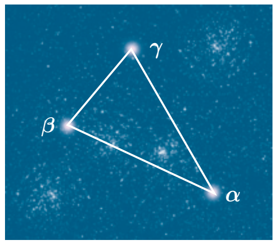
22. (9) What is the length of $\overline{AB}$?

23. (9) What is the length of $\overline{CD}$?

*24. (20) Either Soo-Lin takes cheese sandwiches to school or she takes a salad, but not both or neither. On days when she takes cheese sandwiches, she takes milk. Sometimes she takes milk even if she does not take cheese sandwiches. Consider the statements "Soo-Lin takes cheese sandwiches," "Soo-Lin takes a salad," and "Soo-Lin takes milk." Write two disjunctions that are true, each using two of these statements.

25. (15) Is this conditional statement true? If so, explain why. If not, give a counterexample.
If a polygon is regular, then it is equiangular.

26. (18) **Astronomy** This star chart shows the constellation Triangulum. Given that there is an almost perfect right angle at β ("beta"), what can you say about the angles at the other two vertices?



*27. (10) **a.** State the converse of the statement, "If the month is December, then it has 31 days."
b. Determine whether the statement is true, and whether its converse is true.

28. (8) **Woodworking** Scarlet measures two sides of a board of wood as 14 inches and 7 inches. If she wants to find out the perimeter of the board, what formula should she use? What is the perimeter of the board?

29. (11) Determine the midpoint of each side of triangle KLM with coordinates $K(-3, 8)$, $L(1, 4)$, and $M(5, 2)$.

30. (Inv 1) If a pair of parallel lines is cut by a transversal, and the alternate exterior angles measure $(160 - 2x)°$ and $(5x + 55)°$, what is the measure of each angle?

INVESTIGATION

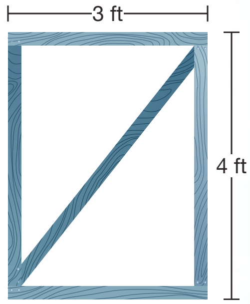
2

Proving the Pythagorean Theorem

Reading Math

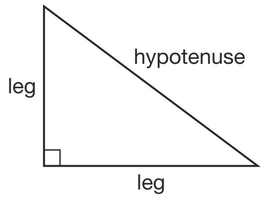
Recall that a **right angle** measures exactly 90°.

A carpenter has built a rectangular frame with dimensions of 3 feet by 4 feet. To help secure this frame and ensure that the corners are at 90° angles, she places a brace across the diagonal between two opposite corners.



1. What type of triangle is formed when the diagonal brace is positioned across the frame?

Recall from Lesson 13 that a triangle with one right angle is classified as a right triangle. In right triangles, the side of the triangle that is opposite the right angle is called the **hypotenuse**. The other two sides of the triangle that form the right angle are called the **legs of a right triangle**.

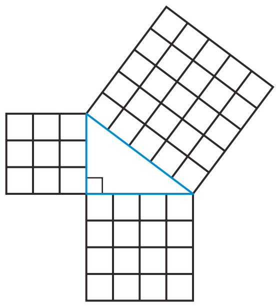


Math Reasoning

Analyze Besides the right angle, what do you know about the other two angles in right triangles?

The carpenter can determine the length of the brace if she knows the lengths of the triangle's legs by using the Pythagorean Theorem. The Pythagorean Theorem states that for a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

The concept of the Pythagorean Theorem can be demonstrated using a model. In the diagram below, each side of the triangle is also a side of a square with each side congruent to that side of the triangle. All three squares are illustrated as blocks to make it easier to calculate the area of each one.



In the figure, the length of the sides of the triangle are 3, 4, and 5 units and the area of each square is 9, 16, and 25 square units, respectively.

2. Draw a right triangle with side lengths of 6 centimeters, 8 centimeters, and 10 centimeters, using a protractor to ensure that one angle is exactly 90°. Now make and cut out three squares; one with 6-centimeter long sides, one with 8-centimeter long sides, and one with 10-centimeter long sides. Place each square next to the corresponding sides of the triangle.

   Which side is the hypotenuse in your diagram?

Hint

Recall that the area of a square is found by multiplying the length of a side by itself, since the sides of a square are all equal.

3. Which square has the largest area?

4. Try to fit the squares you made for the two legs of the triangle into the square for the hypotenuse. Do the two smaller squares fit exactly into the third square if you cut up the smaller squares? What can you say about the relationship between the areas of the squares?

5. Calculate the area of each of the squares that you constructed. Does this support your answer for part 4?

6. **Write** How do these results demonstrate the relationship in the Pythagorean Theorem?

Algebraically, the Pythagorean Theorem can be expressed as $a^2 + b^2 = c^2$ where a and b represent the lengths of the legs and c represents the length of the hypotenuse.

Math Reasoning

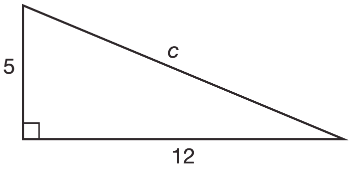
Generalize Is the length of a hypotenuse in a right triangle always longer than each of the legs of the triangle?

The length of the brace that the carpenter needs can be calculated algebraically. If the brace in the carpenter's frame represents the hypotenuse of a right triangle, c, and the length and width of the frame represent the legs of a right triangle, a and b, the equation $a^2 + b^2 = c^2$ can be used.

$$a^2 + b^2 = c^2$$
$$3^2 + 4^2 = c^2$$
$$25 = c^2$$
$$\sqrt{25} = \sqrt{c^2}$$
$$5 = c$$

Therefore, a 5-foot long brace is needed.

7. Determine c for this triangle.



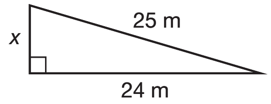
8. A right triangle has a hypotenuse that measures 20 centimeters and one leg that is 16 centimeters. What is the length of the other leg?

9. **Landscaping Design** Jillian has a corner garden in her backyard and wants to place decorative edging along the front edge. The garden is 6 feet along the back fence and 8 feet along the side fence. The two fences meet at a right angle. How much edging will Jillian need?

Investigation Practice

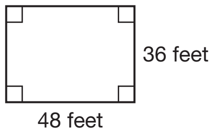
a. If the length of one leg of a right triangle increases while the length of the other leg is constant, what happens to the length of the hypotenuse in a right triangle? the measure of the right angle? the measure of the acute angles?

b. Find the length of side x in the triangle.



c. A right triangle has a hypotenuse of 15 inches and one leg that measures 12 inches. What is the length of the third side?

A builder is framing a house and wants to make sure that the walls will be at 90° angles to each other. The diagram shows the length measurements of two walls. Use this figure to answer the following questions.



d. How could the Pythagorean Theorem be used to determine if the walls meet at a right angle?

e. If the builder strings a rope across the diagonal of the floor, what length would the rope be if the walls were at right angles to each other?