LESSON 21

Laws of Detachment and Syllogism

Warm Up

1. (10) **Vocabulary** The statement below is called a(n) __________ statement. (***conditional***, ***inductive***)
If it is almost 8 p.m., then my favorite television show is about to start.

For each statement below, indicate if each conclusion is true or false.

2. (10) *If all the students on a school bus are athletes from a local school, then all the school's athletes are on the school bus.*
3. (17) *If* $x - 6 = 9$, *then* $x = 15$.
4. (17) *All odd numbers are prime numbers.*
5. (17) *All prime numbers greater than 2 are odd numbers.*

New Concepts

Deductive reasoning is the process of using logic to draw conclusions from given facts, definitions, and properties.

When two related statements are true, deductive reasoning can be used to make a conclusion. For example:

The bakery makes fresh bread every morning. It is morning.
Therefore, the bakery is making fresh bread.

Example 1 Using Deductive Reasoning

Use deductive reasoning to form a "Therefore" concluding statement from the given statements.

a. *All human beings need to breathe. Marla is a human being.*

SOLUTION
Therefore, Marla needs to breathe.

b. *All the chess team members won their opening match in the last tournament. Jeffery is on the chess team.*

SOLUTION
Therefore, Jeffery won his opening match in the last tournament.

c. *All the women in the royal family were wearing hats at the ball. Melissa is in the royal family.*

SOLUTION
Therefore, Melissa was wearing a hat at the ball.

Math Reasoning

Analyze If the second statement in this example was instead *"Marla needs to breathe,"* could you conclude that she is a human being?

Law of Detachment
For two statements p and q, when "If p, then q" is a true statement and p is true, then q is true.

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The Law of Detachment is a form of deductive reasoning that can be used to draw valid concluding statements. When the given facts are true, then correct logic can lead to a valid conclusion.

For example:
If it is Monday, then Marc will go to work.
Today is Monday.
Therefore, Marc will go to work today.

In the statements above, p represents the phrase "*it is Monday*," and q represents the phrase "*Marc will go to work*."

Caution

Be sure to draw conclusions based on the information given. Do not make assumptions that are not logically sound or which have not been stated in the question.

Example 2 Using the Law of Detachment

For the following statements, use the Law of Detachment to write a valid concluding statement. Assume each conditional statement is true.

a. *When it is cold outside, I wear my warm jacket. It is cold outside today.*

SOLUTION

Therefore, I will wear my warm jacket today.

b. *If an angle is acute, then it cannot be obtuse. Angle* D *is acute.*

SOLUTION

Therefore, angle D *cannot be obtuse.*

c. *If a number is even, then it can be divided by 2. The number 104 is even.*

SOLUTION

Therefore, 104 can be divided by 2.

Law of Syllogism
When "If p, then q" and "If q, then r" are true statements, then "If p, then r" is a true statement.

The Law of Syllogism is another form of deductive reasoning. In this case, a third conditional statement is based on two conditional statements in which the conclusion of one is the hypothesis of the other.

This law poses that an intermediate truth is a valid progression from the original statement to a valid conclusion. For example:

If there is a power outage, then the freezer does not work.
AND
If the freezer does not work, then the ice cream will eventually melt.
THEN
If there is a power outage, then the ice cream will eventually melt.

Example 3 Using the Law of Syllogism

Use the Law of Syllogism to write a third conditional statement based on the statements below.

If Annika jumps higher than 5 feet 3 inches in this event, then she will win first place.

If Annika wins first place, then she will receive a medal.

SOLUTION

"*Annika jumps higher than 5 feet 3 inches in this event*" is p. "*She will win first place*" is q. "*She will receive a medal*" is r. Using the Law of Syllogism, write "if p, then r."

If Annika jumps higher than 5 feet 3 inches in this event, then she will receive a medal.

Math Reasoning

Write Write about one instance in the past few days where you have used the Law of Detachment or the Law of Syllogism to draw a conclusion about something in your life.

Example 4 Using the Laws of Detachment and Syllogism

For each of the given statement sets, draw a valid conclusion. Identify which law is used to reach the conclusion. Assume each conditional statement is true.

a. *If Maria wants to see a movie, then she goes to the theater. If Maria goes to the theater, then she buys popcorn.*

SOLUTION

If Maria wants to see a movie, then she buys popcorn. The Law of Syllogism is used. The first statement is of the form "If p, then q." The second statement is of the form "If q, then r." The conclusion follows, "If p, then r."

b. *If it is raining, then I will take an umbrella to school. Today, it is raining.*

SOLUTION

Therefore, today I will bring an umbrella to school. The Law of Detachment is used. The first statement is of the form "If p, then q." The second statement is of the form "p is true," which leads to the conclusion, "then q."

c. *All bibbles are bobbles. All bobbles play bubbles.*

SOLUTION

Recall that conditional statements are not always in 'if-then' form, but they can be rewritten that way. In 'if-then' form, the given statements read as follows:

If something is a bibble, then it is also a bobble. If something is a bobble, then it plays bubbles.

If something is a bibble, then it plays bubbles. The Law of Syllogism is used. The first statement is of the form "If p, then q." The second statement is of the form "If q, then r." The conclusion follows, "If p, then r."

Lesson Practice

Use deductive reasoning to form a concluding statement from the given information.
(Ex 1)

a. *All the girls on the swim team are left-handed. Lorissa is on the swim team.*

b. *When it is below 32°F for at least a week, the pond freezes. It has been below 32°F for a week.*

c. *When every answer on a math test is correct, a student will get a perfect score on the test. Michael got every answer correct on the last math test.*

d. *When employees work more than 40 hours in a week, they get paid overtime. Dominiqua worked 43 hours this week.*

e. (Ex 2) Use the Law of Detachment to write a valid conclusion to the statements below.
If the gift I bought for my cousin is a toy truck, then it has four wheels.
The gift I bought for my cousin is a toy truck.

f. (Ex 3) Write a third conditional statement using the Law of Syllogism:
If Nafeesa enrolls in an elective, then she will enroll in Orchestra.
If Nafeesa enrolls in Orchestra, then she will play the violin this semester.

g. (Ex 4) What conclusion can be drawn from the following set of statements?
If I oversleep tomorrow morning, then I will miss my bus.
If I miss my bus, then I will be late for my appointment.

h. (Ex 4) Which law was used to reach the conclusion in problem **g**?

Use detachment or syllogism to draw a valid conclusion to the following statements. Identify which law was used in reaching the conclusion.
(Ex 4)

i. *If a gumble is hungry, it craves gloop. If a gumble craves gloop, he must hunt for gloop.*

j. *If a vehicle is a unicycle, then it has only one wheel. This vehicle is a unicycle.*

Practice Distributed and Integrated

* **1.** (21) **Analyze** What conclusion can be drawn from these statements?
If the quality of the granite is high, then our company will purchase it.
The quality of the granite is high.

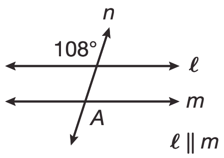
2. (8) What is the length of a rectangle with a perimeter of 300 feet and a width of 32 feet?

3. (16) **Error Analysis** Tim states that the slope between the points (1, 1) and (5, 5) is (4, 4). Yasmini states that the slope is $\frac{4}{4} = 1$. Who is correct? Explain.

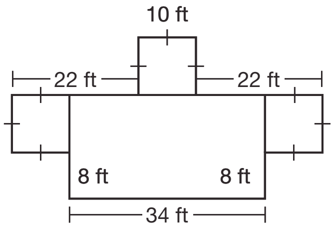
4. (20) **Probability** Write the converse of this statement and state whether the converse is always true. If both the statement and its converse are always true, write it as a biconditional statement.

If it is certain that a sock I pull out of the drawer will be red, then all the socks in the drawer are red.

5. (Inv 1) **Verify** Genji says that angle A measures 108°. Is she correct? What theorem can be used to justify her answer?



*** 6.** (19) **Landscaping** A landscape designer needs to fence in the area indicated in the diagram. How much fencing will be needed for the whole area?

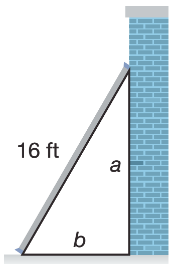


7. (9) Find the distance between the points $A(4, 5)$ and $B(7, 1)$.

*** 8.** (20) **Multiple Choice** A defense lawyer in court states in her cross examination of a witness, "You were either at the scene of the accident or you were not. Which is it?" What type of statement is she using?

A a compound statement | **B** a disjunction
C both **A** and **B** are correct | **D** neither **A** nor **B** is correct

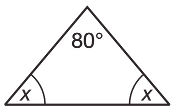
9. (Inv 2) **Safety** It is recommended that the ratio of $a{:}b$ in the figure at right be 4:1 to prevent the ladder from shifting. According to this ratio, how far from the base of the wall should you place the foot of a 16-foot ladder? Round to the nearest inch.



10. (14) **Write** Disprove the statement by writing a counterexample.

All numbers that are either one more or one less than a multiple of 6 are prime.

***11.** (18) Use the Triangle Angle Sum Theorem to find the missing angles in the triangle.

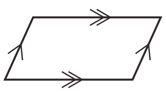


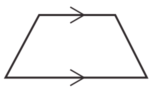
Write the converse of each given statement, then determine whether the converse is true.

12. (17) **Analyze** *If* $3x + 4 = 10$*, then* $x = 2$.

13. (17) **Analyze** *If a number is prime, then it is odd.*

14. (19) Classify the following quadrilaterals.

a. 

b. 

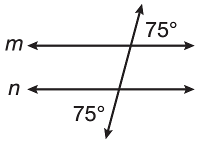
15. (11) Find the midpoint of the points $(-2, 5)$ and $(6, 9)$.

***16.** (21) Use the Law of Detachment to draw a valid concluding statement from the following:

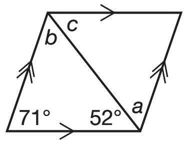
If the bread has just come out of the oven, then it is fresh. The bread has just come out of the oven.

17. (14) Disprove the statement, "All irregular polygons are concave polygons," by drawing a counterexample.

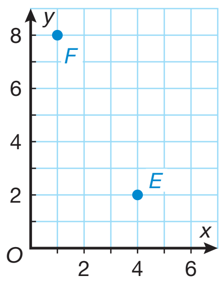
18. (12) **Justify** Are the lines m and n parallel? Explain.



19. (Inv 1) Find the missing angles in the figure on the right.



20. (16) **Algebra** Find the slope of the line segment that joins points E and F.

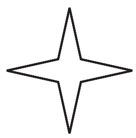


***21.** (20) Convert the following statements into a compound statement with a disjunction.

a. *Some numbers between 20 and 30 are prime numbers. The other numbers between 20 and 30 are composite numbers.*

b. *The lunch special can come with a soup. The lunch special can come with a salad.*

22. (15) Is this polygon concave or convex? Explain.



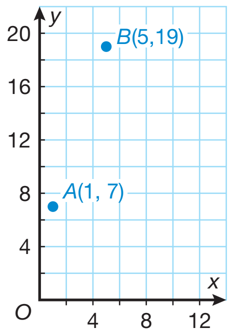
***23.** (21) Write a third conditional statement using the Law of Syllogism:

If the season is rainy, then the crops will flourish.
If the crops flourish, then there will be plenty of corn for the livestock.

***24.** (21) Write a statement that can be deduced from the following statements.

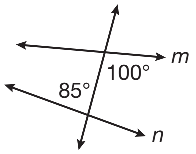
All the children in the Dument family are the tallest in their classes at school.
Lucy is a child in the Dument family.

25. (16) **Formulate** Find the equation of the line that passes through points A and B on the graph.



26. (13) **Sewing** Thomas is sewing a tent with a triangular flap for a door. If the height of the door is 4 feet and the door is $4\frac{1}{2}$ feet wide, how many square feet of fabric will he need to make the door?

27. (12) **Justify** Are the lines m and n parallel? Explain.

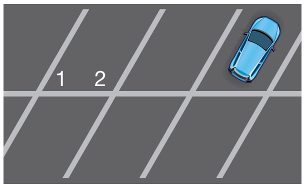


***28.** (20) Convert the following statements into compound statements with a conjunction.

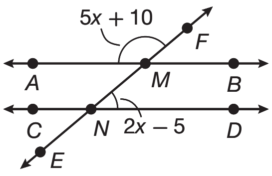
a. *Today it is cold. Today it is snowing.*

b. *Later this week is the big game at our school. Anita is playing in the big game.*

29. (Inv 1) **Parking** A construction firm is designing a large parking lot. The lines that mark the widths of the spaces will be parallel. For optimum parking safety and visibility when backing out, the measure of ∠1 should be half the measure of ∠2. What are the measures of both angles?



30. (Inv 1) Find the measure of each angle in this figure. Lines AB and CD are parallel.



LESSON **22**

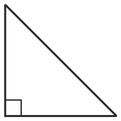
Finding Areas of Quadrilaterals

Warm Up

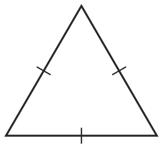
1. (13) **Vocabulary** A segment from a vertex that forms a right angle with the line that contains the base is called the ________ of the triangle. (***base***, ***height***, ***hypotenuse***)

2. (13) Identify each type of triangle.

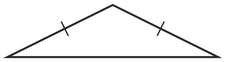
a.



b.

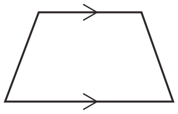


c.

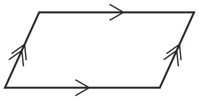


3. (19) Identify each type of quadrilateral.

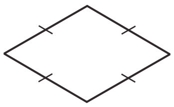
a.



b.



c.

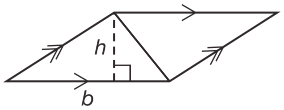


4. (SB 14) Evaluate the expression, $2x^2 + 3xy - y^2$, for $x = -2$ and $y = 1$.

New Concepts

Recall from Lesson 13 that the area of a triangle can be determined using the formula $A = \frac{1}{2}bh$ where b represents the base, h represents the height, and A is in square units.

If a diagonal is drawn in a parallelogram, two identical triangles are formed.



The area of the parallelogram is twice the area of one of the triangles, so an expression to find the area of a parallelogram could be written $A = 2\left(\frac{1}{2}bh\right)$. Simplifying this expression results in the formula below, which can be used to find the area of a parallelogram.

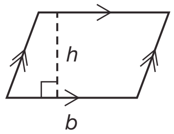
Math Language

In a parallelogram, the **height** is a segment perpendicular to both bases. Its length is used to calculate the area of the parallelogram.

Area of a Parallelogram

To find the area of a parallelogram (A), use this formula, where b is the length of the base, and h is the height.

$$A = bh$$



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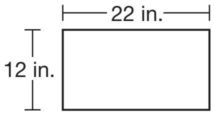
Since rectangles, rhombuses, and squares are all types of parallelograms, the areas of these shapes can also be found using this formula.

Example 1 Finding Areas of Parallelograms

Find the area of each parallelogram.

a.



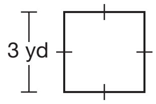
SOLUTION

$A = bh$

$= (22 \text{ in.})(12 \text{ in.})$

$= 264 \text{ in.}^2$

b.



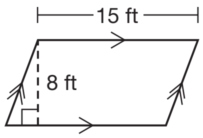
SOLUTION

$A = bh$

$= (3 \text{ yd})(3 \text{ yd})$

$= 9 \text{ yd}^2$

c.



SOLUTION

$A = bh$

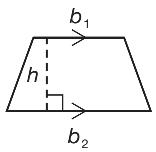
$= (15 \text{ ft})(8 \text{ ft})$

$= 120 \text{ ft}^2$

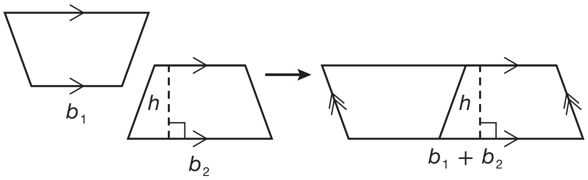
Hint

Always label the answer to an area problem with the correct unit. Area is measured in square units.

Recall that a trapezoid is a quadrilateral with exactly one pair of parallel sides. The height of a trapezoid is a segment that is perpendicular to both parallel sides of the trapezoid. The parallel sides of a trapezoid are known as the bases, b_1 and b_2.



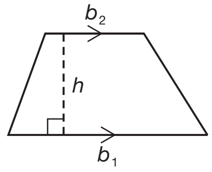
Two congruent trapezoids can be connected to form a parallelogram with side lengths equal to $b_1 + b_2$, illustrated in the following diagram.



The area of the parallelogram is found by multiplying the length of its base by its height. Since the base of the parallelogram is $b_1 + b_2$, the area is $A = (b_1 + b_2)h$. Since this is the area of two trapezoids put together, the area of one trapezoid will be half this amount. The following formula can be used to find the area of any trapezoid.

Area of a Trapezoid

To find the area of a trapezoid (A), use the following formula, where b_1 is the length of one base, b_2 is the length of the other base of the trapezoid, and h is the trapezoid's height.

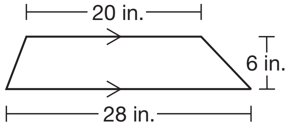


$$A = \frac{1}{2}(b_1 + b_2)h$$

Example 2 Finding Areas of Trapezoids

Find the area of each trapezoid.

a.



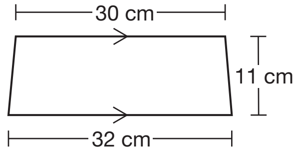
SOLUTION

$$A = \frac{1}{2}(b_1 + b_2)h$$

$$= \frac{1}{2}(20 \text{ in.} + 28 \text{ in.})(6 \text{ in.})$$

$$= 144 \text{ in}^2$$

b.



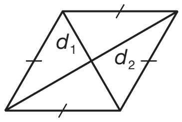
SOLUTION

$$A = \frac{1}{2}(b_1 + b_2)h$$

$$= \frac{1}{2}(30 \text{ cm} + 32 \text{ cm})(11 \text{ cm})$$

$$= 341 \text{ cm}^2$$

Recall that a rhombus is a type of parallelogram with four congruent sides. There are a few different methods that can be used to find the area of a rhombus. One method is to apply the formula developed for a parallelogram, $A = bh$. A second method uses the diagonals of the rhombus, which are labeled d_1 and d_2.



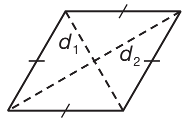
The area of a rhombus is equal to one-half the product of its diagonals. The formula below can be used to find the area of any rhombus.

Verify

A rhombus's diagonals are perpendicular and bisect each other. For a rhombus with diagonals *x* and *y*, show that the area of the four triangles in the rhombus's interior is equal to the area found by using the formula given here.

Area of a Rhombus

To find the area of a rhombus (A), use the following formula, where d_1 is the length of one diagonal, and d_2 id the length of the other diagonal of the rhombus.

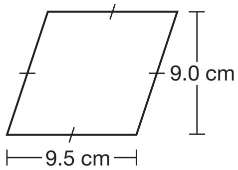


$$A = \frac{1}{2}d_1d_2$$

Example 3 Finding Areas of Rhombuses

Find the area of each rhombus.

a.



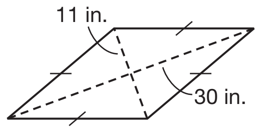
SOLUTION

$A = bh$

$= 9.5 \text{ cm} \times 9.0 \text{ cm}$

$= 85.5 \text{ cm}^2$

b.



SOLUTION

$A = \frac{1}{2}d_1d_2$

$= \frac{1}{2}(11 \text{ in.} \times 30 \text{ in.})$

$= \frac{1}{2}(330 \text{ in}^2)$

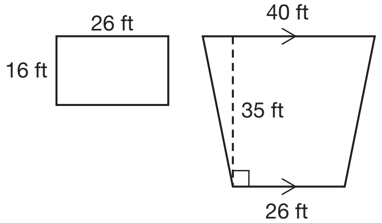
$= 165 \text{ in}^2$

Example 4 Application: Carpeting

Caution

This problem asks for the total area that is to be carpeted. To find this, you will have to combine the areas of both figures.

Two areas of a day care need to be carpeted. The play area is shaped like a trapezoid, and the supplies area is shaped like a rectangle. Use the diagram of these two areas to determine the total area that needs to be carpeted.



SOLUTION

For the rectangular supplies area,

$A = lw$

$= 26 \times 16$

$= 416 \text{ ft}^2$

For the trapezoidal play area,

$A = \frac{1}{2}(b_1 + b_2)h$

$= \frac{1}{2}(40 \text{ ft} + 26 \text{ ft})(35 \text{ ft})$

$= \frac{1}{2}(66 \text{ ft})(35 \text{ ft})$

$= 1155 \text{ ft}^2$

The total area can be found by adding these two areas:

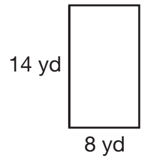
$(416 + 1155) \text{ ft}^2 = 1571 \text{ ft}^2$

Therefore, a total area of 1571 square feet needs to be carpeted.

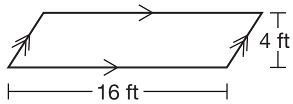
Lesson Practice

Find the area of each parallelogram.
(Ex 1)

a.

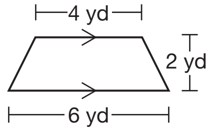


b.



c. *(Ex 2)* Find the area of a trapezoid with parallel sides measuring 14 centimeters and 21 centimeters and a height of 13 centimeters.

d. *(Ex 2)* Find the area of this figure.



e. *(Ex 3)* Find the area of a rhombus that has diagonal lengths of 8 inches and 11 inches.

f. *(Ex 3)* Find d_2 of a rhombus if d_1 is 6 meters and it has an area of 12 square meters.

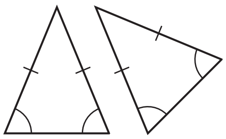
g. *(Ex 4)* **Carpeting** Dakota needs to put new flooring in the cafeteria of a high school. The cafeteria is shaped like a diamond with four congruent sides. What measurements should Dakota take to find the total area of the cafeteria?

Practice Distributed and Integrated

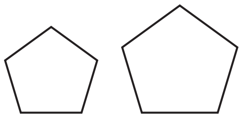
* **1.** *(22)* Find the area of a rectangle with a diagonal measuring 25 inches and one side that measures 20 inches.

2. *(15)* **Analyze** Are the following pairs of polygons congruent? Explain.

a.



b.



3. *(8)* **Multiple Choice** The exact conversion from Celsius to Fahrenheit is °F = $\frac{9}{5}$(°C) + 32. The actual temperature at which the two temperature scales have the same numerical value is

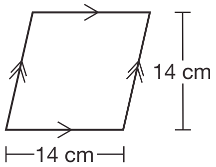
A −30 degrees **B** 30 degrees
C −40 degrees **D** 40 degrees

4. *(21)* Use deductive reasoning to write a valid conclusion to this statement.

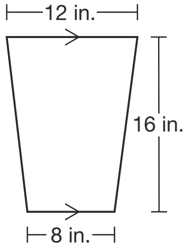
All games played at the baseball tournament were decided by one run. Allentown played Holdenville at the baseball tournament.

* **5.** Find the area of the following quadrilaterals.
(22)

a.



b.

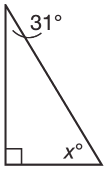


6. **Justify** Write the converse to each conditional statement. State if each converse is true.
(10)

a. *If* $x + 11 = 5$, *then* $x = -6$.

b. *If it rains, then I bring my umbrella to work.*

7. **Justify** What is the value of x? Explain how you found it.
(18)



8. State whether the following statements are true or false.
(6)

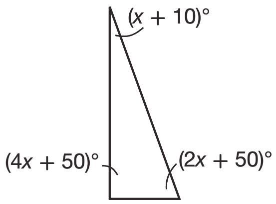
a. Linear pairs of angles are supplementary.

b. Complementary angle pairs can be linear.

c. The adjacent angles formed by two intersecting lines are called vertical angles.

d. If two angles are in the same plane and share a vertex and a side, but no interior points, then the two angles are adjacent angles.

xy^2 * **9.** **Algebra** Determine the measure of each angle.
(18)



***10.** Use the Law of Detachment to write a concluding statement.
(21)

If today is Thursday, then it is Mrs. Wu's turn to drive the children to their piano lesson. Today is Thursday.

11. **Urban Planning** On a scale map of a state, each square represents 5 square miles. The water works company needs to connect two purification stations that are located at the points (3, 11) and (14, 26) on the grid. About how many miles apart are these two stations?
(9)

***12.** Write each set of statements as a biconditional statement.
(20)

a. Two angles are congruent if they have the same measure. Two angles that have the same measure are congruent.

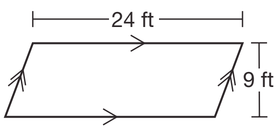
b. A triangle is equilateral if it has three lines of symmetry. A triangle that has three lines of symmetry is equilateral.

13. The following statement is an example of a ___________. (***conjunction, disjunction***)
(20)

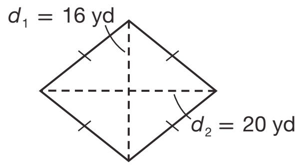
A set of two statements can often be joined as a conjunction or a disjunction.

***14.** Find the areas of the following shapes.
(22)

a.



b.



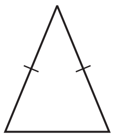
15. (16) Find the equation of the line that:

a. passes through the point (0, 5) with a slope of -2.

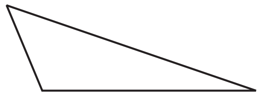
b. passes through the point (4, 11) with a slope of $\frac{1}{2}$.

16. (13) Classify each triangle according to its sides.

a.



b.

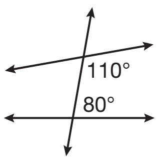


17. (13) **Yachting** A race course for a yachting regatta is in the form of a right triangle. If the two legs of the triangle are 6 miles and 8 miles long, how much distance does the race cover?

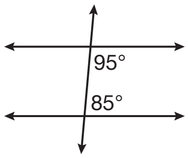
***18.** (16) **Error Analysis** Veronica thought the equation of the line that passes through the points (1, 5) and (5, 17) is $y = 2x + 3$, but then she found out that her answer was incorrect. What has she done wrong and what is the correct equation?

19. (12) **Analyze** Are the following pairs of lines parallel? Explain.

a.

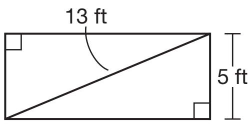


b.



20. (15) **Write** Draw an equiangular octagon with sides that are each 2 centimeters long. Is it regular? Explain.

***21.** (19) **Landscaping** The following diagram represents a patio that is to be covered in stones. Determine the total area of the patio.



22. (11) **Urban Planning** Three transformer stations exist on a grid at positions *A*, *B*, and *C*. The power company needs to locate the three points that are exactly halfway between each of the three stations to determine where new stations must be built. If the existing stations are located at grid points $A(5, 9)$, $B(4, -4)$, and $C(8, 17)$, find the locations of the new stations.

23. (16) Graph the line $y = 3x + 1$ using a table of values.

24. (10) Identify the hypothesis and the conclusion in each conditional statement.

a. *If today is Wednesday, then Jasmine needs to take out the trash.*

b. *If* $x - 3 = 5$, *then* $x = 8$.

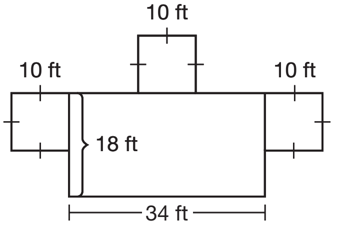
25. (9) **Emergency Response** Two fire stations are located at grid points $A(4, 8)$ and $B(17, 29)$. The closest fire station should respond to any emergency call. Which station should respond to a fire located at the grid point (9, 21)?

*26. (21) Use the Law of Syllogism to write a third conditional statement.

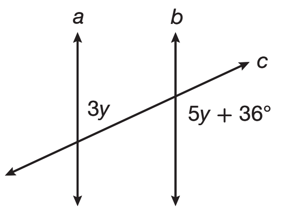
If Mark's pen runs out of ink, then he must get a new pen to finish his assignment.
If Mark must get a new pen to finish his assignment, then he must go to the store.

27. (8) An approximate method for converting from Celsius to Fahreneit is to double the Celsius temperature and add 30. Use this method to convert 25° C and −30° C to Fahrenheit.

*28. (22) **Landscaping** A landscape architect needs to place sod on the area within the shape shown below. How many square feet of sod are needed to cover the area?



29. (Inv 1) Find the measure of each acute angle and each obtuse angle in the diagram. Line a is parallel to line b.



30. (5) **Predict** Draw a line l, and a point P not on l. Then, construct a line parallel to l through point P. Can you construct a different line that is parallel to l through the same point P? Write a conjecture about the number of lines parallel to l through point P.

LESSON 23 Introduction to Circles

Warm Up

1. (8) **Vocabulary** The sum of the side lengths of a closed polygon is called the __________.
2. (SB 14) Evaluate $2x + 5y$ for $x = 3$ and $y = -2$.
3. (21) For the following statements, use the Law of Detachment to write a valid concluding statement.
   If I forget to close the front door, then the dog will get out. I forgot to close the front door.

Evaluate each expression. Round to the nearest hundredth. (SB 6)

4. $2(15)^2$
5. $3.27(6.5)^2$

New Concepts

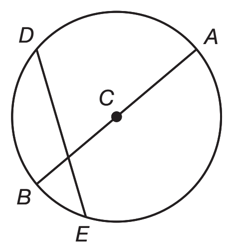
A **circle** is the set of points in a plane that are a fixed distance from a given point. This point is called the **center** of the circle. To name a circle, use the $\odot$ symbol and the center point. For example, $\odot A$ is read, "circle A."

All the points within the circle are called the **interior** of the circle. Any segment whose endpoints are the center of the circle and a point on the circle is called a **radius**. Any segment with both endpoints on the circle that passes through the center is called a **diameter**. The length of a diameter is always twice the length of a radius.

Two circles are congruent if they have congruent radii.

Example 1 Naming Parts of a Circle

Identify a diameter, a radius, and the center of the circle at right.



SOLUTION

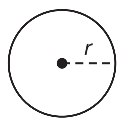
$\overline{AB}$ is a diameter, $\overline{AC}$ and $\overline{BC}$ are both radii, and the center of the circle is point C.

> **Math Reasoning**
>
> **Write** Name a few objects you see regularly that are circles. Do they have easily identifiable centers, radii, and diameters?

As seen in Lesson 8, the sum of the side lengths of a closed figure is called the perimeter. The perimeter of a circle is the distance around the circle, and is called the **circumference**.

Circumference of a Circle

To find the circumference (C) of a circle, use the formula below, where r is the circle's radius.



$$C = 2\pi r$$

Graphing Calculator Tip

Your calculator has a π key. Use it for a more precise value of pi.

Math Reasoning

Formulate What is the mathematical relationship between the diameter and radius of a circle?

Caution

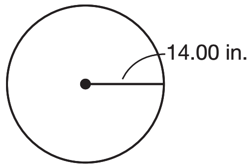
When finding the area, be sure to square the radius before multiplying by pi. A common error is to multiply the radius by pi before squaring it.

Pi, represented by the symbol π, is an irrational number that is defined as the ratio of the circumference of a circle to its diameter. There are several ways to approximate the value of pi. Some common approximations are 3.14 and the fraction $\frac{22}{7}$. Most problems in this book will tell you which approximation to use, or will instruct you to leave your answer in terms of π. If no approximation is specified, use the π key on your calculator for a more precise answer.

Note that, given the relationship between radius and diameter, the formula for the circumference of a circle can also be expressed as $C = \pi d$, where d is the diameter.

Example 2 Finding Circumference

Find the circumference of the circle to the nearest hundredth of an inch. Use 3.14 for π.



SOLUTION

The radius of the circle is 14.00 inches.

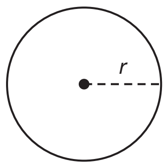
$$C = 2\pi r$$
$$\approx 2(3.14)(14.00)$$
$$\approx 87.92$$

Therefore, the circumference is approximately 87.92 inches.

Area of a Circle

To find the area (A) of a circle, use the formula below, where r is the circle's radius.

$$A = \pi r^2$$

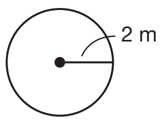


Another way to think of the area of a circle is the area that is swept out by rotating a radius of the circle one full rotation.

Example 3 Finding Area

Find the area of each circle to the nearest hundredth of a square unit. Use 3.14 for π.

a.



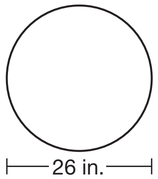
SOLUTION

The radius of the circle is 2 meters.

$$A = \pi r^2$$
$$\approx (3.14)(2)^2$$
$$\approx 12.56$$

Therefore, the area is approximately 12.56 m^2.

b.



SOLUTION

Divide the diameter by 2 to determine the radius measurement.

$$r = \frac{26}{2}$$
$$= 13$$

The radius of 13 inches can then be substituted into the formula.

$$A = \pi r^2$$
$$\approx (3.14)(13)^2$$
$$\approx 530.66$$

Therefore, the area is approximately 530.66 in^2.

> **Math Reasoning**
>
> **Verify** Check that the formula $A = \frac{\pi d^2}{4}$ can be used to solve Example 3b, instead of finding the radius first. Why does this formula work?

Example 4 Application: Urban Design and Planning

A dog park is being constructed with a circular fence surrounding the park. The fence has a radius that is 50 yards long. Use 3.14 for π.

a. What is the distance around the fence to the nearest yard?

SOLUTION

To find the total distance around the fence, the circumference must be calculated.

$$C = 2\pi r$$
$$\approx 2(3.14)(50)$$
$$\approx 314$$

Therefore, the total distance around the fence is approximately 314 yards.

b. Approximately how many square yards of sod would be needed to completely cover the area inside the fence with grass?

SOLUTION

The area must be calculated in order to determine the amount of sod required.

$$A = \pi r^2$$
$$\approx (3.14)(50)^2$$
$$\approx 7850$$

Therefore, the total area to be covered with sod is approximately 7850 yd^2.

Lesson Practice

a. Draw $\odot P$ with a radius, a diameter, and the center labeled.
(Ex 1)

b. Find the circumference of a circle with a radius of 0.5 meters. Use 3.14 for π and round to the nearest hundredth of a meter.
(Ex 2)

c. Find the area of a circle with a radius of 31 centimeters. Use 3.14 for π and round to the nearest hundredth of a square centimeter.
(Ex 3)

d. Find the area of a circle with a diameter of 1 yard. Use 3.14 for π and round to the nearest hundredth of a square yard.
(Ex 3)

e. The lid to a sewer access opening is 35 inches in diameter. If you roll it in a straight line along the ground for three rotations, how much distance would it cover? Use $\frac{22}{7}$ for π.
(Ex 4)

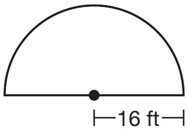
Practice Distributed and Integrated

* **1.** To the nearest kilometer, how far would you go if you traveled along the circumference of the equator, assuming the equator is a circle with a radius of 6378 kilometers? Use 3.14 for π.
(23)

2. **Write** Explain what it means for a mathematical statement to be a conjecture and give an example.
(7)

3. **Algebra** Find the hypotenuse of a right triangle with legs measuring $3x$ and $4x$.
(Inv 2)

* **4.** **Handball** A semicircular area of the gym floor has been marked off with tape to make room for a handball game. If the radius of the semicircle is 16 feet, about how much tape is needed? Use 3.14 for π and round to the nearest tenth of a foot.
(23)



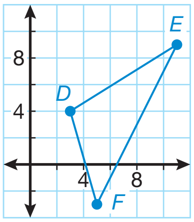
5. **Multi Step** Find the distance between the point (5, 7) and the midpoint of the points (1, 4) and (11, 6).
(9)

6. Find the area of a trapezoid that is 12 centimeters tall, and whose bases are 12 centimeters and 10 centimeters long.
(22)

7. Give three possible sets of dimensions for a right triangle with an area of 36 square inches.
(13)

8. Can the number of sides of a polygon be different than its number of vertices?
(15)

9. **Coordinate Geometry** Find the midpoint of each side of triangle *DEF*.
(11)



10. Draw a regular convex octagon.
(15)

***11.** **Write** Use the Law of Detachment to draw a conclusion to the following statements.
(21)

If I have the correct combination, then I can open the safe.
I have the correct combination.

12. (Inv 2) **Algebra** Find the hypotenuse of a right triangle with congruent leg lengths that are both b.

***13.** (20) Name the conditional statement and the converse that are true if the following biconditional statement is true.

The rooms in the arena will get a new coat of paint if and only if new funding from the local council is approved.

14. (15) **Multiple Choice** Which of these choices *cannot* describe a polygon?

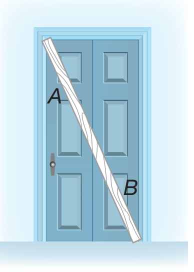
A more than 12 sides **B** a curved side
C closed **D** concave

15. (6) State the measure of the angle that is supplementary to each angle.

a. 75° **b.** 126°

***16.** (19) Draw two different rectangles, each with a perimeter of 100 centimeters, and calculate the area of each.

***17.** (12) **Carpentry** A carpenter is going to use a protractor to determine if the sides of the door she just installed are parallel to the corresponding sides of the door frame. To do this, she attaches a brace to the door, as shown, and measures the indicated angles. How can she use the values of these two angles to determine if the sides of the door are aligned correctly?



18. (21) **Multiple Choice** In this set of statements using syllogism, one of the statements is missing. Which one of the given statements below would correctly complete this syllogism?

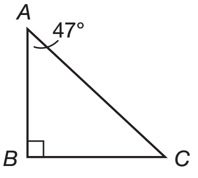
If I plant a seedling in my yard, then a tree will grow.
AND

____

THEN
If I plant a seedling in my yard, then my house will be worth more.

A *If I don't plant a seedling in my yard, then my house will be worth less.*
B *If a tree grows in my yard, then my house will be worth more.*
C *If my house is worth more, then a tree will grow in my yard.*
D *If my house is worth more, then I will plant a seedling in my yard.*

19. (18) **Error Analysis** Dustin reasons that the missing angle in this triangle is 53°. Is he correct? If not, what is the measure of the missing angle?



***20.** (23) **Sewing** Rhonda is making a circular pillow with a diameter of 18 inches, and she needs to sew trim around the edge. To the nearest tenth of an inch, how much trim would Rhonda need to sew around the edge of the pillow? Use 3.14 for π.

***21.** (17) **Write** Write a true conditional statement whose inverse is also true.

22. (13) **Painting** If one can of spray paint can cover 32 square feet, how many cans of spray paint are needed to paint a wall in the shape of an isosceles triangle with a base of 18 feet and a height of 12 feet?

23. (16) **Surveying** Civil engineers are drafting plans for a road with a slope that cannot exceed a value of 0.08. If the total rise of the road from the bottom to the top point needs to be 24 meters, how long must the horizontal distance of the road be to maintain the 0.08 slope?

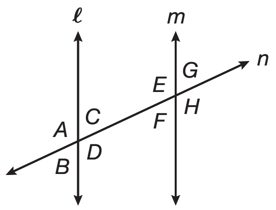
24. (14) Disprove the conditional statement by giving a counterexample.

If a figure is formed by connecting five points with line segments, then the figure is a polygon.

***25.** (23) Two circles on the ground at the baseball field are to be painted red for the upcoming playoffs. If the diameter of each circle is 6 feet, what is the total area to be painted? Use 3.14 for π and round to the nearest tenth of a square foot.

26. (13) The coordinates of the vertices of a right triangle are $R(3, 4)$, $S(9, 4)$, and $T(3, 11)$. If each unit on the grid represents one foot, determine the approximate perimeter and exact area of the resulting triangle.

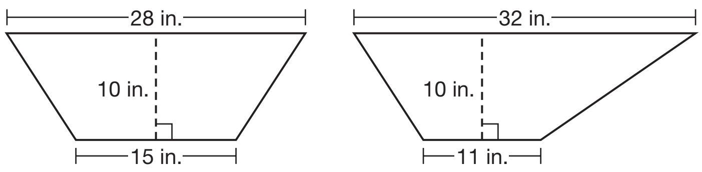
27. (Inv 1) Ana knows that $\angle B$ measures 72°. She says that if lines l and m are parallel, then $\angle G$ also measures 72°. What theorem supports Ana's claim?



***28.** (13) A right triangle has side lengths of 15, 36, and 39 centimeters. What is the area of the triangle?

29. (22) **Algebra** Find the area of a trapezoid with a height of $\frac{1}{4}x$ and base lengths of $3x$ and $5x$.

30. (19) **Justify** Compare the areas of these two trapezoids. Do the figures have the same area? Explain your reasoning.



LESSON

24

Algebraic Proofs

Warm Up

1. (21) **Vocabulary** The statement below is an example of the Law of ________. ***(Detachment, Syllogism)***

   If $1 + 1 = 2$ *and* $2 = a$, *then* $1 + 1 = a$.

2. (SB 2) State the opposite operation to each of the following:

   a. addition

   b. division

   c. finding the square root

3. (SB 14) **Multiple Choice** Which value of x is valid for the equation $3x - 2 = 2x - 3$?

   A -1 **B** -2

   C 1 **D** 2

4. (17) Write the converse and the inverse of the statement below.

   If all the sides of a parallelogram are congruent, then it is a rhombus.

New Concepts

A **proof** is an argument that uses logic to show that a conclusion is true. Since you have already learned to solve an algebraic equation, you have already performed a proof. An algebraic proof uses properties of equality to solve an equation. These properties are listed in the table below.

> **Hint**
>
> See the Skills Bank for a list of other properties of arithmetic that can be used as justifications in an algebraic proof.

Properties of Equality

Property	Example
Addition Property of Equality	If $a = b$, then $a + c = b + c$.
Subtraction Property of Equality	If $a = b$, then $a - c = b - c$.
Multiplication Property of Equality	If $a = b$, then $ac = bc$.
Division Property of Equality	If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$
Symmetric Property of Equality	If $a = b$, then $b = a$.
Reflexive Property of Equality	$a = a$
Transitive Property of Equality	If $a = b$ and $b = c$, then $a = c$.
Substitution Property of Equality	If $a = b$, then b can be substituted for a in any expression.

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> **Hint**
>
> Think of each justification in a proof as the mathematical answer to the question, "what did you do in this step?"

An algebraic proof shows step-by-step how a problem is solved. Each step has to be justified with one of the properties above, or by a property of arithmetic. For example, if a step required adding a number to both sides of an equation, it would be justified by the Addition Property of Equality.

Whenever a step requires that you perform basic mathematical operations on a single side of the equation (like addition, subtraction, multiplication, or division), the step is justified by the term, "Simplify."

Most proofs begin by presenting the facts of a problem. The first line restates what you have already been told, and is justified as "Given." When you are given an equation to solve for an algebraic proof, the original equation is the "Given" statement.

Example 1 Writing an Algebraic Proof

a. Solve this equation. Provide a justification for each step.

$$2(x + 1) = x + 9$$

SOLUTION

$2(x + 1) = x + 9$	Given
$2x + 2 = x + 9$	Distributive Property
$2x + 2 - 2 = x + 9 - 2$	Subtraction Property of Equality
$2x = x + 7$	Simplify.
$2x - x = x + 7 - x$	Subtraction Property of Equality
$x = 7$	Simplify.

b. Solve this equation. Provide a justification for each step.

$$\frac{3x - 1}{5} = \frac{2x + 3}{3}$$

SOLUTION

$\frac{3x - 1}{5} = \frac{2x + 3}{3}$	Given
$15\left(\frac{3x - 1}{5}\right) = 15\left(\frac{2x + 3}{3}\right)$	Multiplication Property of Equality
$\frac{15}{5}(3x - 1) = \frac{15}{3}(2x + 3)$	Associative Property of Multiplication
$9x - 3 = 10x + 15$	Distributive Property
$9x - 3 + 3 = 10x + 15 + 3$	Addition Property of Equality
$9x = 10x + 18$	Simplify.
$9x - 10x = 10x + 18 - 10x$	Subtraction Property of Equality
$-x = 18$	Simplify.
$\frac{-x}{-1} = \frac{18}{-1}$	Division Property of Equality
$x = -18$	Simplify.

Math Reasoning

Verify Explain how the solution to example 1a can be checked.

Example 2 Verifying Algebraic Reasoning

The steps of the algebraic proof for solving the equation $2(a + 1) = -6$ are given below in the correct order. However, the justifications for each step are out of order. Determine the correct order for the justifications.

$2(a + 1) = -6$	Simplify.
$2a + 2 = -6$	Distributive Property
$2a + 2 - 2 = -6 - 2$	Division Property of Equality
$2a = -8$	Given
$\frac{2a}{2} = \frac{-8}{2}$	Subtraction Property of Equality
$a = -4$	Simplify.

Math Reasoning

Justify Edwin performed the same algebraic proof as Example 2, but his started with the Division Property of Equality. Is Edwin also correct? Explain.

SOLUTION

The correct order of the proof is:

$2(a+1) = -6$	Given
$2a + 2 = -6$	Distributive Property
$2a + 2 - 2 = -6 - 2$	Subtraction Property of Equality
$2a = -8$	Simplify.
$\frac{2a}{2} = \frac{-8}{2}$	Division Property of Equality
$a = -4$	Simplify.

Example 3 Application: Finding Dimensions

The area of a rectangular patio is 28 square feet. The patio's length is $(3x + 1)$ feet and the patio's width is $2x$ feet. Find the dimensions of the patio. Provide a justification for each step.

SOLUTION

The formula for the area of a rectangle is $A = lw$, so

$A = 28, l = (3x+1), w = 2x$	Given
$A = lw$	Area formula for a rectangle
$28 = (3x+1)(2x)$	Substitution Property of Equality
$28 = 6x^2 + 2x$	Distributive Property
$6x^2 + 2x = 28$	Symmetric Property of Equality
$\frac{6x^2 + 2x}{2} = \frac{28}{2}$	Division Property of Equality
$3x^2 + x = 14$	Simplify.
$3x^2 + x - 14 = 14 - 14$	Subtraction Property of Equality
$3x^2 + x - 14 = 0$	Simplify.
$(3x+7)(x-2) = 0$	Factor.

There are two solutions to this factorization, $3x + 7$, and $x - 2$. However, the solution to $3x + 7$ is negative. It does not make sense for a side of the rectangle to have a negative length, so that solution is thrown out.

Therefore,

$x - 2 = 0$	Given
$x - 2 + 2 = 0 + 2$	Addition Property of Equality
$x = 2$	Simplify.

Now, substitute $x = 2$ into the expressions for length and width of the rectangle to find the dimensions.

$$\text{length} = 3x + 1 = 3(2) + 1 = 7$$

$$\text{width} = 2x = 2(2) = 4$$

Therefore, the patio is 7 feet long and 4 feet wide.

Math Reasoning

Verify Explain how you could verify that the solution to Example 3 is valid.

Lesson Practice

a. Solve the equation $x + 5 = 4x + 2$. Provide a justification for each step.
(Ex 1)

b. Solve the equation $\frac{4x + 5}{3} = \frac{5x + 7}{4}$. Provide a justification for each step.
(Ex 1)

c. The steps of the proof below are given in the correct order. However, the justifications for each step are out of order. Determine the correct order of the justifications.
(Ex 2)

$\frac{2}{3}x + 6 = 4 - 2x$	Given
$2x + 18 = 12 - 6x$	Subtraction Property of Equality
$2x = -6 - 6x$	Multiplication Property of Equality
$8x = -6$	Addition Property of Equality
$x = -\frac{2}{3}$	Division Property of Equality

d. The area of the rectangular floor of a shed is 40 yd^2. The length of the shed is $(x + 2)$ yd and the width is $(x - 1)$ yd. Find the dimensions of the shed. Provide a justification for each step.
(Ex 3)

Practice Distributed and Integrated

1. (Inv 2) **Monitors** Computer monitors are usually measured in terms of the lengths of their diagonals. If a 20-inch monitor has a horizontal/vertical aspect ratio of 4:3, what are the horizontal and vertical dimensions of the monitor?

2. (12) **Write** Explain how to check if two shelves are parallel by dropping a string and measuring the angles between the string and the shelves.

3. (Inv 1) Write the converse of the theorem, "If two parallel lines are cut by a transversal, then the same-side interior angles are supplementary." Is the converse true?

*** 4.** (24) **Error Analysis** Minh and Soon Yu are asked to solve the expression $2(x + 1) = 12$. Here are their two solutions. Is Minh's solution correct? Is Soon Yu's solution correct? Explain.

Minh's solution:

$2(x + 1) = 12$	Given
$2x + 2 = 12$	Distributive Property
$2x = 10$	Subtraction Property of Equality
$x = 5$	Division Property of Equality

Soon Yu's solution:

$2(x + 1) = 12$	Given
$x + 1 = 6$	Division Property of Equality
$x = 5$	Subtraction Property of Equality

5. (17) Write the converse of the following statement. *If the window is open, then it is not closed.* Is the converse true?

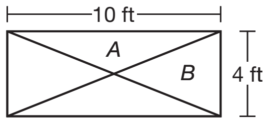
*** 6.** (24) **Multiple Choice** If $x = 3$ and $3 = y$, then $x = y$. This is an example of which property of equality?

A Symmetric **B** Reflexive **C** Transitive **D** Comparative

7. (8) **Multi Step** What is the width of a rectangular driveway with an area of 2592 square feet and a length of 27 yards?

8. (18) **Generalize** What is the relationship between the interior angle and its adjacent exterior angle at any vertex of any polygon?

9. (13) **Art** Tony wants to paint a triangular pattern on a rectangle which is 10 feet long and 4 feet wide. He divides the rectangle into four triangles by stretching wire across the diagonals. Determine the area of triangles *A* and *B*.

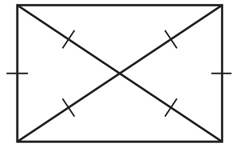


10. (15) Draw a convex hexagon with all sides congruent and a concave hexagon with all sides congruent.

*11. (20) **Write** Write the converse of the following statement and state whether it is true. If the converse is true, write the two statements as a biconditional.

If all of the students in the school achieve over 70% on the state exams, then the school will receive a new computer lab from the district.

12. (13) **Justify** If the diagonals of a rectangle form two equilateral triangles, classify the other two triangles by their angles. How do you know?



*13. (24) Outline the solution and justification for each step in solving the equation $2x - 1 = 5$.

14. (14) **Multiple Choice** Suzanne suggests that multiples of 3 never end in a 4, but her friend Maxine disproves her by stating that 24 is a multiple of 3. What has Maxine just used to disprove Suzanne's suggestion?

A a conjecture　　**B** inductive reasoning
C a theorem　　**D** a counterexample

*15. (22) **Algebra** Miguel painted an equal number of squares with sides measuring 6 centimeters and 12 centimeters, respectively. How many of each type of square did he paint if the squares cover 4500 cm^2?

16. (16) Graph the equation $2x + 3y - 15 = 0$ using the slope and *y*-intercept.

17. (22) **Landscaping** A patio in the shape of a trapezoid is to be covered in flagstone. The parallel sides of the trapezoid measure 20 feet and 32 feet respectively, and are separated by a perpendicular distance of 14 feet. How many square feet need to be covered?

*18. (23) A figure skater is practicing her figure-eights by repeatedly skating around two circles that join to form a figure "8." If the diameter of the circles is 30 feet each, how much total distance, to the nearest foot, will she cover in 5 full trips around the figure?

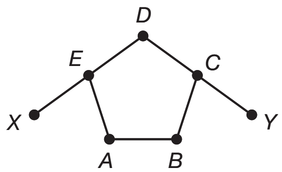
19. (8) A right triangle has a side length of 6 inches and a hypotenuse of 10 inches. What is the length of the third side?

20. (17) Write the inverse of the following statement. Then state whether the inverse is true.

If it is raining, then I will use my umbrella.

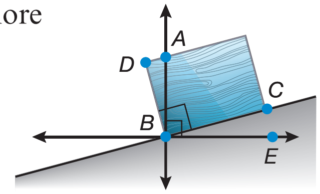
***21.** A right triangle has side lengths of 48 meters, 20 meters, and 52 meters. What is the area of this triangle?
(13)

22. Name an interior and an exterior angle of the figure at right.
(15)



23. The midpoint of $(4, 5)$ and a second point is $(7, 1)$. What is the second point?
(11)

24. **Balancing** A rectangular box sitting on a ramp must not be inclined more than 30° from vertical or it will slide down the ramp. What is the relationship between $\angle ABD$ and $\angle CBE$? How do you know? What is the greatest value $\angle EBC$ can have before the box slides?
(6)



***25.** Jack is drawing circles on a playground for a game. If he draws a circle with a radius 3 feet, what is the area of the circle, to the nearest square foot?
(23)

26. **Fitness Membership** At a fitness club, the equation for the cost of a membership is given by $C = 30m + 200$, where C is dollars and m is months. In this application of a linear equation, what is the slope and what does it represent? How much does it cost to be a member of this gym for 5 months?
(16)

27. **Error Analysis** Explain why the following inductive reasoning is not a valid proof.
(7)
All the birds I have seen in this park have brown feathers. All the birds in this park must have brown feathers.

***28.** For any given angle, if the angle is obtuse, then it cannot be acute. A given angle is found to be obtuse. What conclusion can be made from this and which law was used to reach this conclusion?
(21)

29. Write the equation of a line with slope $\frac{1}{2}$ that passes through (3, 4).
(16)

***30.** **Verify** To solve the linear equation $3(x + 5) = 9$, Cynthia begins her solution like this:
(24)

$$3(x + 5) = 9$$
$$3(x + 5) - 5 = 9 - 5$$
$$3x = 4$$

Is Cynthia proceeding correctly? If not, what mistake has she made?

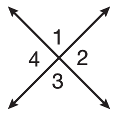
LESSON
25 Triangle Congruence: SSS

Warm Up

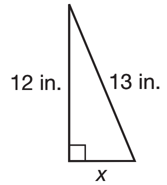
1. (13) **Vocabulary** A triangle with no congruent sides is a(n) ________ triangle. (***equilateral, isosceles, scalene***)

2. (SB 4) Solve for x: $\frac{3.5}{x} = \frac{17.5}{40}$.

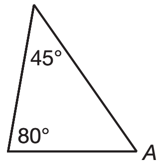
3. (6) Two lines intersect as shown. Which angle is equal to $\angle 2$?



4. (Inv 2) What is the length of side x in this right triangle?



5. (18) What is the measure of $\angle A$?



New Concepts

Corresponding sides and **corresponding angles of polygons** are those that are in the same position in two different polygons with the same number of sides. These corresponding parts are indicated by the names of the polygons.

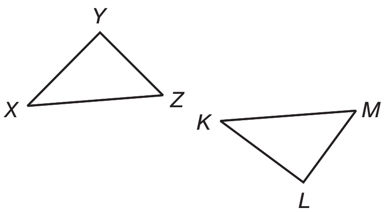
When naming congruent polygons, it is important that the order of the points, or vertices, in the names correspond.

Math Reasoning

Model Is it possible to make two different quadrilaterals with four sides of the same length? Make a sketch to illustrate your answer.

Example 1 Identifying Corresponding Parts

Identify the corresponding angles and sides for $\triangle XYZ$ and $\triangle KLM$.



SOLUTION

The names of the triangles show that $\angle X$ corresponds to $\angle K$, $\angle Y$ corresponds to $\angle L$, and $\angle Z$ corresponds to $\angle M$. Since we know which angles correspond with one another, the sides enclosed by those angles must also correspond. Side $\overline{XY}$ corresponds to side $\overline{KL}$, side $\overline{XZ}$ corresponds to side $\overline{KM}$, and side $\overline{YZ}$ corresponds to side $\overline{LM}$.

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Triangles are said to be **congruent triangles** when all of their corresponding sides and angles are congruent. One way to determine if two triangles are congruent is to use the Side-Side-Side Congruence Postulate.

Postulate 13: Side-Side-Side (SSS) Congruence Postulate
If three sides of one triangle are congruent to three sides of another triangle, then the triangles are congruent.

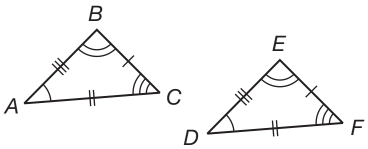
Exploration: Exploring the SSS Postulate

In this exploration, you will work with a partner to find out if three congruent line segments can form two different triangles.

1. Your teacher will give three segments of different lengths to each student. Make sure that your segments are the same lengths as your partner's.
2. Assemble a triangle using the segments provided.
3. Compare the triangle that you assembled with your partner's triangle. Are they the same or different? If they are different, in what way(s) are they different?
4. Working with your partner, try to assemble two triangles that are different than the ones originally formed. What do you notice about the triangles you assemble?

Side-side-side congruence indicates that if all the sides of a triangle are of a fixed length, the triangle can have only one size and shape. This is called **triangle rigidity.**

If triangles $\triangle ABC$ and $\triangle DEF$ are congruent, their relationship can be shown by the congruence statement $\triangle ABC \cong \triangle DEF$.

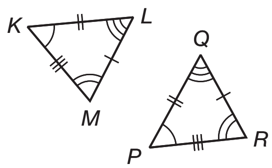


Example 2 Naming Congruent Triangles

Math Reasoning

Verify In Example 2, is the congruence statement $\triangle LMK \cong \triangle QRP$ true? Explain.

Write a congruence statement for the two triangles below.



SOLUTION

In these two triangles, M corresponds to R, K corresponds to P, and L corresponds to Q. Therefore, $\triangle MKL \cong \triangle RPQ$.

CPCTC
An abbreviation for the phrase "**C**orresponding **P**arts of **C**ongruent **T**riangles are **C**ongruent."

Math Reasoning

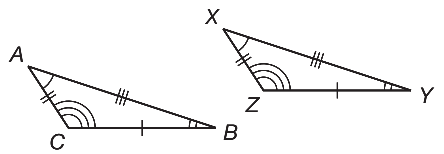
Verify If the congruence statement in this example had been $\triangle ABC \cong \triangle FED$, which of the angle congruence statements listed here would still be true?

When two triangles are congruent, CPCTC states that the corresponding angles and sides of those triangles will also be congruent. For example, if $\triangle ABC \cong \triangle DEF$, then by CPCTC, all of the following congruence statements can be written.

Congruent Angles	Congruent Sides
$\angle A \cong \angle D$	$\overline{AB} \cong \overline{DE}$
$\angle B \cong \angle E$	$\overline{BC} \cong \overline{EF}$
$\angle C \cong \angle F$	$\overline{AC} \cong \overline{DF}$

Example 3 Writing Congruence Statements

Identify the congruent sides and angles of the two triangles below and write six congruence statements.



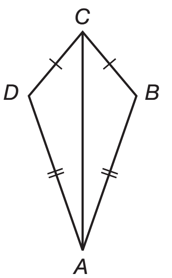
SOLUTION

In these congruent triangles, A corresponds to X, B corresponds to Y, and C corresponds to Z. Therefore,

Congruent Angles	Congruent Sides
$\angle A \cong \angle X$	$\overline{AB} \cong \overline{XY}$
$\angle B \cong \angle Y$	$\overline{BC} \cong \overline{YZ}$
$\angle C \cong \angle Z$	$\overline{AC} \cong \overline{XZ}$

Example 4 Application: Making a Kite

Regina is making her own kite. It is made of two perpendicular pieces of wood, to which she will attach a plastic kite shape. The kite shape is made of two congruent triangles as shown in the picture below. Regina has already found the measures that two of the angles need to be so that the kite can fit on the wooden frame. These measures are: $m\angle DAB = 40°$ and $m\angle DCB = 80°$. What should the measure of $\angle B$ be?

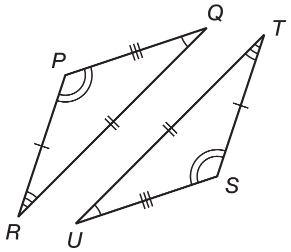


SOLUTION

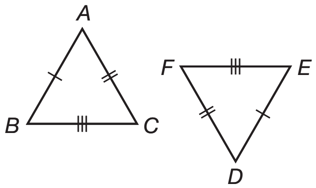
Since the two triangles are congruent, their corresponding parts are congruent. This means that $\angle BAC \cong \angle DAC$ and, since together they measure 40°, each one must measure 20°. Similarly, each of the angles that make up $\angle DCB$ must be 40°. Using the Triangle Angle Sum Theorem, $\triangle ABC$'s angles must add up to 180°, so $m\angle B = 180° - 40° - 20° = 120°$. So $m\angle B = 120°$.

Lesson Practice

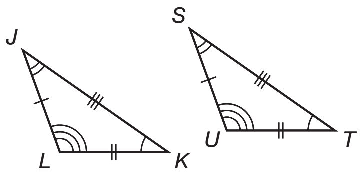
a. Identify the corresponding angles and sides.
(Ex 1)



b. Write a congruence statement for the two triangles below.
(Ex 2)



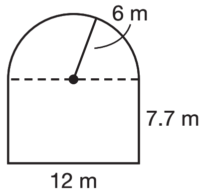
c. Identify the congruent sides and angles of the two triangles below and write six congruence statements.
(Ex 3)



d. **Kites** Imagine you are making a kite, as in Example 4, with two congruent triangles that make up the kite shape. You know that one obtuse angle of the kite shape is 110°. What is the measure of the other obtuse angle? What will be the total measure of the kite's other two angles?
(Ex 4)

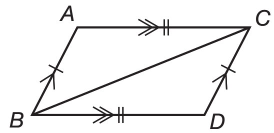
Practice Distributed and Integrated

1. **Art** A painting covers a wall as shown. What is the perimeter of the painting, to the nearest tenth of a meter?
(8)

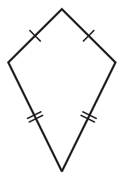


*** 2.** An area of 400 square meters is to be roped off for a farmers' market that is to be in the shape of a circle. What will be the diameter of this circle, to the nearest tenth of a meter?
(23)

*** 3.** Give the congruence statement for the triangles shown.
(25)



4. Classify the quadrilateral shown here.
(19)



*** 5.** **Baseball** The bases on a baseball diamond form a square that is 90 feet on each side. What is the minimum distance that would be covered when running around all four bases?
(22)

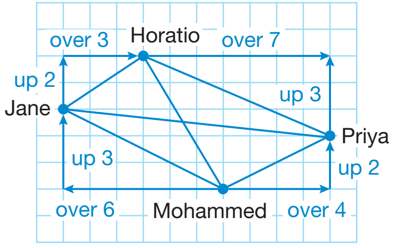
6. (Inv 1) **Railroads** Railway tracks consist of two parallel steel rails. If an inspector places a meterstick across the rails and determines that the measures of the alternate interior angles differ by several degrees, is there anything wrong? Explain.

*** 7.** (25) **Model** Draw two congruent triangles that correspond to the following statements: $\angle E \cong \angle N$, $\angle F \cong \angle L$, $\angle G \cong \angle M$, $\overline{EF} \cong \overline{NL}$, $\overline{EG} \cong \overline{NM}$, and $\overline{FG} \cong \overline{LM}$.

8. (14) Find a counterexample to prove that the following conjecture is false.

For every natural number n, $4n > n^2$.

Use the grid to determine how far each friend lives from the other. Each grid unit is 0.1 mile. Round to the nearest hundredth of a mile.



9. (9) How far apart do Jane and Horatio live?

10. (9) How far apart do Horatio and Priya live?

***11.** (25) **Error Analysis** Polly determines that for $\triangle ABC$ and $\triangle XYZ$, $\angle A \cong \angle X$, $\angle B \cong \angle Y$, and $\angle C \cong \angle Z$. She also concludes that $\overline{AB} \cong \overline{XY}$, $\overline{BC} \cong \overline{XZ}$, and $\overline{AC} \cong \overline{YZ}$. Are her conclusions correct? Explain.

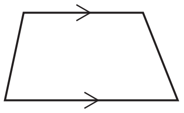
***12.** (24) **Algebra** Triangles *ABC* and *DEF* are congruent. If $AB = 2x + 10$ and $DE = 4x - 20$, find the value of x and include justifications for each step in the solution.

***13.** (21) **Multiple Choice** Use the Law of Detachment to identify a valid concluding statement.

If we get more than 12 inches of snow in the next hour, then the roads will be closed.
The weather network predicts that there will be 15 inches of snow in the next hour.

A *Based on the weather network, the roads will be open in the next hour.*
B *The snow should stop after 15 inches of snow has fallen.*
C *Based on the weather network, the roads will be closed in the next hour.*
D *It will continue to snow, even after 12 inches of snow has fallen.*

14. (19) Classify the quadrilateral.



15. (21) **Write** What can be deduced from the following statements?

The oldest person in each class in the school is wearing a blue shirt.
Kelly is the oldest person in one of the classes in the school.

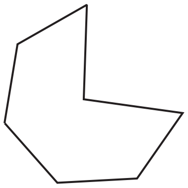
16. (20) Is the statement below a biconditional statement? If so, are both conditional statements true?

An angle is a right angle if and only if it measures 90°.

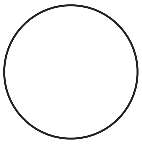
17. (24) Solve for x in the equation $\frac{x + 5}{3} = 3$. Justify each step.

Determine whether each figure is a polygon. If it is, name it.

18. (15)



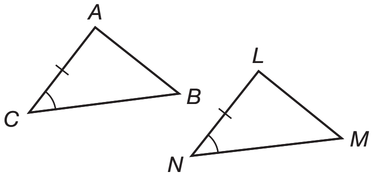
19. (15)



20. (19) **Justify** Is the following statement true or false? If true, provide a theorem to support it. If false, provide a counterexample.

If the sides of a quadrilateral have equal length, then it is a square.

***21.** (25) In the triangles shown, $\angle C \cong \angle N$ and $\overline{AC} \cong \overline{LN}$. If $\triangle ABC \cong \triangle LMN$, provide four more congruence statements using CPCTC.



***22.** (24) **Algebra** Find the value of x for two supplementary angles if the larger angle is 20° more than three times the smaller angle. Justify each step.

23. (16) **Urban Planning** Find the slope between two grid points marked on a road: $A(-3, 4)$ and $B(6, -4)$. Is the road going uphill or downhill from A to B? Explain.

24. (20) Identify whether the statement is conditional or biconditional.

A phone rings if and only if someone is calling.

25. (14) Find a counterexample to prove that the conjecture is false.

$\frac{1}{n}$ *is a rational number for every whole number.*

26. (23) **Construction** Workers are stringing caution tape around a circular parking lot that has just been repaired. To the nearest foot, how much tape is needed if the parking lot has a diameter of 40 feet?

***27.** (21) **Formulate** What conclusion can be drawn from these statements? How do you know?

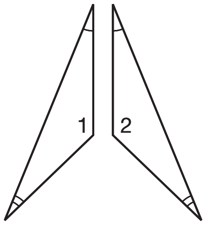
If the team wins the remainder of the games, then they will make the playoffs.
The team won the remainder of their games.

28. (17) **Write** What is the contrapositive of the following statement?

If a bee is the queen bee, then it does not leave the hive.

29. (Inv 1) **Model** Line ℓ_1 and ℓ_2 are parallel lines. Point P lies on ℓ_1, and point M lies on ℓ_2. Draw a line through P and M. Label one pair of alternate interior angles, and label one pair of corresponding angles.

30. (18) Find m∠1 and m∠2 if $m\angle 1 = 2x^2 + 2x + 3$ and $m\angle 2 = 3x^2 - 5x + 3$.



LESSON 26

Central Angles and Arc Measure

Warm Up

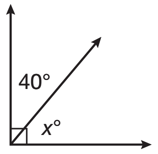
1. (23) **Vocabulary** A ______ is a line segment with the center of a circle as one endpoint and any point on the circle as the other endpoint.

2. (23) **Multiple Choice** Which of these formulas can be used to find the area of a half circle?

   A $2\pi r$ **B** πr

   C πd **D** $\frac{\pi r^2}{2}$

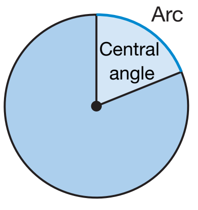
3. (6) Find the value of x.



4. (SB 4) What percent of a full circle is a rotation of 45°?

New Concepts

An **arc** is a part of a circle consisting of two points on the circle, called endpoints, and all the points on the circle between them. When two arcs on a circle share exactly one endpoint, they are called **adjacent arcs**. An angle whose vertex is at the center of a circle is called a **central angle**. The diagram below shows a central angle that intercepts an arc on the circle.



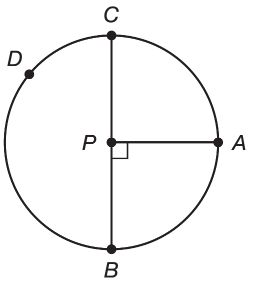
> **Math Reasoning**
>
> **Generalize** What is the relationship between the arc measures of a minor arc and the major arc that share endpoints?

Arcs of a Circle

A **minor arc** is an arc that is smaller than half a circle. The **measure of a minor arc** is the same as the measure of its central angle. The measure of a minor arc must be greater than 0° and less than 180°. All minor arcs are named using the two endpoints of the arc.	A **major arc** is an arc that is larger than half a circle. The **measure of a major arc** is the difference of 360° and the measure of the associated minor arc. The measure of a major arc must be greater than 180°and less than 360°. All major arcs are named using the two endpoints of the arc and a point on the circle between the endpoints.	A **semicircle** is an arc equal to half a circle. The measure of a semicircle is 180°. Like major arcs, semicircles can be named with the two endpoints of the semi-circle and a point on the circle between the endpoints.

Example 1 Identifying Arcs and Angles

Identify a central angle, minor arc, major arc, and semicircle in $\odot P$.

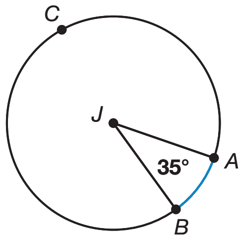


SOLUTION

Two central angles are pictured: $\angle APB$ and $\angle APC$. Each central angle forms a minor arc: $\overset{\frown}{AB}$ and $\overset{\frown}{AC}$. There are also two major arcs: $\overset{\frown}{ABC}$ and $\overset{\frown}{ACB}$. Finally, there are two semicircles: $\overset{\frown}{BAC}$ and $\overset{\frown}{BDC}$.

Example 2 Finding Arc Measures

What is $m\overset{\frown}{AB}$?



SOLUTION

The central angle's measure is 35°, so the measure of the arc is also 35°.

> **Math Reasoning**
>
> **Predict** In Example 2, what would be the measure of the major arc $\overset{\frown}{ACB}$?

Two arcs that are in the same circle or in congruent circles and that have the same measure are called **congruent arcs.**

Example 3 Congruent Arcs

The measure of $\overset{\frown}{DE}$ is given by the expression $3x + 10$, and the measure of $\overset{\frown}{HJ}$ is given by the expression $5x - 40$. It is given that $\overset{\frown}{DE} \cong \overset{\frown}{HJ}$. Determine the value of x and the measure of each arc.

SOLUTION

Since the two arcs are congruent, the expressions for their measures must be equal.

Therefore,

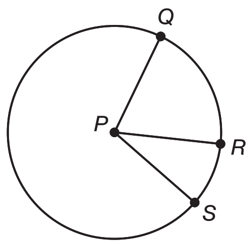
$$\begin{aligned} 3x + 10 &= 5x - 40 \\ 3x + 10 - 10 &= 5x - 40 - 10 \\ 3x &= 5x - 50 \\ 3x - 5x &= 5x - 50 - 5x \\ -2x &= -50 \\ \frac{-2x}{-2} &= \frac{-50}{-2} \\ x &= 25 \end{aligned}$$

Therefore, $m\overset{\frown}{DE} = 3(25) + 10$
$= 75 + 10$
$= 85°$

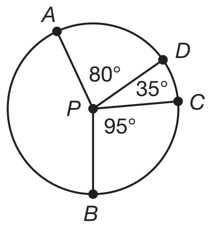
Since the arcs are congruent, $m\overset{\frown}{HJ} = 85°$.

Postulate 14: Arc Addition Postulate
The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.

The Arc Addition Postulate says that if two arcs are adjacent, the sum of their measures is equal to the measure of the larger arc that they form. In the diagram, $m\overset{\frown}{QR} + m\overset{\frown}{RS} = m\overset{\frown}{QS}$.



Example 4 Using the Arc Addition Postulate



a. Use the Arc Addition Postulate to write an expression that represents $m\overset{\frown}{AC}$.

SOLUTION

While there are several arcs indicated, the only two that are adjacent and make up the same arc as $\overset{\frown}{AC}$ are $\overset{\frown}{AD}$ and $\overset{\frown}{DC}$. Therefore, $m\overset{\frown}{AD} + m\overset{\frown}{DC} = m\overset{\frown}{AC}$.

b. Find $m\overset{\frown}{AC}$.

SOLUTION

Since $m\overset{\frown}{AC} = m\overset{\frown}{AD} + m\overset{\frown}{DC}$

$= 80° + 35°$

$= 115°$

Therefore, $m\overset{\frown}{AC}$ is 115°.

> **Math Reasoning**
>
> **Analyze** For Example 4, Rahel suggested $m\overset{\frown}{AB} + m\overset{\frown}{BC} = m\overset{\frown}{AC}$. Is she correct? Explain.

Example 5 Application: Surveillance Cameras

A surveillance camera has a viewing angle of 42°. How many surveillance cameras would be needed to cover a semicircle of a room, with minimal overlap of the area to be viewed? How much of an overlap would these cameras produce?

SOLUTION

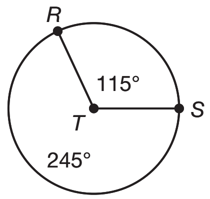
The Arc Addition Postulate can be used to determine the number of cameras to be used. A circle is 360°, so a semicircle is 180°. To find the number of cameras needed, divide 180° by the viewing angle of a camera: $\frac{180°}{42°} = 4.3$. Four cameras would not quite cover the area, so 5 cameras are needed. To find the overlap, multiply the number of cameras by the viewing angle and subtract 180°: $(42° \cdot 5) - 180° = 30°$.

> **Hint**
>
> The cameras' overlap will be any number of degrees over 180° that they cover when their viewing angles are added together.

Lesson Practice

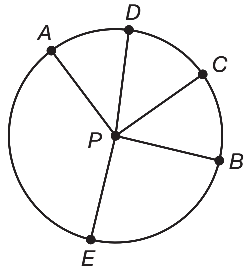
a. Draw a diagram of a circle, identifying a central angle, a minor arc, and a major arc.
(Ex 1)

b. Identify the measure of the minor arc.
(Ex 2)

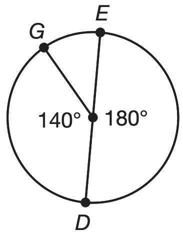


c. The measure of $\overset{\frown}{JK}$ is given by the expression $2x - 15$, and the measure of $\overset{\frown}{LM}$ is given by the expression $x + 30$. It is given that $\overset{\frown}{JK} \cong \overset{\frown}{LM}$. Determine the value of x and the measure of each arc.
(Ex 3)

d. Use the Arc Addition Postulate to write an expression that represents $m\overset{\frown}{AB}$.
(Ex 4)



e. Find $m\overset{\frown}{DEG}$.
(Ex 4)



f. **Outdoor Lighting** A lamp projects a beam of light over a 100° arc. How many lamps facing outward from the center of a circle would be needed to form a full circle of light at the center of a park? What would be the overlap of these beams?
(Ex 5)

Practice Distributed and Integrated

1. Find the slope of the line containing points $C(7, 3)$ and $D(12, -2)$.
(16)

*** 2.** **Analyze** What conclusion can be drawn from the following statements?
(21)
All plants require water to develop. A cactus is a plant.

3. **Algebra** Triangle ABC has side lengths of $AB = x^2 - x + 5$, $BC = 5x - 4$, and $AC = 3x^2 - 2x - 1$. If $\triangle ABC$ is isosceles where $AB = BC$, find the lengths of all sides.
(13)

*** 4.** **Geography** The Kentland Impact Crater in Indiana is in the shape of a circle whose diameter is approximately 13 kilometers. To the nearest kilometer, how far would Farah walk if she were to travel the full distance around the outside of the crater?
(23)

5. Construct a truth table for not P and not Q.
(20)

*** 6.** What is the measure of a central angle of a circle if the associated minor arc has an angle measure of 65°?
(26)

7. **Multiple Choice** Which of the following classifications allows you to know the measures of the triangle's angles with no additional information? What are the angle measures of such a triangle?
(13)

A equilateral　　**B** equiangular
C isosceles　　**D** both **A** and **B**

8. (20) **Geography** Use A and B to determine the truth value of the statement, "A and not B".

A: New York City is the most populous city in the United States.
B: New York City is near the Pacific Ocean.

9. (6) **Algebra** The supplement of an angle is four times more than its complement. Find the measure of the angle.

10. (14) Find a counterexample to the conjecture $n^3 < 3n$.

11. (16) **Algebra** If the slope of $\overline{XY}$ is 0, and $X(2, 3)$ and $Y(-1, k)$, find k.

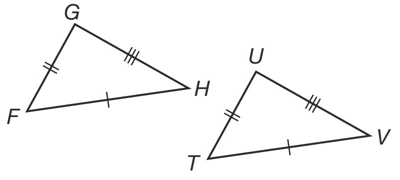
12. (Inv 1) Angles A and B are a pair of alternate interior angles on a pair of parallel lines intersected by a transversal, and $\angle A$ and $\angle C$ are a pair of same-side interior angles on the same figure. If $m\angle B = 30°$, what is $m\angle C$?

***13.** (25) Which side of $\triangle PRG$ corresponds to $\overline{XT}$, if $\triangle XTW \cong \triangle PRG$?

14. (14) **Error Analysis** A student wrote the following conjecture. Find a counterexample and explain the error.

If x is a positive even number and y is any integer, then the product xy will be positive.

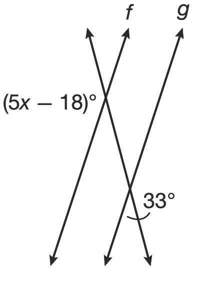
***15.** (25) Write the congruence statement for the given triangles.



16. (17) **Write** What is the inverse of the following statement?

If a building has more than four floors, then it has an elevator.

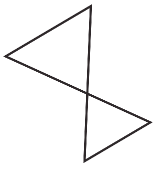
17. (Inv 1) If lines f and g are parallel, find x.



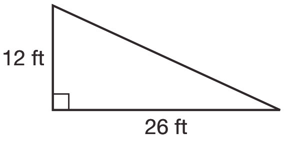
***18.** (26) **Multiple Choice** If there are a total of 6 non-overlapping adjacent minor arcs in a circle, the Arc Addition Postulate gives

A a total arc measure of 90°.
B a total arc measure of 180°.
C a total arc measure of 360°.
D a total that is the sum of the measures of the adjacent arcs.

19. (15) Is this figure a polygon? If so, name it. If not, explain why not.



20. (13) What is the area of this triangle?



21. (24) Janie has just solved an equation for x and found $4 = x$. She writes her answer as $x = 4$. What property of equality is she using to do this?

22. (16) **Phone Plans** To calculate monthly charges, a cell phone provider uses the equation $C = 0.25m + 60$, with C being dollars and m representing minutes used beyond the ones provided with the plan. In this linear equation, what is the value of the vertical intercept and what does it represent?

xy^2 ***23.** (26) **Algebra** Two congruent arcs have central angle measures that are $3x + 10$ and $50 - 2x$. Find x and the measure of the arcs.

***24.** (25) Write six congruence statements for $\triangle ABC \cong \triangle WXY$.

25. (17) **Write** Write the contrapositive of the following statement.

If a polygon is a triangle, then it has three sides.

***26.** (24) What property of equality is used as a justification in solving for x in the problem below?

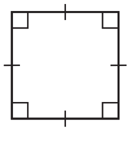
$$x + 5 = 11$$
$$x + 5 - 5 = 11 - 5$$
$$x = 6$$

27. (Inv 2) **Geography** Roderick calculated the distance from New York City to Montreal using indirect measurement. On his map, New York City is 188 miles from Boston. Boston is 253 miles from Montreal, Canada. Assuming the angle from New York City to Boston to Montreal is 90°, what is the distance between New York City and Montreal, to the nearest mile?

***28.** (26) A major arc has a measure of 227°. What is the corresponding minor arc measure?

29. (7) **Write** The Fibonacci Sequence is a famous pattern which begins: 1, 1, 2, 3, 5, 8, 13, 21, … Describe the pattern and give the next four terms.

30. (19) What are four different names for this quadrilateral?



LESSON

27 Two-Column Proofs

Warm Up

1. (24) **Vocabulary** A process that uses logic to show that a conclusion is true is called a _____.

2. (24) Solving the equation $3x = 6$ yields the answer $x = 2$. Which property of equality was used to solve for x?

3. (24) In solving $6 = 3x$, Sofia obtains an answer of $2 = x$. She writes it as $x = 2$. Which property of equality allows her to write the solution this way?

4. (13) **Multiple Choice** In a right triangle, one of the acute angles measures 43°. What is the measure of the second acute angle?
   A 37° **B** 57°
   C 43° **D** 47°

New Concepts

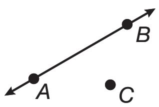
In a proof, deductive reasoning is used to develop a logical argument from given information to prove a conclusion. Proofs in geometry must be done step by step, and each step must have a justification. These justifications can include the given information, definitions, postulates, theorems, and properties, as seen in the two-column proofs in this lesson.

Hint

Review Lesson 24 for some of the justifications that can be used in a proof. You may wish to make a list of justifications for your reference. See the Skills Bank for more properties that can be used as reasons in a proof.

Example 1 Justifying Statements in a Two-Column Proof, Part 1

Fill in the justifying statements to support the proof of Theorem 4-2: If there is a line and a point not on the line, then exactly one plane contains them.



Given: Point C is not on $\overleftrightarrow{AB}$.
Prove: Exactly one plane contains $\overleftrightarrow{AB}$ and C.

Statements	Reasons
1. Point C is noncollinear with $\overleftrightarrow{AB}$.	1.
2. Exactly one plane contains points A, B, and C.	2.
3. Exactly one plane contains $\overleftrightarrow{AB}$ and C.	3.

SOLUTION

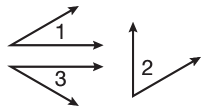
The missing justifying statements are:
1. Given
2. Through any three noncollinear points there exists exactly one plane. (Postulate 6)
3. If two points lie in a plane, then the line containing the points lies in the plane. (Postulate 8)

Online Connection
www.SaxonMathResources.com

Example 2 Justifying Statements in a Two-Column Proof, Part 2

Prove Theorem 6-1: If two angles are complementary to the same angle, then they are congruent.

Given: $\angle 1$ is complementary to $\angle 2$.
$\angle 3$ is complementary to $\angle 2$.



Prove: $\angle 1 \cong \angle 3$

Statements	Reasons
1. $\angle 1$ is complementary to $\angle 2$. $\angle 3$ is complementary to $\angle 2$.	1. Given
2. $m\angle 1 + m\angle 2 = 90°$ $m\angle 3 + m\angle 2 = 90°$	2.
3. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 2$	3.
4. $m\angle 1 + m\angle 2 - m\angle 2 = m\angle 3 + m\angle 2 - m\angle 2$	4.
5. $m\angle 1 = m\angle 3$	5.
6. $\angle 1 \cong \angle 3$	6.

Hint

A proof does not always have to begin by restating the given, but doing so may help you understand what the next step should be.

SOLUTION

The missing justifying statements are:

2. Definition of complementary angles
3. Substitution Property of Equality
4. Subtraction Property of Equality
5. Simplify
6. Definition of congruent angles

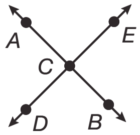
Two-column proofs have a format that is composed of five parts.

Five Parts of a Two-Column Proof
1. Given statement(s): The information that is provided.
2. Prove statement: The statement indicating what is to be proved.
3. Diagram: A sketch that summarizes the provided information. Sometimes you will need to draw the sketch yourself based on given information.
4. Statements: The specific steps that are written in the left-hand column.
5. Reasons: Postulates, theorems, definitions, or properties written in the right-hand column, which justify each statement.

Example 3 Writing a Two-Column Proof, Part 1

Prove Theorem 6-4: If two angles are vertical angles, then they are congruent. (Vertical Angles Theorem)

Given: $\overleftrightarrow{AB}$ and $\overleftrightarrow{DE}$ intersect at point C
Prove: $\angle ACD \cong \angle BCE$



SOLUTION

Proof:

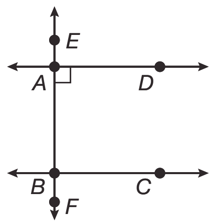
Statements	Reasons
1. $\overleftrightarrow{AB}$ and $\overleftrightarrow{DE}$ intersect at point C	1. Given
2. $m\angle BCD + m\angle ACD = 180°$	2. Linear Pair Theorem
3. $m\angle BCD + m\angle BCE = 180°$	3. Linear Pair Theorem
4. $m\angle BCD + m\angle ACD = m\angle BCD + m\angle BCE$	4. Transitive Property of Equality
5. $m\angle ACD = m\angle BCE$	5. Subtraction Property of Equality
6. $\angle ACD \cong \angle BCE$	6. Definition of congruent angles

Math Reasoning

Verify Prove that the other pair of vertical angles in Example 3 are also equal.

Example 4 Writing a Two-Column Proof, Part 2

Prove Theorem 5-3: If a transversal is perpendicular to one of two parallel lines, then it is perpendicular to the other one.



Given: $\overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$
$\overleftrightarrow{EB} \perp \overleftrightarrow{AD}$
Prove: $\overleftrightarrow{EB} \perp \overleftrightarrow{BC}$

SOLUTION

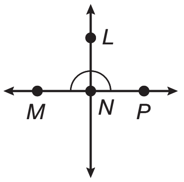
Proof:

Statements	Reasons
1. $\overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$	1. Given
2. $\angle EAD \cong \angle ABC$	2. Postulate 11: corresponding angles are congruent
3. $m\angle EAD = 90°$	3. Definition of perpendicular
4. $m\angle ABC = m\angle EAD$	4. Definition of congruent angles
5. $m\angle ABC = 90°$	5. Transitive Property of Equality
6. $\overline{EB} \perp \overline{BC}$	6. Definition of perpendicular

Lesson Practice

a. If a triangle is obtuse, what can you conclude about the measures of its two non-obtuse angles? Justify your answer.
(Ex 1)

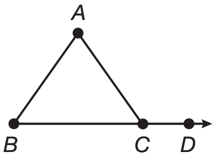
b. Fill in the reasons of the proof of Theorem 5-5: If two lines form congruent adjacent angles, then they are perpendicular.
(Ex 2)



Given: $\angle LNM \cong \angle LNP$
Prove: $LN \perp MP$

Statements	Reasons
1. $\angle LNM \cong \angle LNP$	1.
2. $m\angle LNM = m\angle LNP$	2.
3. $m\angle MNP = 180°$	3.
4. $m\angle LNM + m\angle LNP = m\angle MNP$	4.
5. $2m\angle LNM = 180°$	5.
6. $m\angle LNM = 90°$	6.
7. $LN \perp MP$	7.

c. Given $\triangle ABC$ with exterior angle $\angle ACD$, write a two-column proof to prove the Exterior Angle Theorem.
(Ex 3)



Given: $\angle ACD$ is an exterior angle of $\triangle ABC$.
Prove: $m\angle ACD = m\angle CAB + m\angle ABC$

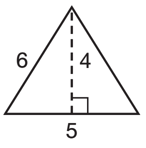
Practice Distributed and Integrated

1.* **Write Explain the difference between a minor arc and a major arc.
(26)

2. Is the statement, "A square is a rhombus," sometimes, always, or never true?
(19)

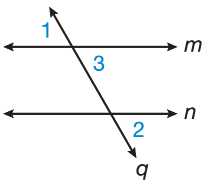
3. **Error Analysis** Parveer calculated the area of the given triangle as follows. Is he correct? Explain.
(13)

$$A = \frac{1}{2}bh$$
$$= \frac{1}{2}(5)(6)$$
$$= 15$$



4. Prove Theorem 10-2: If two parallel lines are cut by a transversal, then the alternate exterior angles are congruent.
(27)

Given: $m \parallel n$, transversal
Prove: $\angle 1 \cong \angle 2$



* **5.** (23) **Landscaping** A sprinkler system used for watering a field sprays water in a circular pattern with a radius of 20 feet. To the nearest square foot, what area is covered by this sprinkler?

* **6.** (27) **Verify** If M is the midpoint of $\overline{AB}$, write a two-column proof to show that $AM = \frac{1}{2}AB$.

7. (11) **Coordinate Geometry** Find the coordinates of the midpoints of $\overline{AB}$ and $\overline{CD}$ given the following coordinates: $A(8, 1)$, $B(14, 7)$, $C(7, 11)$, and $D(5, 5)$.

8. (21) **Write** Use the Law of Syllogism to write a valid conclusion.

If all the math classes are full, then Raymond must enroll in an elective.
If Raymond must enroll in an elective, then he will enroll in theater.

9. (23) **Furniture Design** The seating surface of a stool is a woolen circle with a diameter of 14 inches. To the nearest tenth of a square inch, how much material is used for the top surface of the stool?

10. (8) **Woodworking** If a rectangular table with an area of 35 square feet has a width that is 2 feet less than the length, what are the dimensions of the table?

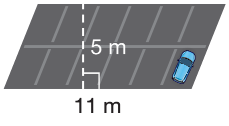
11. (20) Write the disjunction of the statements "Priyanka takes the bus," and "Priyanka goes to work early."

12. (27) Prove Theorem 5-6: All right angles are congruent.

Given: Angle 1 and angle 2 are right angles.
Prove: $\angle 1 \cong \angle 2$

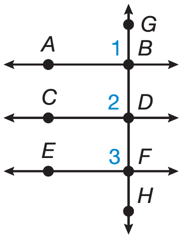
13. (Inv 1) Adrienne claims that for transversals of two lines, all the acute angles generated by the transversal are congruent and all the obtuse angles are congruent. Is she correct? Explain.

14. (22) **Road Repair** In the diagram, a parallelogram-shaped area of a parking lot is to be repaved. How many square meters are to be repaved?



***15.** (27) Prove Theorem 5-7: If two lines are parallel to the same line, then they are parallel to each other.

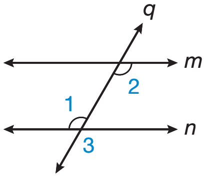
Given: $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ and $\overleftrightarrow{EF} \parallel \overleftrightarrow{CD}$
Prove: $\overleftrightarrow{AB} \parallel \overleftrightarrow{EF}$



16. (19) Draw a quadrilateral with no two sides parallel. Classify this shape.

17. (12) Prove Theorem 12-1: If two lines are cut by a transversal and the alternate interior angles are congruent, then the lines are parallel.

Given: $\angle 1 \cong \angle 2$
Prove: $m \parallel n$



18. (23) What is one-fourth of the circumference of a circle with a diameter of 10 meters?

19. (25) **Construction** A construction worker is framing a new roof and needs to make a triangular structure congruent to one already on the roof. If she only has tools for linear measurement, what measurements should she take to ensure that the triangles are congruent?

***20.** (24) **Justify** Write an algebraic proof to solve $4x = 8 - 2x$ for x.

21. (16) **Algebra** Solve for y, if the slope between points U and V is $-\frac{2}{3}$, with points $U(-2, -3)$ and $V(-5, y)$.

***22.** (25) Which angle corresponds to $\angle R$, if $\triangle XTW \cong \triangle PRG$?

23. (13) **Multiple Choice** Which triangle classification pertains to the lengths of the sides of a triangle?

A scalene　　**B** equiangular

C acute　　**D** right

24. (19) **Algebra** Quadrilateral $PQRS$ is a rhombus. If $PQ = 4x + 7$ and $QR = 8x - 5$, what is the length of $\overline{QR}$?

***25.** (26) Two congruent arcs have measures $3(x + 4)°$ and $4(x - 5)°$. What is the value of x?

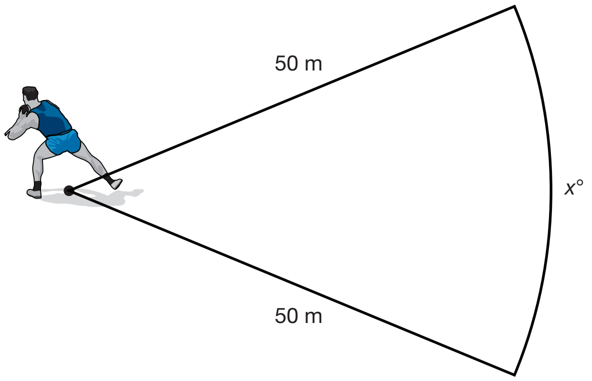
26. (Inv 1) Construct and label a diagram based on the following information:

Transversal k intersects lines m and n to produce corresponding angles $\angle 1$ and $\angle 2$ that are both acute, alternate interior angles $\angle 3$ and $\angle 4$ that are both obtuse, and alternate exterior angles $\angle 1$ and $\angle 5$ that are both acute.

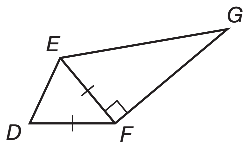
27. (14) Jameer states that the difference between any two even integers is positive. Give a counterexample to disprove this conjecture.

***28.** (24) Which property of equality supports the statement if $x = k$ and $k = y$, then $x = y$?

***29.** (26) **Track and Field** The area used for shot-put events is a sector of a circle with a central angle of 45°. What would be the arc measure (x) 50 meters away from the center of the area?



30. (13) Classify $\triangle DEF$ by its angles and sides.



LESSON 28

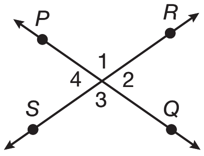
Triangle Congruence: SAS

Warm Up

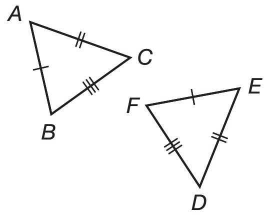
1. (25) **Vocabulary** Two triangles that have the same shape and size are said to be __________.

2. (6) What angle is vertical to ∠2?

3. (6) **Multiple Choice** What is the measure of the complement of a 25° angle?
   A 75° **B** 155°
   C 65° **D** 25°



4. (25) Explain why "$\triangle ABC \cong \triangle DEF$" is a false statement. Write a true congruence statement for the triangles.

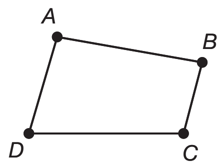


New Concepts

Sides and angles of polygons have special relationships to each other. The angle formed by two adjacent sides of a polygon is called an **included angle.** The common side of two consecutive angles of a polygon is called an **included side.**

Example 1 Identifying Included Angles and Sides

What is the included side of ∠A and ∠B? What is the included angle of $\overline{BC}$ and $\overline{CD}$?



SOLUTION

Angles A and B share the side $\overline{AB}$, so $\overline{AB}$ is the included side. The angle between $\overline{BC}$ and $\overline{CD}$ is ∠C, so ∠C is the included angle.

Online Connection
www.SaxonMathResources.com

Caution

In the SAS Postulate, the included angle is the angle that is formed by the two congruent sides. Remember that in the SAS Postulate, the *A* is between the two *S*'s, showing that the angle is between the two sides.

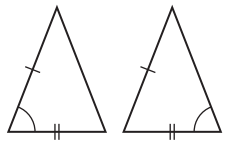
You learned in Lesson 25 how to prove triangles congruent by using the SSS Triangle Congruence Postulate. This lesson presents another way to prove that triangles are congruent. It uses two sides and the included angle.

Side-Angle-Side (SAS) Triangle Congruence Postulate

If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent by side-angle-side congruence.

Example 2 Using the SAS Postulate to Determine Congruency

Determine whether the pair of triangles is congruent by the SAS Postulate.



SOLUTION

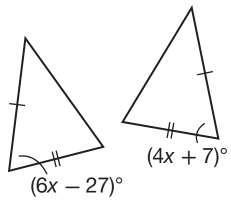
The two indicated triangles are not necessarily congruent, even though they have two congruent sides and one congruent angle. In the second triangle, the angle that is congruent is not the included angle of the two congruent sides.

Math Reasoning

Analyze The two triangles that are made by drawing a diagonal in a square are congruent by the SSS Postulate. Could the SAS Postulate also be used?

Example 3 Finding Missing Angle Measures

Find the value of x that makes the triangles congruent.



SOLUTION

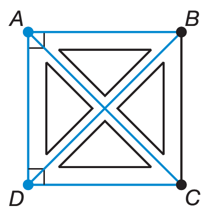
For the two triangles to be congruent, the measures of the included angles must be equal. Therefore,

$$6x - 27 = 4x + 7$$
$$6x = 4x + 34$$
$$2x = 34$$
$$x = 17$$

The triangle congruence postulates and theorems will be used as justifications in proofs. An example is given below.

Example 4 Using the SAS Postulate in a Proof

Triangles make an "X" design on this barn door. Use the SAS Postulate to write a two-column proof.



Given: $\overline{AB} \cong \overline{DC}$

Prove: $\triangle ABD \cong \triangle DCA$

SOLUTION

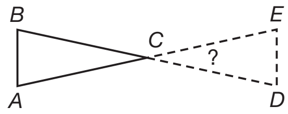
1. $\overline{AB} \cong \overline{DC}$ $\angle ADC$ and $\angle DAB$ are right angles	1. Given
2. $m\angle DAB = m\angle ADC$	2. All right angles are congruent.
3. $\overline{AD} \cong \overline{AD}$	3. Reflexive Property of Congruence
4. $\triangle ABD \cong \triangle DCA$	4. SAS Postulate

Hint

Recall that in a proof, every algebraic statement must be justified using properties of equality. See Lesson 24 and the Skills Bank for lists of these properties.

Example 5 Application: Design

An artist is designing patterned wallpaper made of congruent triangles. He starts by drawing $\triangle ABC$, shown below. He wants to design a mirror image of $\triangle ABC$, shown as $\triangle EDC$ below. How can he make sure that this new triangle is congruent to $\triangle ABC$ using the SAS pattern of triangle congruence?



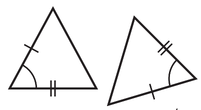
SOLUTION

To ensure that the two triangles are congruent, he should first measure $\overline{BC}$ and $\overline{AC}$. He can then extend both segments at points E and D, respectively, such that C is the midpoint of both $\overline{AE}$ and $\overline{BD}$. Since $\angle BCA$ and $\angle ECD$ are vertical angles, they are congruent, and the triangles must also be congruent by the SAS Postulate.

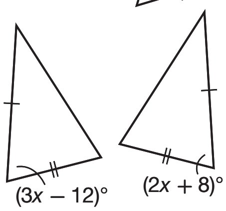
> **Math Reasoning**
>
> **Verify** The two triangles that are formed by drawing a diagonal through a parallelogram can be proven congruent by the SSS Postulate. Could the SAS Postulate also be used? Explain.

Lesson Practice

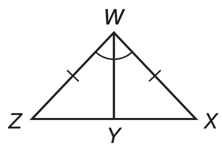
a. *(Ex 1)* Determine whether the pair of triangles is congruent by the SAS Postulate.



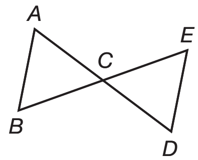
b. *(Ex 2)* Find the value of x that makes the triangles congruent.



c. *(Ex 3)* Use the SAS Postulate to prove $\triangle WXY \cong \triangle WZY$ if $\overline{WZ} \cong \overline{WX}$ and $\angle ZWY \cong \angle XWY$.



d. *(Ex 4)* A land dispute over two triangular parcels of land can be ended by showing that the parcels have the same shape and size. In the figure, two sides of each parcel of land were measured and it was found that point C is the midpoint of both $\overline{AD}$ and $\overline{BE}$. End the dispute by proving $\triangle ABC \cong \triangle DEC$.



Practice Distributed and Integrated

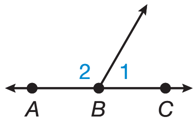
1. *(Inv 1)* Henry wants to hang a picture frame on to the wall but does not have a leveling tool. However, the striped wallpaper on the wall has parallel lines that form 45-degree angles to the floor and the ceiling. How can Henry use a protractor to hang the picture frame so that it is not tilted?

2. *(21)* **Justify** Is the following conclusion valid? Why or why not?

Some skates that fit well are expensive. For my feet to be comfortable, I need skates that fit well. As a result, it is expensive to keep my feet comfortable.

* **3.** (26) **Algebra** Two congruent arcs have measures $(x^2 - 4x - 1)°$ and $(x^2 - 3x + 10)°$. What is the value of x?

* **4.** (27) If two angles form a linear pair, then they are supplementary.

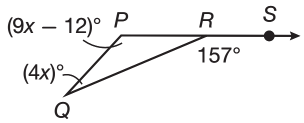


Given: $\angle 1$ and $\angle 2$ are a linear pair.
Prove: $m\angle 1 + m\angle 2 = 180°$

5. (8) **City Planning** A rectangular city block measures 105 meters by 88 meters. The city council wants to divide the block into two triangles of equal area. What is the length of the hypotenuse of each triangular area?

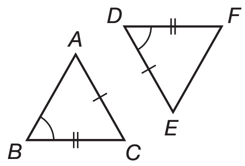
6. (26) A minor arc has a measure of 122°. What is the measure of the corresponding major arc?

7. (18) Find the measure of $\angle P$ in the figure shown.



8. (7) **Generalize** The numbers in the geometric sequence $1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \ldots$ continue to get smaller in value. Make a general formula for the sequence. State whether a term of the sequence will ever have a value of 0.

* **9.** (28) **Error Analysis** Is the following proof for congruent triangles correct? Explain.

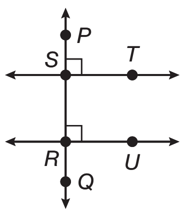


1. $\overline{AC} \cong \overline{DE}$	1. Given
$\angle ABC \cong \angle EDF$	
$\overline{BC} \cong \overline{DF}$	
2. $\triangle ABC \cong \triangle EFD$	2. SAS Postulate

10. (11) **Multiple Choice** What are the coordinates of the midpoint of a segment that connects the points (1, 1) and (−3, 5)?

A (2, 3) **B** (−1, −3)
C (−1, 3) **D** (4, 6)

***11.** (27) Prove Theorem 5-2: If two lines in a plane are perpendicular to the same line, then they are parallel to each other.



Given: $\overleftrightarrow{ST} \perp \overleftrightarrow{PQ}, \overleftrightarrow{RU} \perp \overleftrightarrow{PQ}$
Prove: $\overleftrightarrow{ST} \parallel \overleftrightarrow{RU}$

12. (17) **Write** What is the inverse of this statement?

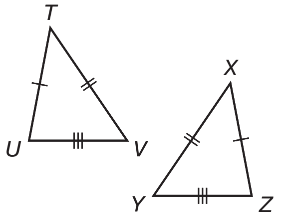
If John is at least 5’10” tall, then he is taller than Herschel.

***13.** (28) **Write** When can the SAS Postulate be used?

14. (23) **Art** Paolo was hired to do an abstract mural on the side of a building. His mural includes three circles. The first circle has a diameter of 34 feet, the second has a diameter of 30 feet, and the third has a diameter of 26 feet. What is the total circumference of the three circles, to the nearest foot?

***15.** (23) Explain this statement: In $\odot P$, with two points anywhere on the circle, K and L, $\overline{KP} \cong \overline{LP}$.

*16. (19) **Algebra** Figure $DEFG$ is a parallelogram. If $m\angle D = 8x + 17$ and $m\angle E = 5x^2 - 2$, what are the measures of the other two angles?



17. (25) Write a congruence statement for the triangles and give a justification.

18. (16) Write the equation of line AB in slope-intercept form if A is at $(-2, -3)$ and B is at $(2, -1)$.

19. (21) Given these statements, is the conclusion valid?

Wilhelm will be in Paris on Friday if he leaves New York by Wednesday.
Wilhelm was not in Paris on Friday.
Wilhelm did not leave New York by Wednesday.

20. (25) **Write** If $\triangle KLM \cong \triangle TUV$ by the SSS Postulate, is $m\angle K \cong m\angle U$? Explain.

21. (22) A rhombus has diagonals that measure 22 inches and 14 inches. What is the area of the rhombus?

22. (21) **U.S. History** Use the Law of Detachment to make a valid conclusion based on these statements.

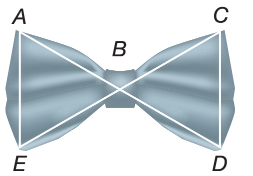
If you live in Arkansas, then you live in the 25th state to enter the Union.
Clark lives in Arkansas.

23. (23) **Farming** A crop disease has started at a central point in a field. The disease is spreading in a circular pattern whose radius increases at a rate of 2 meters per day. At this rate, how large an area will be affected in 3 days, to the nearest square meter?

*24. (28) **Write** If two triangles are proven congruent by the SAS Postulate, what can be said about the third sides of the two triangles?

*25. (16) **Model** Graph the line $y = 2x + 4$.

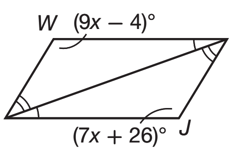
*26. (28) A properly-tied bow tie can be modeled as two congruent triangles. If $\overline{BA} \cong \overline{BD}$ and $\overline{BE} \cong \overline{BC}$, prove $\triangle ABE \cong \triangle DBC$.



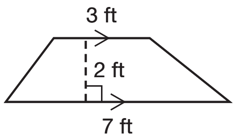
27. (13) **Road Signs** A triangular yield sign is to be rust-proofed with a layer of plastic. Its base is 18 inches and its height is 24 inches. How many square feet of plastic are needed to cover both the front and the back of the sign?

28. (Inv 2) **Algebra** What is the hypotenuse of a triangle whose legs are a and $\sqrt{3}a$?

29. (18) Find $m\angle W$ and $m\angle J$.



30. (22) Find the area of the trapezoid.



LAB 5

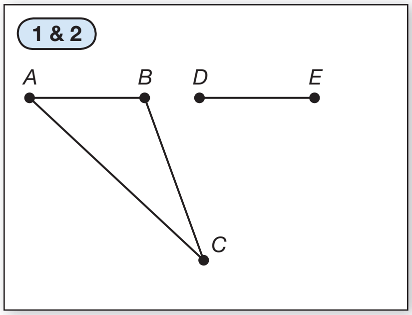
Congruent Triangles

Construction Lab 5 *(Use with Lesson 28)*

In Lesson 28, you learned some triangle congruence patterns. This lab shows you how to use a compass and straightedge to construct a triangle congruent to a given triangle using the SAS Postulate.

1. Sketch a triangle and label it ABC.

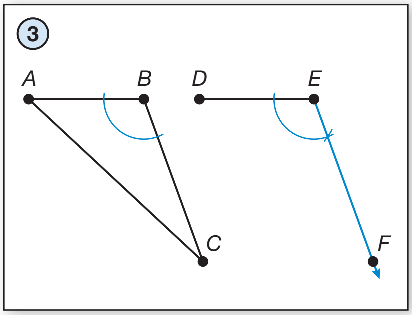
2. Using the skills you learned in Construction Lab 1, construct $\overline{DE}$ congruent to $\overline{AB}$.



Hint

Refer to Construction Lab 1 for a reminder on how to construct congruent angles and segments. To copy this triangle, you will need to use both techniques at once.

3. Again, just like in Construction Lab 1, construct $\angle DEF$ congruent to $\angle ABC$. Use your compass to place point F on the ray so that $\overline{EF}$ is congruent to $\overline{BC}$.



4. Draw segment $\overline{FD}$ to complete $\triangle DEF$.

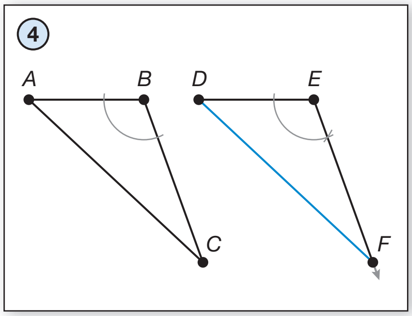
   You constructed the figures so that:

   $\overline{DE} \cong \overline{AB}$

   $\angle DEF \cong \angle ABC$

   $\overline{EF} \cong \overline{BC}$

   Therefore, by the SAS Postulate, $\triangle DEF \cong \triangle ABC$.



Lab Practice

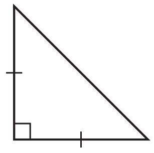
Sketch a triangle using a straightedge, then trade with a partner and construct triangles congruent to each other's sketches. Verify that the triangles are congruent by measuring all the sides and angles using a ruler and protractor.

LESSON
29
Using the Pythagorean Theorem

Warm Up

1. (Inv 2) **Vocabulary** The side that is opposite the right angle in a right triangle is the ___________.

2. (13) Classify the triangle according to its sides and angles.



3. (SB 6) **Multiple Choice** Simplify: $\sqrt{3^2 + 4^2}$
   A 7 **B** 5
   C 49 **D** 25

4. (SB 6) Simplify: $\sqrt{640}$

New Concepts

When the side lengths of a right triangle are nonzero whole numbers that satisfy the Pythagorean Theorem, they form a **Pythagorean triple**.

Pythagorean Triples
A Pythagorean triple is a set of three nonzero whole numbers a, b, and c such that: $a^2 + b^2 = c^2$.

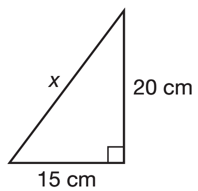
Two of the most well-known sets of Pythagorean triples are (3, 4, 5) and (5, 12, 13). An easy way to find Pythagorean triples is to multiply one of these two sets by a whole number. For example, multiplying the first set by 2 yields (6, 8, 10), which is also a Pythagorean triple.

Math Reasoning

Verify Use your calculator to verify that multiples of these two common Pythagorean triples are also Pythagorean triples.

Example 1 Finding Pythagorean Triples

Find the unknown length in the triangle.
Do the side lengths form a Pythagorean triple?



SOLUTION The length of the hypotenuse is the unknown, $c = x$. The lengths of the legs are 15 centimeters and 20 centimeters, which are represented by a and b. Note that it does not matter which value is a and which is b.

$$a^2 + b^2 = c^2$$
$$(15)^2 + (20)^2 = x^2$$
$$225 + 400 = x^2$$
$$625 = x^2$$
$$25 = x^2$$

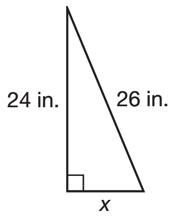
Therefore, the length of the hypotenuse is 25 centimeters. The set (15, 20, 25), which gives the side lengths of this triangle, is the Pythagorean triple (3, 4, 5) multiplied by 5.

Math Reasoning

Justify When solving $x^2 = 625$, remember that $x = 25$ or $x = -25$. However in this type of problem, the negative value of the square root is ignored. Why?

Example 2 Using Pythagorean Triples To Find the Legs

Find the unknown length in the triangle.
Do the side lengths form a Pythagorean triple?



SOLUTION The hypotenuse is known, $c = 26$ inches. The value of 24 inches is substituted for either a or b, with x representing the other leg.

$$a^2 + b^2 = c^2$$
$$(24)^2 + x^2 = (26)^2$$
$$576 + x^2 - 576 = 676 - 576$$
$$x^2 = 100$$
$$x = 10$$

Therefore, the length of the other leg is 10 inches.
Since the side lengths are nonzero whole numbers that satisfy the equation $a^2 + b^2 = c^2$, they form a Pythagorean triple. The set which gives the side lengths of this triangle (10, 24, 26), is the Pythagorean triple (5, 12, 13) multiplied by 2.

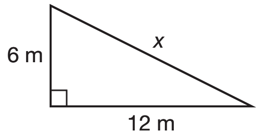
However, not all right triangles are composed of side lengths that are nonzero whole numbers. In such cases, one or more side lengths may be written as a **radical expression**. A radical expression is any expression that contains a root. Typically, a radical expression should be reduced to simplified radical form.

Example 3 Simplifying Radicals

Hint

When substituting numbers into $a^2 + b^2 = c^2$, always substitute the largest number (the hypotenuse) for c. The other two numbers can be substituted for a and b.

a. Find the value of x. Give your answer in simplified radical form.



SOLUTION The length of the hypotenuse is the unknown, x, and the legs are 6 meters and 12 meters in length.

$$a^2 + b^2 = c^2$$
$$6^2 + 12^2 = x^2$$
$$36 + 144 = x^2$$
$$180 = x^2$$
$$\sqrt{180} = x$$

Math Reasoning

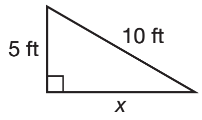
Analyze List the first ten perfect squares. These are examples of factors that you look for to express a radical in simplified form.

To write the answer in simplified radical form, you must factor out all perfect square factors of the number under the radical sign. The largest perfect square that is a factor of 180 is 36, so 180 is factored out as 36×5.

$$\sqrt{36 \times 5} = x$$
$$6\sqrt{5} = x$$

Therefore, the length of the hypotenuse is $6\sqrt{5}$ m.

b. Find the value of x. Give your answer in simplified radical form.



SOLUTION The hypotenuse is 10 feet, one of the legs is 5 feet, and the other leg is x ft.

$$a^2 + b^2 = c^2$$
$$x^2 + 5^2 = 10^2$$
$$x^2 + 25 = 100$$
$$x^2 = 75$$
$$x = 5\sqrt{3}$$

Therefore, the length of the second leg is $5\sqrt{3}$ feet.

Example 4 Application: TV Aspect Ratios

The aspect ratio of a TV screen is the ratio of the width to the height of the image. Find the height and the width of a 42-inch TV screen with an aspect ratio of 4:3 to the nearest tenth of an inch. The length 42 inches refers to the diagonal distance across the screen.

SOLUTION

Understand

The problem asks for the height and width of a TV screen. There are two important pieces of information: the ratio of the TV's width to its height is 4:3, and the diagonal of the TV is 42 inches long.

Plan

The diagonal, width, and height of the screen form a right triangle. Therefore, the Pythagorean Theorem can be used.

Although both a and b are unknown, it is known that the ratio of their lengths is 4:3. The width can be represented by $4x$ and the height by $3x$. Substitute $a = 4x$, $b = 3x$, and $c = 42$ to find the value of x. Then, use the value of x to determine the width and the height.

Solve

$$a^2 + b^2 = c^2$$
$$(4x)^2 + (3x)^2 = 42^2$$
$$25x^2 = 42^2$$
$$x^2 = \frac{42^2}{25}$$
$$x = \frac{42}{5}$$
$$x = 8.4$$

Width: $= 4x$
$= 4(8.4)$
$= 33.6$ inches

Height: $= 3x$
$= 3(8.4)$
$= 25.2$ inches

Check

Check your answer by substituting the calculated height and width into the equation $a^2 + b^2 = c^2$ and solving for c. It should simplify to 42 inches.

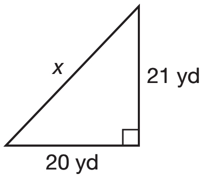
$$a^2 + b^2 = c^2$$
$$(33.6)^2 + (25.2)^2 = c^2$$
$$42 = c$$

Hint

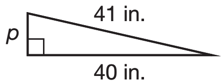
Taking the square root of a fraction is equivalent to taking the square root of both the denominator and the numerator separately. In other words: $\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}}$

Lesson Practice

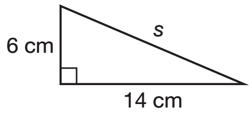
a. Find the hypotenuse of the triangle. Do the side lengths form a Pythagorean triple?
(Ex 1)



b. Find the value of p in the triangle at right. Do the side lengths form a Pythagorean triple?
(Ex 2)

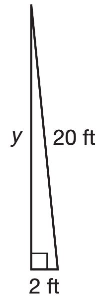


c. Find the value of s in the triangle at right. Give your answer in simplified radical form.
(Ex 3)



d. Find the value of y in the triangle at right. Give your answer in simplified radical form.
(Ex 3)

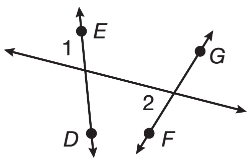
e. A ratio of a TV's width to its height is 16:9. If its width is 32 inches, what is the length of its diagonal?
(Ex 4)



Practice Distributed and Integrated

* **1.** **a.** **Algebra** In the figure shown, if $x = 5$, find $m\angle 1$ and $m\angle 2$ where $m\angle 1 = (x^2 + 3x - 5)°$ and $m\angle 2 = (x^3 - 2x^2 - 2x - 10)°$.
(12)

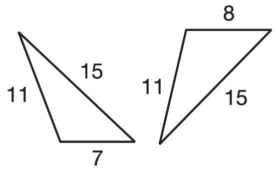
b. Is $\overleftrightarrow{DE}$ parallel to $\overleftrightarrow{FG}$?



2. **Air Traffic Controller** An air traffic controller uses radar to scan the airspace above an area near the airport. A controller's radar covers a circular area of 250,000 square kilometers. What is the radius of the circle, to the nearest kilometer?
(23)

3. Construct a truth table for "s and not t," where s and t are logical statements.
(20)

* **4.** **Write** Are the two triangles congruent? Explain.
(25)

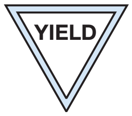


* **5.** **Write** When can SAS congruency be used to conclude two triangles are congruent?
(28)

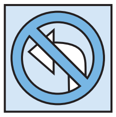
6. In $\triangle DEF$, $m\angle D = 71°$ and the measure of exterior angle F is 107°. Find $m\angle E$.
(18)

7. (15) **Road Signs** While driving, Anil saw the following signs. Which of these is not a regular polygon?

A


B


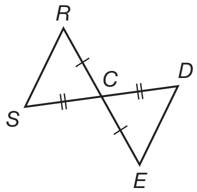
C


D


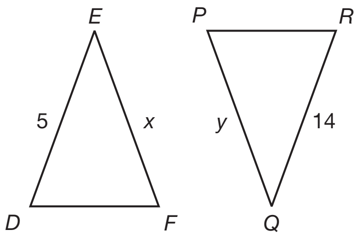
8. (21) What conclusion can be drawn from the following statements using the Law of Syllogism?

My family room is a place where I like to study.
The places where I like to study are quiet.

*** 9.** (28) Write the congruency statement for the triangles shown below.



10. (25) **Justify** The congruence statement for the two triangles is written as $\triangle DEF \cong \triangle PQR$. Find the values of x and y.

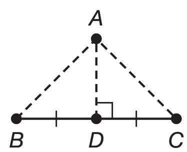


11. (14) **Multiple Choice** Which of the following conjectures is false?

A If x is odd, then $2x$ is even.
B If x is prime, then $x + 1$ is not prime.
C The product of two odd numbers is odd.
D Division of an integer by a prime number gives a rational number.

12. (11) **Geography** On a grid map, Albuquerque is located at $(-10, 4)$ and Oklahoma City is at $(4, 2)$. If Amarillo is the midpoint between the two cities, where is it located?

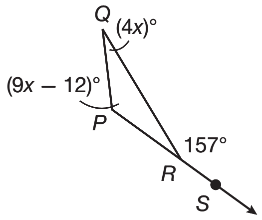
***13.** (27) If a point lies on the perpendicular bisector of a segment, then the point is equidistant from the endpoints of the segment.



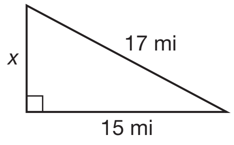
Given: A lies on the perpendicular bisector of $\overline{BC}$.
Prove: $AB = AC$

14. (23) A small car has a turning radius of 22 feet. If the car makes a complete 360° turn at this radius, what is the area of the circle made to the nearest square foot?

15. Find m$\angle Q$.
(18)

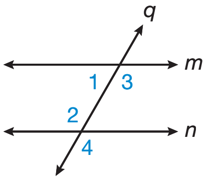


***16.** Find the unknown length of the side in the triangle.
(29)



***17.** If an arc of 120° were to be divided into three equal arcs, what would be the measure of each arc?
(26)

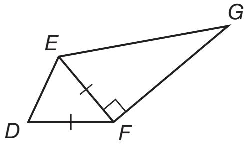
18. Prove Theorem 10-3: If two parallel lines are cut by a transversal, then the same-side interior angles are supplementary.
(Inv 1)
Given: $m \parallel n$, transversal q
Prove: $\angle 1$ is supplementary to $\angle 2$.



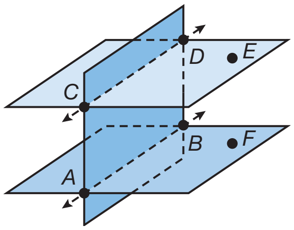
19. **Astronomy** The moon orbits the Earth with an average radius of 380,000 kilometers. Assuming the orbit is circular, what is the distance the moon travels in one orbit, to the nearest kilometer?
(23)

20. Graph the line $y = -3x - 6$.
(16)

21. Classify $\triangle EFG$ by its angles and sides.
(13)



22. Prove Theorem 5-1: If two parallel planes are cut by a third plane, then the lines of intersection are parallel.
(5)
Given: Planes *CDE* and *ABF* are parallel. Plane *ABC* is a transversal intersecting both planes.
Prove: $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$



23. **Landscaping** A school's soccer field is rectangular with dimensions 110 yards by 60 yards. If the field is temporarily fenced and covered with a protective tarp during the off-season, how many yards of fence are needed? How many square yards of tarp are needed?
(22)

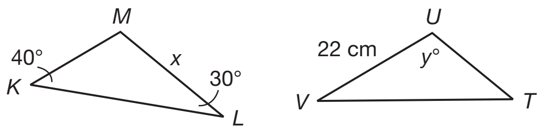
24. (20) **Analyze** Use the statements to determine the truth value for "*A* and not *B*."

A: Thanksgiving is a holiday.

B: Thanksgiving is celebrated in January.

***25.** (29) **Error Analysis** Oshwinder found the solution 13.9 feet when solving for the length of one leg of a right triangle with a hypotenuse of 13 feet and a side length of 5 feet. Is he correct? Explain.

***26.** (28) These two triangles are congruent, $\triangle KLM \cong \triangle TVU$. Find the values of x and y.



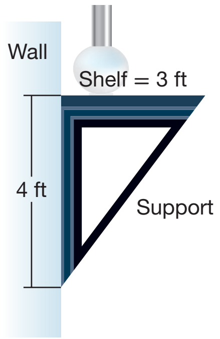
27. (14) **Error Analysis** A student wrote the following as a conjecture. Is he correct? If not, provide a counterexample.

Any sequence starting with $\frac{1}{2}$ *which keeps increasing will eventually include a term greater than 1.*

***28.** (29) Give two Pythagorean triples that are related to the triple (7, 24, 25).

29. (19) Is the statement, "A parallelogram is a rectangle," always, sometimes, or never true?

***30.** (29) **Carpentry** A shelf bracket is needed to secure the shelf in the diagram. Use the Pythagorean Theorem to find the length of the support.



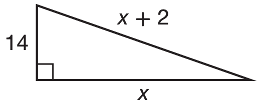
LESSON 30

Triangle Congruence: ASA and AAS

Warm Up

1. (28) **Vocabulary** The angle between two adjacent sides of a polygon is called a(n) __________ angle. (***exterior, vertical, included***)

2. (29) What is the value of x in the triangle?



3. (25, 28) **Multiple Choice** Which of these is *not* used to prove that two triangles are congruent?
   A AAA Postulate
   B SAS Postulate
   C SSS Postulate
   D none of the above

New Concepts

There are many ways to prove triangle congruence. In Lesson 25, you learned that three congruent sides between two triangles prove triangle congruence (SSS Congruence Postulate). In Lesson 28, you learned that congruence of two pairs of corresponding sides and the corresponding included angles of two triangles also proves triangle congruence (SAS Congruence Postulate). In this lesson, you will learn two more triangle congruence patterns—the Angle-Side-Angle Postulate and the Angle-Angle-Side Theorem.

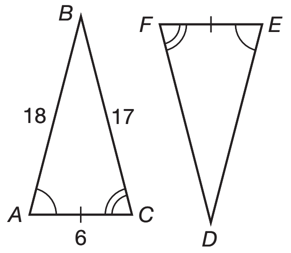
Hint

Recall that two congruent triangles have 3 congruent sides and 3 congruent angles. Although there are 6 congruent parts, only 3 properly chosen ones are needed to prove triangle congruence.

Postulate 16: Angle-Side-Angle (ASA) Congruence Postulate
If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent.

Example 1 Using the ASA Postulate

Use ASA congruence to determine the measure of the sides of $\triangle DEF$.



SOLUTION

In the two triangles, it is given that $\angle C \cong \angle F$, $\overline{AC} \cong \overline{EF}$, and $\angle A \cong \angle E$.

Since two angles and the included side of $\triangle BAC$ are congruent to two angles and the included side of $\triangle DEF$, by the ASA Postulate, $\triangle BAC \cong \triangle DEF$.

Therefore,

since $\overline{AC} \cong \overline{EF}$, $EF = 6$;

since $\overline{CB} \cong \overline{FD}$, $FD = 17$;

and since $\overline{AB} \cong \overline{ED}$, $ED = 18$.

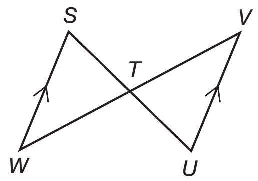
Online Connection
www.SaxonMathResources.com

Caution

Be sure that the congruent side is an included side of the two congruent angles when using ASA congruence.

Example 2 Using the ASA Postulate in a Proof

Prove that $\triangle SWT \cong \triangle UVT$, given that T is the midpoint of $\overline{WV}$ and $\overline{VU} \parallel \overline{WS}$.



SOLUTION

1. T is the midpoint of $\overline{WV}$	1. Given
2. $\overline{WT} \cong \overline{VT}$	2. Definition of midpoint
3. $\angle SWT \cong \angle TVU$	3. If two parallel lines are cut by a transversal, then alternate interior angles are congruent.
4. $\angle WTS \cong \angle VTU$	4. Vertical angles are congruent.
5. $\triangle SWT \cong \triangle UVT$	5. ASA Congruence Postulate

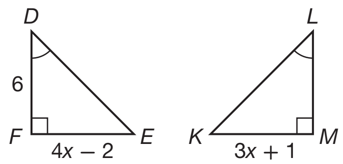
Another way to prove that two triangles are congruent involves two angles and a non-included side.

Theorem 30-1: Angle-Angle-Side (AAS) Triangle Congruence Theorem

If two angles and a nonincluded side of one triangle are congruent to two angles and the corresponding nonincluded side of another triangle, then the triangles are congruent.

Example 3 Using the AAS Congruence Theorem

Given that $\overline{DE} \cong \overline{LK}$, find the area of each triangle shown below.



SOLUTION

It is given that $\overline{DE} \cong \overline{LK}$. It is also given in the illustration that $\angle D \cong \angle L$, and since all right angles are congruent, $\angle F \cong \angle M$. Since two angles and a non-included side of $\triangle DEF$ are congruent to the corresponding angles and non-included side of $\triangle LKM$, $\triangle DEF \cong \triangle LKM$ by the AAS Congruence Theorem.

Therefore, solve for x using CPCTC:

$$EF = KM$$
$$4x - 2 = 3x + 1$$
$$x = 3$$

Since the value of x is 3, the measure of $\overline{EF}$ is $4 \cdot 3 - 2$, or 10. So, $\overline{EF}$ and $\overline{KM}$ each have a length of 10. Since $\overline{DF} \cong \overline{LM}$, they both have a length of 6. The area of each triangle is $\frac{1}{2}bh = \frac{1}{2}(10)(6) = 30$.

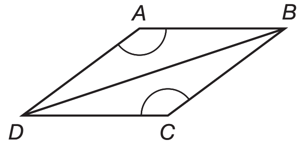
Therefore, the area of each triangle is 30 square units.

Math Reasoning

Justify Write a congruence statement for each of the parts of $\triangle ABD$ and $\triangle CBD$ that are given as congruent. What other congruence statements can you write once you have proven that the two triangles are congruent?

Example 4 Using the AAS Theorem in a Proof

Given: $\overline{BD}$ bisects $\angle ADC$ and $\angle A \cong \angle C$.
Prove: $\triangle ABD \cong \triangle CBD$

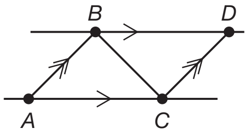


SOLUTION

Statements	Reasons
1. $\angle A \cong \angle C$	1. Given
2. $\angle ADB \cong \angle CDB$	2. Definition of angle bisector
3. $\overline{DB} \cong \overline{DB}$	3. Reflexive Property of Congruence
4. $\triangle ABD \cong \triangle CBD$	4. AAS Theorem

Example 5 Application: Bridges

A diagram of a portion of the truss system of a new bridge is shown below. Prove $\triangle ABC \cong \triangle DCB$.

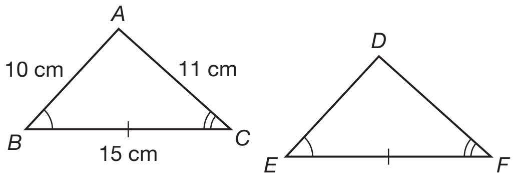


SOLUTION

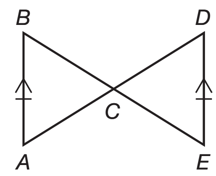
Statements	Reasons
1. $\overline{BD} \parallel \overline{AC}$ $\overline{AB} \parallel \overline{CD}$	1. Given
2. $\angle DBC \cong \angle ACB$	2. If parallel lines are cut by a transversal, then alternate interior angles are congruent (Theorem 10-1).
3. $\angle ABC \cong \angle DCB$	3. Theorem 10-1
4. $\overline{BC} \cong \overline{BC}$	4. Reflexive Property of Congruence
5. $\triangle ABC \cong \triangle DCB$	5. ASA Theorem

Lesson Practice

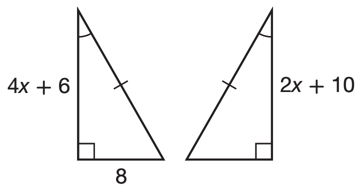
a. *(Ex 1)* State the postulate that can be used to prove the triangles congruent, and state the measure of the sides of $\triangle DEF$.



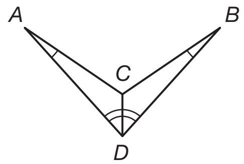
b. Prove that $\triangle ABC \cong \triangle DEC$, given that $\overline{AB} \cong \overline{DE}$ and $\overline{AB} \parallel \overline{DE}$.
(Ex 2)



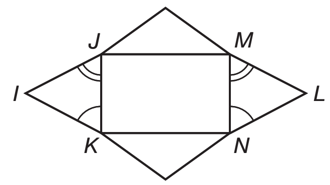
c. If the two triangles are congruent by the AAS Theorem, what is the area of each triangle?
(Ex 3)



d. Prove that $\triangle ADC \cong \triangle BDC$.
(Ex 4)



e. A standard-sized envelope is a $9\frac{1}{2}$-inch by 4-inch rectangle. The envelope is folded and glued from a sheet of paper shaped like the figure shown. Prove that if $JKNM$ is a rectangle, then $\overline{JI} \cong \overline{ML}$.
(Ex 5)



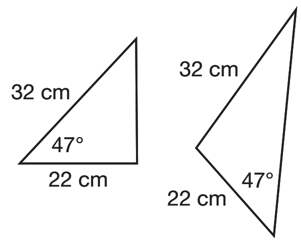
Practice Distributed and Integrated

1. Construct a truth table for the biconditional statement, "(if x, then y) and (if y, then x)."
(20)

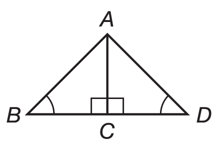
2. **Mapping** After walking 0.5 kilometers north and 0.4 kilometers east of her house, Juanita is one-third the distance to her grandfather's house. If the coordinates of Juanita's house are (1, 2), what are the coordinates of her grandfather's house?
(9)

3. **Landscaping** A newly planted tree will be secured by wires, each attached to the ground 1.5 meters from the base of the tree. If each wire is 4 meters long, how high up the tree should each wire be attached, to the nearest tenth of a meter?
(29)

4. **Justify** Can the SAS Postulate be used to prove these triangles congruent? Explain.
(28)

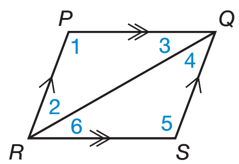


* **5.** Prove that the two triangles are congruent.
(30)



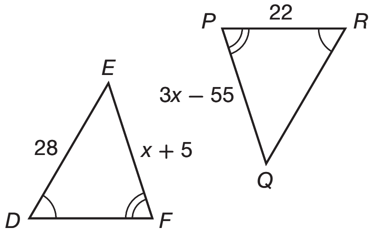
6. **Algebra** Solve the equation $\frac{2x - 5}{5} = 3$, and justify each step.
(24)

7. **Justify** **Given:** $PQRS$ is a parallelogram.
(27)
Prove: $\angle 1 \cong \angle 5$, $\angle 2 \cong \angle 4$ and $\angle 6 \cong \angle 3$.



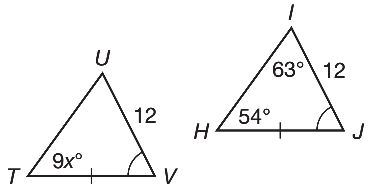
8. (23) **Safety** A lid is needed to cover a well that has a 14-foot circumference. What is the area of the lid that is needed to cover the well, to the nearest tenth of a square foot?

*** 9.** (30) If the triangles are congruent, what is the perimeter of $\triangle PRQ$?

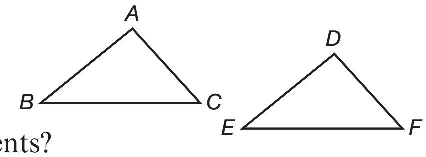


10. (26) **Algebra** The expressions $12x - 62$ and $8x + 86$ represent a pair of congruent arcs. What is the value of x in the expressions?

***11.** (28) Determine the value of x.



***12.** (25) Given that $\triangle ABC \cong \triangle DEF$, write the six congruence statements for the triangles.

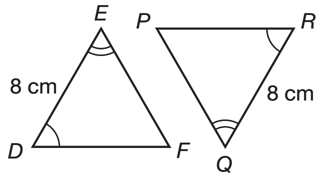


13. (17) **Muiltiple Choice** What is the relationship between these two statements?

If Ming rides the bus to school, then she will not ride with Debra.
If Ming rides with Debra, then she will not ride the bus to school.

A They are converses. **B** They are contrapositives.
C They are inverses. **D** none of the above

***14.** (30) **Write** What theorem or postulate can be used to prove that the two triangles are congruent? Explain.



15. (20) **Justify** Use *a, b,* and *c* to determine the truth value of "*a* or (*b* and *c*)".

a: New York City is the most populous city in the United States.
b: Florida is the most northwestern of all states.
c: The first president of the United States was George Washington.

16. (14) Find a counterexample for the following conjecture.

The difference between x and y is a positive, even integer.

17. (29) **Algebra** Find the length of each leg of an isosceles right triangle with a hypotenuse of 24 inches, to the nearest inch.

18. (21) **Analyze** Draw a valid conclusion from the following statements.

Whoever dropped the bowl made a mess.
The clerk dropped the bowl.

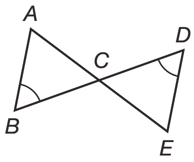
***19.** (26) A minor arc has a measure of 135°. If the corresponding major arc is divided into 5 equal parts, what is the measure of each part?

*20. (29) **Surveying** A road is to be built on an incline to reach a new bridge. The surveyor measures a horizontal distance of 660 yards and a vertical distance of 60 yards for the incline. How long is the actual road surface, to the nearest yard?

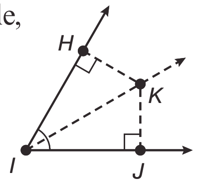
21. (18) **Multi-Step** In $\triangle GHI$, $m\angle G = 33°$ and the measure of an exterior angle at I is 141°. Find $m\angle H$.

22. (22) The diagonals of a rhombus are 14 centimeters and 22 centimeters long, respectively. If the rhombus is cut apart and reassembled into a rectangle with a width of 5 centimeters, what would be the length of the rectangle?

*23. (30) **Error Analysis** Jamal says that $\overline{AB} \cong \overline{DE}$ is needed to prove that $\triangle ABC \cong \triangle DEC$ by the AAS Theorem. Identify and correct Jamal's mistake.



*24. (27) Prove Theorem 6-7: If a point lies on the bisector of an angle, then the point is equidistant from the sides of the angle.
Given: K lies on the angle bisector of $\angle HIJ$.
Prove: $HK = JK$



25. (16) **Multi-Step** Find the slope and the y-intercept of the line $C(1, 1)$ and $D(-2, 3)$.

26. (23) A circular lid has an area of 42 square inches. If the lid rolls 8 times, what distance will it cover, to the nearest inch?

*27. (25) **Model** How many triangles could you construct if you were given three different line segments? Explain.

28. (18) The measure of one acute angle in a right triangle is y. What is the measure of the other acute angle?

29. (22) **Stereo Equipment** The front of a speaker box is shaped like a trapezoid with a height of 2 feet, a lower base of 4 feet, and an upper base of 3 feet. How many square feet of mesh is needed to cover the front of the speaker?

30. (17) What is the relationship between the following two statements?
If a rectangle is regular, then it is a square.
If a rectangle is not regular, then it is not a square.

INVESTIGATION

3

Exploring Angles of Polygons

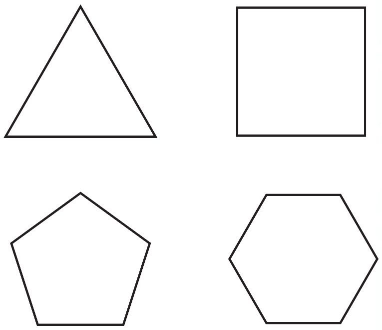
In Lesson 15, you learned to identify interior and exterior angles in polygons. In this investigation, you will explore ways of determining angle measures in regular polygons.

Interior Angles in Regular Polygons

> **Math Language**
>
> Recall that a **regular polygon** has all sides congruent and all interior angles congruent, and that all regular polygons are **convex**.

1. Using a protractor, measure an interior angle of each of the polygons and record your measurements in the second row of the table below.

2. **Analyze** Compare the interior angle measures for each polygon. What patterns do you notice? How does the data support your observations?



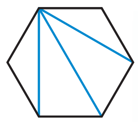
Regular Polygon	Triangle	Quadrilateral	Pentagon	Hexagon
Interior Angle Measure				
Sum of Interior Angle Measures				

3. Since the polygons are regular, how can you calculate the sum of the interior angles for each polygon?

4. Record the sums of the interior angle measures in the table and compare these sums for the different polygons. What do you notice?

> **Math Language**
>
> Recall that a **diagonal of a polygon** is a segment connecting a vertex with a nonadjacent vertex.

Rather than directly measuring interior angles, there is a way to calculate their measures for regular polygons. By drawing all possible diagonals from one vertex, a polygon can be divided into triangles.



The sum of the interior angle measures for all the triangles is the sum of the interior angle measures for the polygon. For any convex polygon, the number of triangles formed is two fewer than the number of sides. The formula for the sum of the interior angles of a polygon is the number of triangles it can be divided into multiplied by 180°.

Formula for the Sum of the Interior Angles of a Polygon
To find the sum of the interior angles of a polygon, use the formula below, where n is the number of sides of the polygon. $(n - 2)180°$

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For example, if you draw a diagonal on a quadrilateral there are 2 triangles formed. The sum of the interior angles of the polygon is equal to 2 times 180°, or 360°.

To calculate the angle measure of a regular quadrilateral:

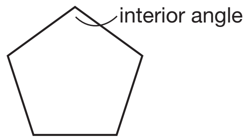
$$\text{measure of each interior angle in the square} = \frac{360°}{4}$$

Thus, the measure of each interior angle is 90°. Since each interior angle measure in a regular polygon is the same, the following formula can be used for any regular polygon.

Formula for Interior Angle Measure of a Regular Polygon

To find the measure of each interior angle, use the formula below, where n is the number of sides of the polygon.

$$\frac{(n-2)180°}{n}$$



5. **Verify** Using the regular polygons for which you measured the interior angles:
   a. Calculate the interior angle measure for each using the formula above.
   b. Compare your measurements with the values in part **a**.

Exterior Angles in Regular Polygons

6. Trace each regular polygon from the previous page, then draw an exterior angle on each. Using a protractor, measure the exterior angle of each of the polygons and record your measurements in the table below.

7. Since the polygons are regular, what conjecture can you make about the exterior angle measures at each vertex for each polygon?

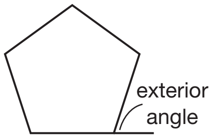
Regular Polygon	Triangle	Quadrilateral	Pentagon	Hexagon
Exterior Angle Measurements				
Sum of Exterior Angle Measurements				

8. **Analyze** Compare the exterior angle measures for each polygon. What do you notice? Does the data support your conjecture?

9. **Generalize** Add the exterior angle measures for each polygon, and record the results in the table. What do you notice?

This illustrates the fact that for all convex polygons, the sum of the exterior angles is 360°. For a regular polygon, each exterior angle measure is equivalent. Therefore, the following formula can be used to calculate the exterior angle measure in regular polygons.

Formula for Exterior Angle Measure of a Regular Polygon
To find the exterior angle measures of a regular polygon, use the formula below, where n is the number of sides in the polygon. $\frac{360°}{n}$



Math Reasoning

Generalize What are some examples of regular polygons that you see every day? Can you identify the interior angles? The exterior angles? How do they compare to the regular polygons you studied in this investigation?

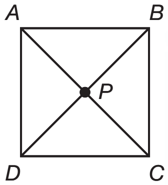
10. **Write** Using the regular polygons for which you measured the exterior angles:

a. Calculate the exterior angle measure for each using the formula above.

b. Compare the results you obtained by direct measurement with your values in part **a**.

Central Angles in Regular Polygons

The **center of a regular polygon** is the point that is equidistant from each of the polygon's vertices. A **central angle of a regular polygon** has its vertex at the center of the polygon and its sides pass through consecutive vertices.



$\angle APB$, $\angle BPC$, $\angle CPD$, and $\angle DPA$ are the four central angles in polygon $ABCD$.

11. For each of the polygons at the beginning of the investigation, sketch all of the central angles and measure each angle formed using a protractor. Record your results in the table.

12. Since the polygons are regular, what conjecture can you make about the central angles in each polygon?

Regular Polygon	Triangle	Quadrilateral	Pentagon	Hexagon
Central Angle Measurements				
Sum of Central Angle Measurements				

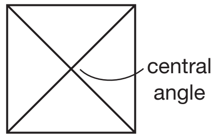
13. **Generalize** Compare the central angle measures for each polygon. What do you notice? Does the data support your conjecture?

14. Add the central angle measurements for each of the polygons and record the results in the table. What is the sum of the central angles for each polygon and what do you notice?

15. How many central angles are in each polygon?

16. What conjecture can you write regarding the number of sides in a regular polygon and the number of central angles?

For each polygon, the sum of the central angles is 360° and the measure of each central angle is directly proportional to the number of sides in the polygon. Therefore, the following formula can be used to calculate central angle measure of a regular polygon.

Formula for Central Angle Measure of a Regular Polygon
To find the measure of each central angle of a regular polygon, use the formula below, where n is the number of sides in the polygon. $\frac{360°}{n}$ 

17. Using the regular polygons for which you measured the central angles:

a. Calculate the central angle measure for each using the formula above.

b. Compare the results you obtained by direct measurement with your values in part **a**.

Investigation Practice

a. As the number of sides increases, what happens to the measure of each interior angle of a regular polygon? What happens to the measure of each exterior angle? The measure of each central angle?

b. What is the interior angle measure of a regular octagon?

c. What is the interior angle measure of a regular 20-gon?

d. Determine the measure of each exterior angle for a regular decagon.

e. Determine the measure of each central angle for a regular 30-gon.

f. **Write** Could you use the same formula to determine the interior angle measures of an irregular polygon as you did for regular polygons? Explain.

g. **Write** Could you use the same formula to determine the central angle measures of an irregular polygon as you did for regular polygons? Explain.