LESSON 31 Flowchart and Paragraph Proofs

Warm Up

1. **Vocabulary** The process of using logic to draw conclusions is called
(21) __________ reasoning.

2. **Multiple Choice** Which set of numbers is a Pythagorean triple?
(29)
   A (1, 2, 3)
   B $(1, 1, \sqrt{2})$
   C (1, 1, 1)
   D (3, 4, 5)

3. Write a conclusion based on these statements. What law was used to make
(21) this conclusion?
If the milk has gone bad, I will go to the store. The milk has gone bad.

New Concepts

A **flowchart proof** is a style of proof that uses boxes and arrows to show the structure of the proof.

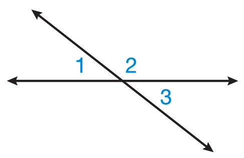
A flowchart proof should be read from left to right or from top to bottom. Each part of the proof appears in a box, while the justification for each step is written under the box. The arrows show the progression of the proof's steps.

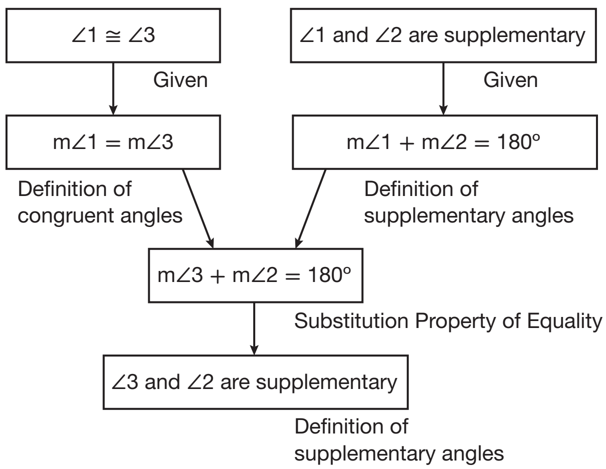
Example 1 Interpreting a Flowchart Proof

Use the given flowchart proof to write a two-column proof.

Given: ∠1 and ∠3 are congruent.
∠1 and ∠2 are supplementary.

Prove: ∠2 and ∠3 are supplementary.





Hint

A flowchart proof has all the same components as a 2-column proof, including a diagram. If you are having trouble writing or interpreting a flowchart proof, try writing it as a 2-column proof first.

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SOLUTION

Write the steps and justifications of the proof as a 2-column proof.

Statements	Reasons
1. ∠1 and ∠3 are congruent.	1. Given
2. ∠1 and ∠2 are supplementary.	2. Given
3. $m\angle 1 = m\angle 3$	3. Definition of congruent angles
4. $m\angle 1 + m\angle 2 = 180°$	4. Definition of supplementary angles
5. $m\angle 3 + m\angle 2 = 180°$	5. Substitution Property of Equality
6. ∠3 and ∠2 are supplementary.	6. Definition of supplementary angles

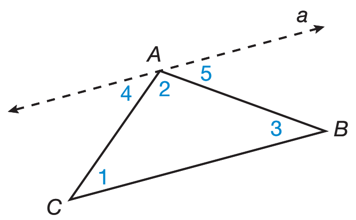
Flowchart proofs are useful when a proof has two different threads that could be performed at the same time, rather than in sequence with one another. Whenever a proof does not proceed linearly from one step to another, a flowchart proof should be considered.

Math Language

Line *a* is an **auxiliary line**. Auxiliary lines are drawn only to aid in a proof.

Example 2 Writing a Flowchart Proof

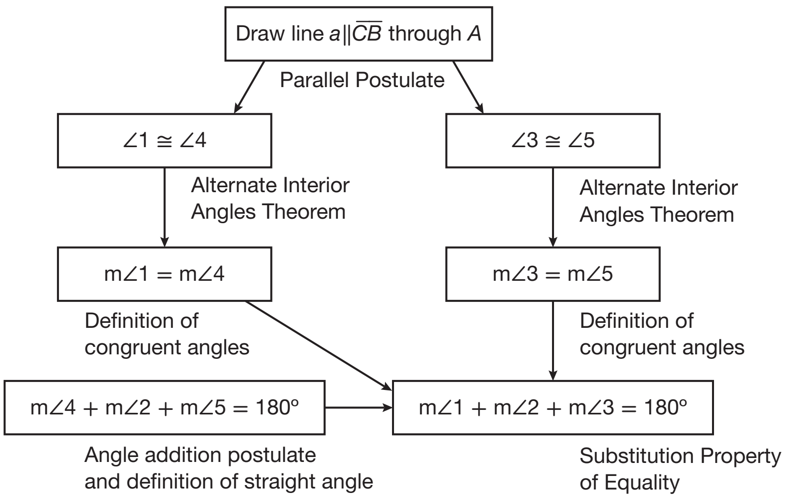
Prove the Triangle Angle Sum Theorem: The sum of the interior angles of a triangle is 180°.



Given: △*ABC*

Prove: $m\angle 1 + m\angle 2 + m\angle 3 = 180°$

SOLUTION

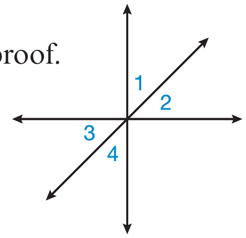


A **paragraph proof** is a style of proof in which statements and reasons are presented in paragraph form.

In a paragraph proof, every step of the proof must be explained by a sentence in the paragraph. Each sentence contains a statement and a justification.

Example 3 Reading a Paragraph Proof

Use the given paragraph proof to write a two-column proof.
Given: ∠1 and ∠2 are complementary.
Prove: ∠3 and ∠4 are complementary.



∠1 and ∠2 are complementary, so m∠1 + m∠2 = 90° by the definition of complementary angles. Angle 1 is congruent to ∠4, and ∠2 is congruent to ∠3, by the Vertical Angles Theorem. So m∠1 = m∠4, and m∠2 = m∠3. By substitution, m∠4 + m∠3 = 90°. Therefore, ∠3 and ∠4 are complementary by the definition of complementary angles.

SOLUTION
Put the steps in the paragraph proof into a two-column proof:

Statements	Reasons
1. ∠1 and ∠2 are complementary.	1. Given
2. m∠1 + m∠2 = 90°	2. Definition of complementary angles
3. ∠1 and ∠4 are congruent. ∠2 and ∠3 are congruent.	3. Vertical Angles Theorem
4. m∠1 = m∠4 and m∠2 = m∠3	4. Definition of congruent angles
5. m∠3 + m∠4 = 90°	5. Substitution Property of Equality
6. ∠3 and ∠4 are complementary.	6. Definition of complementary angles

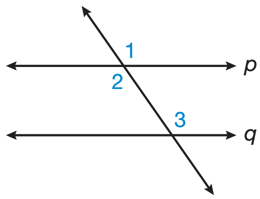
Caution

When writing a paragraph proof, make sure that every statement is accompanied by a justification. If necessary, make a 2-column proof of the most important steps as a plan.

A paragraph proof is good for short proofs where each step follows logically from the one before. Paragraph proofs are usually more compact than two-column proofs.

Example 4 Writing a Paragraph Proof

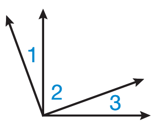
Prove Theorem 10-1: If two lines are cut by a transversal, then alternate interior angles are congruent.
Given: Lines p and q are parallel.
Prove: ∠2 ≅ ∠3

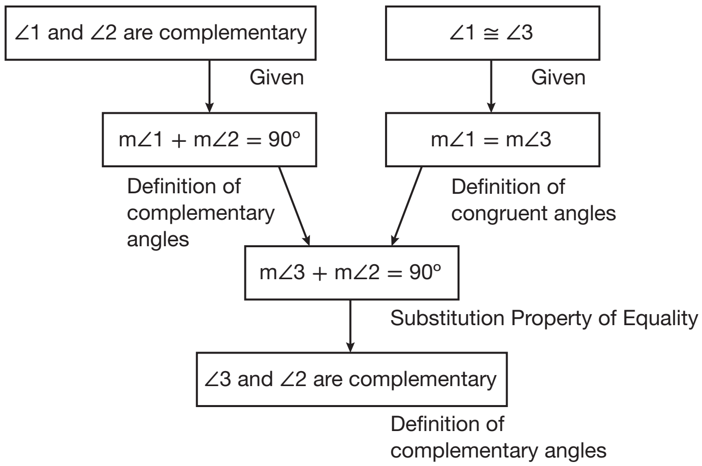


SOLUTION
It is given that lines p and q are parallel. It is known that ∠1 ≅ ∠3, by the Corresponding Angles Postulate (Postulate 11). By the Vertical Angles Theorem, ∠2 ≅ ∠1, so by the Transitive Property of Congruence, ∠2 ≅ ∠3.

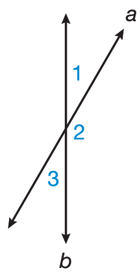
Lesson Practice

a. *(Ex 1)* Write a two-column proof from the given flowchart proof.
Given: ∠1 and ∠2 are complementary.
∠1 and ∠3 are congruent.
Prove: ∠2 and ∠3 are complementary.

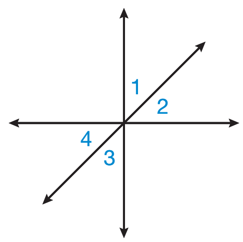




b. *(Ex 2)* Prove the Vertical Angles Theorem using a flowchart proof.
Given: a and b are intersecting lines.
Prove: ∠1 and ∠3 are congruent.

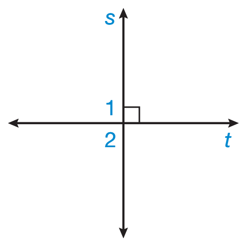


c. *(Ex 3)* Write a two-column proof from the given paragraph proof.
Given: ∠1 and ∠4 are complementary.
Prove: ∠2 and ∠3 are complementary.



∠1 and ∠4 are complementary, so m∠1 + m∠4 = 90° by the definition of complementary angles. Angle ∠1 ≅ ∠3, and ∠2 ≅ ∠4, by the Vertical Angles Theorem. Therefore, m∠1 = m∠3 and m∠2 = ∠4 by the definition of congruent angles, and m∠2 + m∠3 = 90° by substitution. Therefore, ∠2 and ∠3 are complementary by the definition of complementary angles.

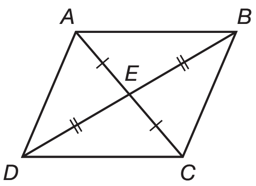
d. *(Ex 4)* Prove Theorem 5-4: If two lines are perpendicular, then they form congruent adjacent angles.
Given: Lines s and t are perpendicular.
Prove: Angles 1 and 2 are congruent adjacent angles.



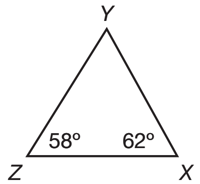
Practice Distributed and Integrated

* **1.** *(Inv 3)* **Baseball** Vernon is standing on the pitcher's mound of a baseball field. He turns from looking toward home plate to first base. If the pitcher's mound is at the center of the regular polygon made by the bases, how many degrees did Vernon rotate?

2. (28) **Analyze** In the diagram given, prove that $\overline{AD} \cong \overline{BC}$, using a two-column proof.



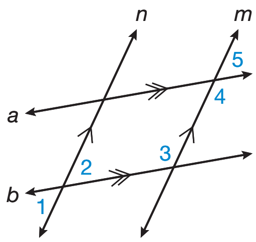
3. (18) Determine m$\angle Y$ in $\triangle XYZ$.



4. (22) **Algebra** Erin drew a parallelogram with an area of 18 square centimeters and a base of 6 centimeters. What was the height?

5. (26) If m$\widehat{AB} = 70°$, what is the measure of the associated major arc?

* **6.** (31) Write a paragraph proof.
Given: $a \parallel b$, $n \parallel m$
Prove: $\angle 1 \cong \angle 5$.



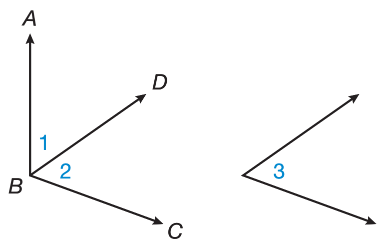
7. (25) **Algebra** $\triangle ABC$ and $\triangle DEF$ are congruent triangles with m$\angle A = 90°$, $AB = 7$, and $DF = 24$. Based on the given information, what is the length of BC?

* **8.** (29) **Home Repair** A contractor arrives at a house with a ladder that is 4 meters long. If the closest he can safely place the ladder to the house is 1.5 meters, will the top of the ladder reach the edge of the roof, which is 3.8 meters above the ground?

* **9.** (Inv 3) **Design** Whitney is making a stop sign, a regular octagon. What should be the measure of each angle of her sign?

10. (22) **Kites** Mohinder wants to make a rhombus-shaped kite with an area of 224 square inches and a frame using one diagonal crosspiece that is 14 inches long. What is the length of the other diagonal crosspiece?

***11.** (31) Write a flowchart proof.
Given: $\overrightarrow{BD}$ bisects $\angle ABC$ and $\angle 1$ is congruent to $\angle 3$.
Prove: Angle 2 is congruent to $\angle 3$.



12. (25) **Justify** If $\triangle ABC \cong \triangle LMN$ and $\triangle LMN \cong \triangle XYZ$, write the congruence statements between $\triangle ABC$ and $\triangle XYZ$. What property justifies these conclusions?

13. (27) **Landscaping** Prove that if the area of a rectangular patio $ABCD$ is 25 square units and one of the lengths is 5 units, then $ABCD$ is a square.

14. (17) Write the converse of the following statement.

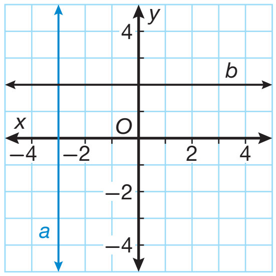
If two numbers have midpoint 0 on a number line, then they are opposites.

Is the original statement true? Is the converse?

***15.** (29) **Multiple Choice** The length of a leg of a right triangle with a hypotenuse of 20 centimeters and another leg of 12 centimeters is __________.

A 6 centimeters **B** 4 centimeters
C 16 centimeters **D** 10 centimeters

16. (16) Determine the equation of each line graphed on this grid.



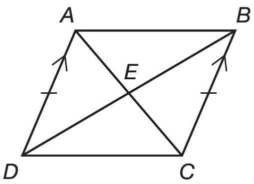
17. (21) **Meteorology** When cumulus clouds appear, there is a good chance of rain later in the day. When it rains, the air cools and descends, creating wind. What can students conclude from these statements if they see cumulus clouds forming?

***18.** (30) **Error Analysis** Suresh and Gayle are having a debate. Suresh believes that since each angle in an equilateral triangle measures 60°, all equilateral triangles are congruent. Gayle disagrees since she can draw two equilateral triangles that are different sizes. Who is correct? Explain.

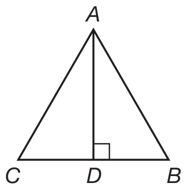
19. (14) **Algebra** Find a counterexample to the conjecture.

A linear equation in one unknown has exactly one solution.

***20.** (30) Using the diagram given, prove that $\overline{AE} \cong \overline{CE}$.



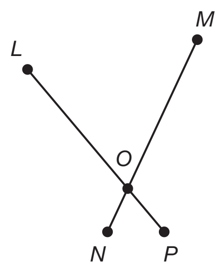
***21.** (31) Write a paragraph proof.
Given: $\overline{AD}$ bisects $\overline{CB}$ and $\overline{AD} \perp \overline{CB}$.
Prove: $\triangle ACD \cong \triangle ABD$.



22. (24) City Hall is located at (1, 1) on a coordinate grid of a city. The police station is located at point $(x - 3, 4x + 1)$. If the police station is 5 units away from City Hall and x is positive, what are the numeric coordinates of the police station? Provide a justification for each step.

23. (16) Write an equation for the line passing through (−3, 2) that has slope 4.

***24.** (31) Write a flowchart proof.
Given: $\overline{LP}$ is congruent to $\overline{MN}$ and $\overline{LO}$ is congruent to $\overline{MO}$.
Prove: $\overline{OP}$ is congruent to $\overline{ON}$.



***25.** (28) **Generalize** If $\triangle ABC$ and $\triangle DEF$ are right triangles, $\angle B$ and $\angle E$ are right angles, $AC = DF$, and $AB = DE$, is it possible to conclude that $\triangle ABC \cong \triangle DEF$? If yes, use the SAS Theorem to prove it. If no, explain why.

26. (13) **a.** A right triangle has a base length of 1.5 meters and a height of 1.2 meters. Determine its area.

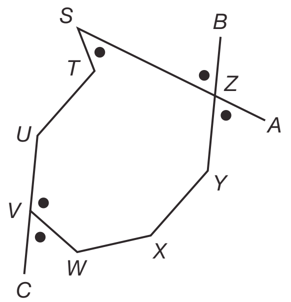
b. The other side of the same triangle has a length of 0.81 meters. Determine the triangle's perimeter.

***27.** (20) **a.** **Generalize** Rewrite the statement, "If p, then q" as a compound statement involving p, q, and/or their negations.

b. Write the negation of the statement, "p or q" as a compound statement involving p, q, and/or their negations.

c. Write the negation of your answer to part **a** as a compound statement involving p, q, and/or their negations.

28. (15) In the figure below, name each angle marked with a dot and identify it as an interior or exterior angle.



29. (23) **Verify** For a circle with a diameter of 13 centimeters, Sal calculated the circumference to be 81.64 centimeters. Is Sal's calculation correct?

xy^2 ***30.** (30) **Algebra** If in $\triangle ABC$, $AB = (3x + 11)$, $m\angle ABC = 45°$, $m\angle BCA = 75°$, and $BC = 15$, and in $\triangle DEF$, $DE = (7x - 9)$, $m\angle DEF = 45°$, $m\angle EFD = 75°$, and $EF = 15$, what is the value of x?

LESSON 32

Altitudes and Medians of Triangles

Warm Up

1. **Vocabulary** A triangle with three congruent angles is called a(n) __________ triangle. (13)

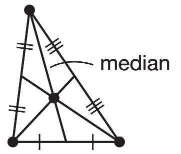
2. Is this statement true or false? (13)
The height of a triangle is always a segment in the interior of the triangle.

3. What is the midpoint of the points (−3, 4) and (2, 8)? (11)

4. What is the midpoint of the points (4, 2) and (−1, 3)? (11)

New Concepts

The **median of a triangle** is a segment whose endpoints are a vertex of the triangle and the midpoint of the opposite side.



centroid
intersection of the medians

Every triangle has one median for each vertex, or three medians total. The point of concurrency of these three medians also has a special name. The **centroid of a triangle** is the point of concurrency of the three medians of a triangle. This point is also called the center of gravity. The three medians of a triangle are always concurrent lines.

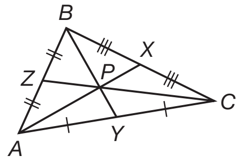
Math Language

A **point of concurrency** is a point where three or more lines intersect. When three or more lines intersect at one point, they are called **concurrent lines**.

Caution

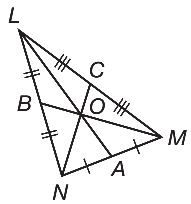
Be careful not to flip the equalities given by the Centroid Theorem. Remember that the centroid is always closer to the side than to the vertex.

Theorem 32-1: Centroid Theorem
The centroid of a triangle is located $\frac{2}{3}$ the distance from each vertex to the midpoint of the opposite side. In $\triangle ABC$: $CP = \frac{2}{3}CZ$ $BP = \frac{2}{3}BY$ $AP = \frac{2}{3}AX$



Example 1 Using the Centroid to Find Segment Lengths

In $\triangle LMN$, $LA = 12$ and $OC = 3.1$. Find LO.



SOLUTION

$LO = \frac{2}{3}LA$ Centroid Theorem

$LO = \frac{2}{3}(12)$ Substitution Property of Equality

$LO = 8$ Simplify.

Find the length of $\overline{NC}$.

SOLUTION

$NO = \frac{2}{3}NC$ Centroid Theorem

$NO + OC = NC$ Segment Addition Postulate

$\frac{2}{3}NC + 3.1 = NC$ Substitution Property of Equality

$3.1 = \frac{1}{3}NC$ Subtraction Property of Equality

$9.3 = NC$ Multiplication Property of Equality

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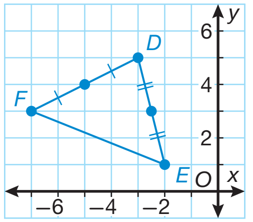
Example 2 Finding a Centroid on the Coordinate Plane

Find the centroid of $\triangle DEF$ with vertices at $D(-3, 5)$, $E(-2, 1)$, and $F(-7, 3)$.

SOLUTION



Graph the triangle. Start by finding the midpoint of each segment of the triangle.

The midpoint of $\overline{DF}$ is $(-5, 4)$ and the midpoint of $\overline{DE}$ is $(-2.5, 3)$.

Since all three medians meet at the same point, the intersection of any two will give the location of the centroid.

Use the two points on each median to find an equation for the medians. The slope of the median that extends from E to $(-5, 4)$ is -1.

$y - y_1 = m(x - x_1)$	Point-slope formula
$y - 4 = -1(x + 5)$	Substitute.
$y = -x - 1$	Simplify.

The equation of the median from F to $(-2.5, 3)$ is $y = 3$. Solve these two equations as a system.

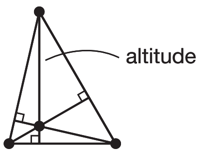
$$y = -x - 1 \qquad\qquad y = 3$$
$$3 = -x - 1$$
$$x = -4$$

We already know the y-coordinate is 3, so the centroid is located at $(-4, 3)$.

Hint

Try to pick a median with a simple equation to make it easier to find the centroid. In this example, one of the medians is horizontal, so the equation, $y = 3$, is easy to find.

The **altitude of a triangle** is a perpendicular segment from a vertex to the line containing the opposite side. The **orthocenter of a triangle** is the point of concurrency of the three altitudes of a triangle.

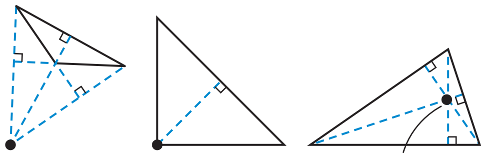


orthocenter
intersection of the altitudes

Example 3 Locating the Orthocenter of a Triangle

Draw an acute triangle, a right triangle, and an obtuse triangle. Sketch the altitudes of each triangle and find their orthocenters. Is the orthocenter always in the interior of a triangle?

SOLUTION



The diagram shows an acute, obtuse, and right triangle. From the diagram, the orthocenter of the obtuse triangle is in the exterior of the triangle. So orthocenters are not always in the interior of triangles.

Example 4 Application: Balancing Objects

The centroid of a triangle is the point where the triangle can be balanced on a point. Suppose a student wants to perform a balancing act with triangles as part of a talent show. Each of the triangles are the same size, and the three medians are 4.2 inches, 7.2 inches, and 8.1 inches long, respectively. What is the distance from each vertex to the centroid of each of these triangles?

SOLUTION

Use the Centroid Theorem. The distance from each vertex to the centroid is two-thirds the length of the median.

$4.2 \times \frac{2}{3} = 2.8$

$7.2 \times \frac{2}{3} = 4.8$

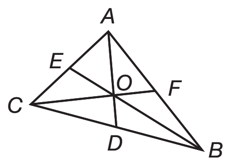
$8.1 \times \frac{2}{3} = 5.4$

The balancing point, or center of gravity, for each triangle is 2.8 inches, 4.8 inches, and 5.4 inches away from their respective vertices.

> **Hint**
>
> Remember that the centroid is also known as the center of gravity. This is due to the fact that a triangle suspended or held up by the centroid will be perfectly level.

Lesson Practice

a. *(Ex 1)* In $\triangle ABC$, $AD = 5$ and $EO = 4.2$. Use the Centroid Theorem to find the lengths of $\overline{OD}$ and $\overline{BE}$ to the nearest hundredth.



b. *(Ex 2)* Use the coordinate plane to find the centroid of $\triangle JKL$ with vertices $J(-9, 1)$, $K(-1, 5)$, and $L(-5, 9)$.

c. *(Ex 3)* Where is the orthocenter of a right triangle located?

d. *(Ex 4)* Clara is hanging triangles from a mobile. She needs to find the centroid of each triangle for the mobile to hang correctly. If the triangles have medians of 3.6 inches, 6.9 inches, and 4.5 inches respectively, how far is the centroid from each vertex?

Practice Distributed and Integrated

1. *(26)* **Formulate** An arc measure of 180° creates a semicircle. Use this fact to derive a formula for the area of a semicircle.

2. *(19)* Three sides of convex quadrilateral *ABCD* are congruent, but the fourth is not. What possible shape(s) could *ABCD* be?

3. *(22)* xy^2 **Algebra** A trapezoid has an area of 22, a height of 4, and bases of $x + 2$ and $3x - 3$. Solve for x.

4. *(7)* Make a conjecture about the next item in the list and explain the basis of your conjecture.

Triangle, square, pentagon, hexagon, heptagon…

*** 5.** *(32)* A median of a triangle connects a vertex and the ________ of the opposite side.

6. *(15)* **Marketing** In the diagram, a corporate logo is designed in the shape shown at right.

a. Name the polygon.

b. Determine whether the polygon is equiangular, equilateral, regular, irregular, or more than one of these.

c. Determine whether the polygon is concave or convex. Explain.

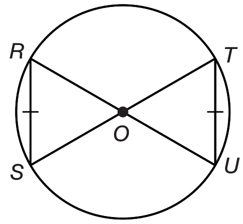
7. *(14)* Consider the conjecture.

If △MNO *is an acute triangle,* D *lies on* $\overline{MN}$*, and* E *lies on* $\overline{MO}$*, then* △MDE *is an acute triangle.*

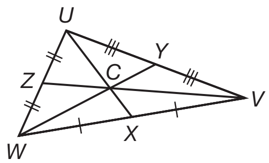
a. What is the hypothesis of the conjecture? What is its conclusion?

b. Find a counterexample to the conjecture.

8. *(27)* In the diagram given, $\overline{RU}$ and $\overline{ST}$ are diameters of the circle and O is the center of the circle. Prove that $m\angle SRO = m\angle UTO$.



Use the diagram to answer the next two questions.

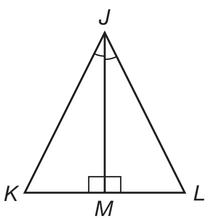


*** 9.** *(32)* If the length of $\overline{ZV}$ is 72, determine the length of $\overline{VC}$.

***10.** *(32)* If the length of $\overline{CX}$ is 18, determine the length of $\overline{UC}$.

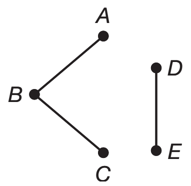
11. *(28)* **Designing** Spencer is making two triangular flags, $\triangle ABC$ and $\triangle DEF$, for the upcoming football game. Given that if $\triangle ABC$ is a right triangle at $\angle B$ and $\triangle DEF$ is a right triangle at $\angle E$ with $\overline{AB} \cong \overline{DE}$ and $\overline{BC} \cong \overline{EF}$, which congruence postulate would prove $\triangle ABC \cong \triangle DEF$?

***12.** *(30)* Using the diagram shown, prove $KM = LM$.



13. *(22)* **Art** Using a technique known as vanishing point to give a three-dimensional perspective to the painting, an artist drew a rectangular table as a trapezoid with a height of 3 inches and with upper and lower bases of 3 and 4 inches, respectively. What is the area of the painting of the table?

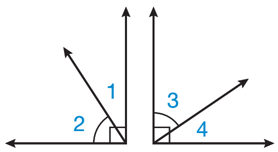
***14.** Write a paragraph proof.
(31)
Given: $\overline{AB} \cong \overline{BC}$ and $\overline{BC} \cong \overline{DE}$
Prove: $\overline{AB} \cong \overline{DE}$



15. Prove that if all angles in $\triangle PQR$ are equal, then each angle is 60°.
(27)

16. **Error Analysis** In calculating the length of a hypotenuse, Beatrice and Amelie both got an answer of $\sqrt{360}$. However, when asked to provide their answer in simplified radical form, Beatrice's answer was $36\sqrt{10}$ and Amelie's was $6\sqrt{10}$. Who is incorrect and why?
(29)

***17.** Write a paragraph proof.
(31)
Given: Angle 1 and ∠2 are complementary.
Angle 3 and ∠4 are complementary.
∠2 ≅ ∠3
Prove: ∠1 ≅ ∠4



18. **Algebra** Two vertical angles measure $(4x - 4)°$ and $(3x + 7)°$. What is the measure of each angle? Provide a justification for each step.
(24)

19. Consider the following conjecture.
(14)

If the product of three numbers is positive, then all three numbers are positive.

a. What is the hypothesis of the conjecture? What is its conclusion?
b. Find a counterexample to the conjecture.

20. Use the Law of Detachment to fill in the missing statement in this conjecture.
(21)

____

M is the midpoint of $\overline{WX}$.
Therefore, $WM = MX$.

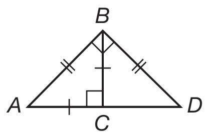
21. **Verify** A triangle has an exterior angle measuring 98°. One remote interior angle measures 49° and the other remote angle is congruent to it. Verify the Exterior Angle Theorem for this exterior angle.
(18)

22. **Machinery** Two gears are interlocked. One has a radius of 10 centimeters and for each complete rotation, it rotates the second gear 0.77 of a full turn.
(23)
a. What is the relationship between the circumferences of the two gears?
b. What is the second gear's radius, to the nearest centimeter?

23. Prove that if the heights of two parallelograms, h_1 and h_2, lie on the same parallel lines and have the same base, b, then the parallelograms are equal in area.
(27)

***24.** **Multiple Choice** If the length of a median of a triangle equals 4.5, what is the distance from the centroid to the opposite side?
(32)
A 4.5 **B** 3
C 2.25 **D** 1.5

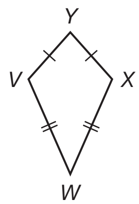
25. (28) **Analyze** In the diagram given, prove that $\triangle ABC \cong \triangle BDC$ using the SAS Triangle Congruence Theorem. *Hint: Show m∠BAC = m∠DBC.*



26. (17) **Generalize** Complete the table by writing the contrapositive of each statement.

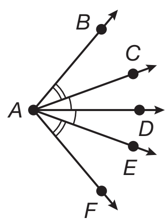
Statement	Contrapositive
If p, then q	If $\sim q$, then $\sim p$
If $\sim p$, then q	
If p, then $\sim q$	
If $\sim p$, then $\sim q$	

***27.** (30) **Design** Ian is making a kite he has labeled *VWXY*. He has already made $m\angle VWY = m\angle XWY$. What pair of angles can he make equal to ensure $\triangle VWY \cong \triangle XWY$?



28. (25) $\triangle HIJ \cong \triangle LKN$ and $m\angle H = 50°$, $m\angle K = 100°$, and $HI = 5$ centimeters. What is $m\angle J$?

***29.** (31) Write a flowchart proof. Refer to the figure shown.
Given: $m\angle BAC = m\angle EAF$, $m\angle CAD = m\angle DAE$
Prove: $m\angle BAD = m\angle DAF$



30. (20) **Generalize** What is the negation of the statement, "If p, then q?" Write a truth table to help you.

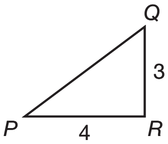
LESSON

33 Converse of the Pythagorean Theorem

Warm Up

1. (10) **Vocabulary** A statement formed by exchanging the hypothesis and conclusion of a conditional statement is called the _________.

2. (Inv 2) Find the length of $\overline{PQ}$.



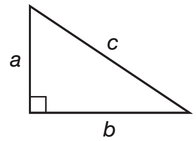
3. (10) Write the converse of the conditional statement. Is the converse true?
If two angles are vertical angles, then they are congruent.

New Concepts

In Investigation 2, the Pythagorean Theorem is presented. The Pythagorean Theorem states that for any right triangle, the sum of the squares of the lengths of its two shortest sides is equal to the square of the length of its longest side.

As an equation, the Pythagorean Theorem reads: $a^2 + b^2 = c^2$.

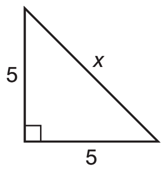
Each of the shorter sides is called a leg, and the longest side is called the hypotenuse.



Example 1 Applying the Pythagorean Theorem

Find the value of x. Write the answer in simplified radical form.

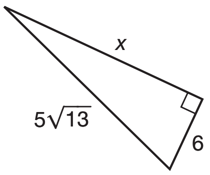
a.



SOLUTION

$a^2 + b^2 = c^2$	Pythagorean Theorem
$5^2 + 5^2 = c^2$	Substitute.
$50 = c^2$	Simplify.
$c = \sqrt{50} = 5\sqrt{2}$	Solve for c.

b.



SOLUTION

$a^2 + b^2 = c^2$	Pythagorean Theorem
$6^2 + b^2 = \left(5\sqrt{13}\right)^2$	Substitute.
$36 + b^2 = 325$	Simplify.
$b = 17$	Solve for b.

Hint

Remember that you can simplify radicals by factoring out perfect squares.

$2^2 = 4$	$6^2 = 36$
$3^2 = 9$	$7^2 = 49$
$4^2 = 16$	$8^2 = 64$
$5^2 = 25$	$9^2 = 81$

The converse of the Pythagorean Theorem is true and can be used to determine whether or not a triangle is a right triangle.

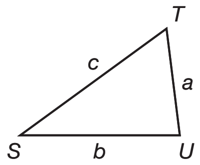
Theorem 33-1: Converse of the Pythagorean Theorem
If the sum of the squares of the two shorter sides of a triangle is equal to the square of the longest side of the triangle, then the triangle is a right triangle.

Hint

In order to do this proof, draw another right triangle with the same side lengths as $\triangle STU$. Sometimes proofs require that a figure be added to the given diagram.

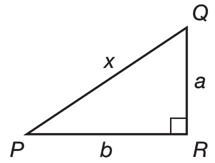
Example 2 Proving the Converse of the Pythagorean Theorem

Given: $a^2 + b^2 = c^2$
Prove: $\triangle STU$ is a right triangle.



SOLUTION

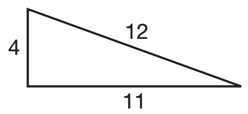
Draw right triangle $\triangle PQR$ with leg lengths identical to $\triangle STU$ and a third side of length x. In $\triangle STU$, it is given that $a^2 + b^2 = c^2$. In $\triangle PQR$, $a^2 + b^2 = x^2$ by the Pythagorean Theorem. Since $a^2 + b^2 = c^2$ and $a^2 + b^2 = x^2$, it follows by substitution that $x^2 = c^2$. Take the positive square root of both sides, and $x = c$. So $ST = PQ$, $TU = QR$, and $SU = PR$. By the definition of congruent segments, $\overline{ST} \cong \overline{PQ}$, $\overline{TU} \cong \overline{QR}$, and $\overline{SU} \cong \overline{PR}$. Therefore, $\triangle STU \cong \triangle PQR$ by SSS Triangle Congruence, and $\angle U \cong \angle R$ by CPCTC. Since $\angle R$ is a right angle, $\angle U$ is a right angle by the definition of congruent angles, and $\triangle STU$ is a right triangle by the definition of right triangles.



Example 3 Applying the Converse of the Pythagorean Theorem

Determine whether each triangle is a right triangle.

a.



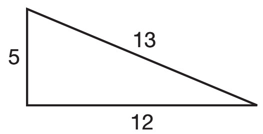
SOLUTION

Use the Pythagorean Theorem.

$a^2 + b^2 = c^2$	Pythagorean Theorem
$4^2 + 11^2 \stackrel{?}{=} 12^2$	Substitute.
$137 \neq 144$	Add and compare.

This triangle is not a right triangle by the Converse of the Pythagorean Theorem.

b.



> **Reading Math**
>
> The symbol $\stackrel{?}{=}$ is used to show that it is not known if an equation is true or false. In the last step of this problem, if the equation is true, the question mark is not used.

SOLUTION

Use the Pythagorean Theorem.

$a^2 + b^2 = c^2$	Pythagorean Theorem
$5^2 + 12^2 \stackrel{?}{=} 13^2$	Substitute.
$169 = 169$	Add and compare.

This triangle is a right triangle by the Converse of the Pythagorean Theorem.

The Pythagorean Theorem can also be used to determine if a triangle is an acute triangle or an obtuse triangle by using inequalities. This is called the Pythagorean Inequality Theorem.

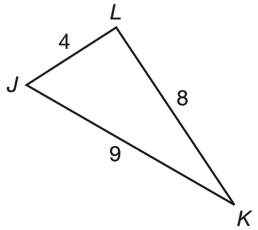
Theorem 33-2: Pythagorean Inequality Theorem

In a triangle, let a and b be the lengths of the two shorter sides and let c be the length of the longest side. If $a^2 + b^2 < c^2$, then the triangle is obtuse. If $a^2 + b^2 > c^2$, then the triangle is acute.

Example 4 Classifying Triangles Using the Pythagorean Inequality Theorem

Determine whether $\triangle JKL$ is an obtuse, acute, or a right triangle.

a.



SOLUTION

The longest side is $\overline{JK}$, so it will represent c, with the other two sides representing a and b.

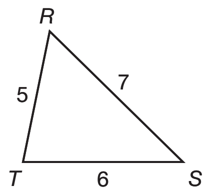
$a^2 + b^2 = c^2$

$4^2 + 8^2 \stackrel{?}{=} 9^2$

$80 < 81$

Since $a^2 + b^2 < c^2$, the triangle is an obtuse triangle.

b.



SOLUTION

$a^2 + b^2 = c^2$

$6^2 + 5^2 \stackrel{?}{=} 7^2$

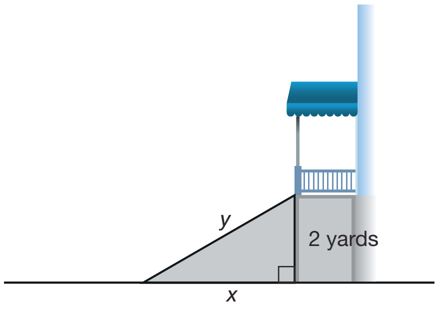
$61 > 49$

Since $a^2 + b^2 > c^2$, the triangle is an acute triangle.

Example 5 Application: Building a Wheelchair Ramp

> **Math Reasoning**
>
> **Justify** Suppose instead that a contractor knows the ramp goes to a platform that is 2 yards high, 8 yards long, and 7 yards along the base. If the ramp does not make a 90° angle with the ground, it needs to be rebuilt. Does the ramp need to be rebuilt? If so, why?

The local library needs a ramp for wheelchair access. If the city says that the ramp should be built with a slope of $\frac{1}{4}$, and must reach a platform that is 2 yards high, what must be the length of the ramp? Round to the nearest hundredth.



SOLUTION

Slope is in the form $\frac{\text{rise}}{\text{run}}$. The ramp rises 2 yards. Let x be its run.

$\frac{\text{rise}}{\text{run}} = \frac{1}{4} = \frac{2}{x}$ Substitute a rise of 2 and a run of x.

$x = 8$ Cross multiply.

Therefore, the run of the ramp x is equal to 8 yards.

Now use the Pythagorean Theorem to solve for y.

$x^2 + 2^2 = y^2$ Pythagorean Theorem

$8^2 + 2^2 = y^2$ Substitute.

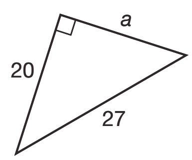
$64 + 4 = y^2$ Simplify.

$y \approx 8.25$ Solve for y.

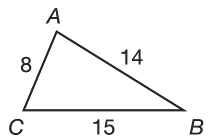
The ramp needs to be about 8.25 yards long.

Lesson Practice

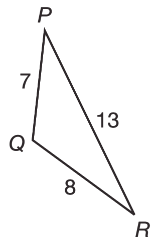
a. Use the Pythagorean Theorem to solve for the missing side length. Give your answer in simplified radical form.
(Ex 1)



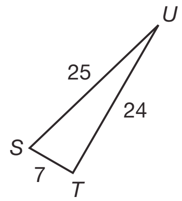
b. Is $\triangle ABC$ a right triangle?
(Ex 3)



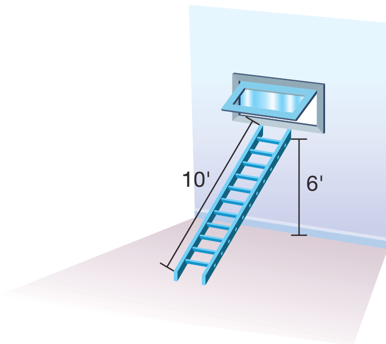
c. Is $\triangle PQR$ obtuse, acute, or right?
(Ex 4)



d. Is $\triangle STU$ obtuse, acute, or right?
(Ex 4)

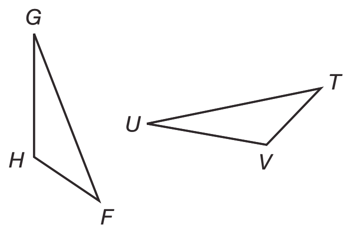


e. A ladder leans against a wall. Its top reaches a window that is 6 feet above the ground and the bottom of the ladder is 7.5 feet from the base of the wall, measured along the ground. Do the ladder, the wall, and the ground form a right triangle? Explain.
(Ex 5)



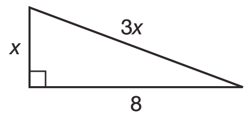
Practice Distributed and Integrated

1. The two triangles at right are congruent. Write the six congruency statements for them.
(25)



2. In $\triangle ABC$, $m\angle BAC = 57°$, $AB = 10$, and $m\angle ABC = 52°$. In $\triangle DEF$, $m\angle EDF = 57°$, $DE = 10$, and $m\angle DFE = 71°$. Is $\triangle ABC \cong \triangle DEF$? Explain.
(30)

*** 3.** **Multi-Step** What is the value for x if the triangle is a right triangle? Write an inequality to show the smallest value for x that makes the triangle an obtuse triangle. Write your answer in simplified radical form.
(33)



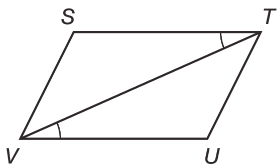
4. Write the converse of the following false statement.
(17)

If a triangle is isosceles, then it is obtuse.

*** 5.** Write a paragraph proof.
(31)

Given: $\angle STV \cong \angle TVU$, $\angle STU \cong \angle UVS$

Prove: $\angle SVT \cong \angle UTV$



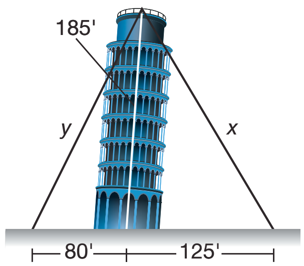
6. The angles of $\triangle ABC$ are $m\angle BAC = 9x - 77°$, $m\angle ABC = 5x + 17°$, and $m\angle ACB = 6x - 33°$. If $m\angle DEF = 90°$, $\overline{AB} \cong \overline{DE}$, and $\overline{BC} \cong \overline{EF}$, is $\triangle ABC \cong \triangle DEF$?
(28)

7. Find an expression for the area of a parallelogram with a height of $2y$ and a base of $3x$.
(22)

*** 8.** **Architecture** The Leaning Tower of Pisa in Italy would be 185 feet tall if it were standing vertically.
(33)

a. If you are standing 80 feet from the base of the tower, and the tower is leaning away from you, what type of triangle would be formed? What is the minimum length of y?

b. If you are standing 125 feet from the base of the tower, and the tower is leaning toward you, what type of triangle would be formed? What is the maximum length of x?

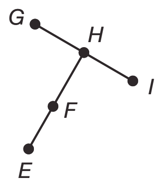


9. **Verify** Write the inverse of the statement, "If a triangle does not have an obtuse angle, then it is acute." Which is true: the statement, its inverse, both, or neither? Explain.
(17)

***10.** Write a flowchart proof.
(31)

Given: $\overline{EF} \cong \overline{HI}$, H is the midpoint of $\overline{GI}$

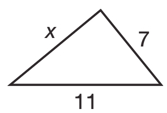
Prove: $\overline{EF} \cong \overline{GH}$



11. **a.** **Generalize** How many different radii does a circle have?
(23)

b. How many different measures do the radii of a circle have?

***12.** **Algebra** Write an inequality to show the largest value for x that makes the triangle obtuse if the longest side length is 11.
(33)



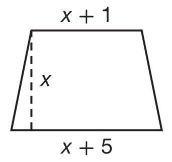
13. (16) **Write** Explain how the slope-intercept form of a line can be used to sketch its graph, without creating a table of values.

14. (20) **Geology** A volcano is active if it has erupted recently and is expected to do so again soon. It is dormant if it is not active, but is expected to erupt again at some future time. It is extinct if it is not ever expected to erupt again.

a. Write the conjunction of the statements, "A volcano is active," and "A volcano is not ever expected to erupt again." Is this conjunction true? Explain.

b. Write the disjunction of the same two statements. Is the disjunction true? Why or why not?

15. (24) **Gardening** Ashanti has a 40-square foot garden in the shape of a trapezoid. If the diagram shows the garden's dimensions, what are the lengths of the two bases? Provide a justification for each step.

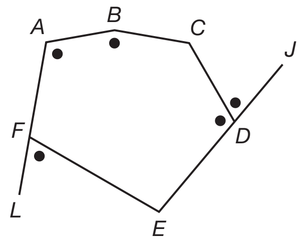


***16.** (32) **Multiple Choice** The altitudes are used to find which of the following?

A orthocenter **B** median

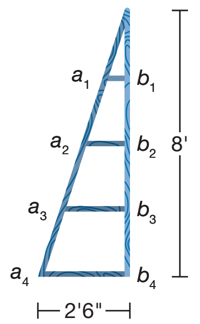
C area **D** centroid

17. (15) In the figure, name each angle marked with a dot and identify it as an interior or exterior angle.



18. (Inv 3) **Algebra** A convex polygon has interior angles $(x + 4)°$, $(2x + 5)°$, $(3x + 6)°$, $(4x + 7)°$, and $(5x + 8)°$. What is the value of x?

***19.** (29, 33) **Furniture** The height of a triangular cabinet is 8 feet. There are four shelves which are evenly spaced. What is the distance from the top of the cabinet to each of the shelves along the hypotenuse, to the nearest inch?

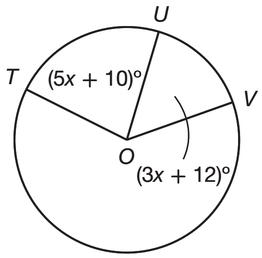


20. (21) **Error Analysis** From the statements, "If a, then b. If c, then b. a is a true statement," a student incorrectly concluded, "c is a true statement." What mistake did the student make? What conclusion should have been made?

21. *(Inv 3)* What is the measure of each exterior angle in a regular dodecahedron?

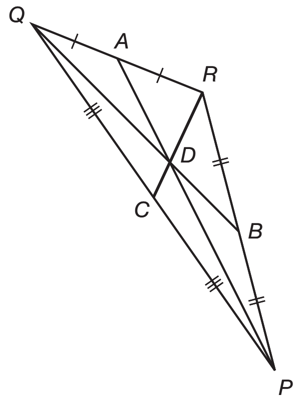
22. *(18)* **Justify** An exterior angle of a triangle measures 136°. One remote interior angle measures 56°. Determine the other two interior angle measures, justifying each step in your reasoning.

23. *(26)* **Algebra** In the diagram, $m\overset{\frown}{TU} = (5x + 10)°$, $m\overset{\frown}{UV} = (3x + 12)°$ and $m\overset{\frown}{TV} = (6x + 50)°$. What is the value of x?



24. *(16)* **Write** Describe how to write the equation of a line, given that its slope is −1 and that the point (4, 5) lies on it.

Use the diagram to answer the following questions.



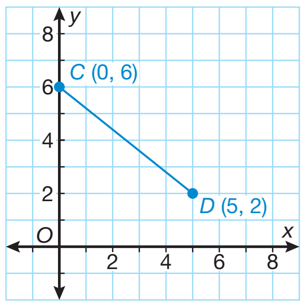
***25.** *(32)* Given that $AP = 123$, determine the length of $\overline{AD}$.

***26.** *(32)* Given that $BD = 63$, determine the length of $\overline{DQ}$.

27. *(29)* **Gardening** Julian installed a total of 10 yards of fence which runs along the perimeter of his square garden. He decides to run a watering hose between two opposite corners. What length of hose will he need, to the nearest tenth of a yard?

28. *(26)* If $m\overset{\frown}{AB} = 23°$ and $m\overset{\frown}{BC} = 33°$, what is the measure of $\overset{\frown}{AC}$ if $\overset{\frown}{AB}$ and $\overset{\frown}{BC}$ are non-overlapping, adjacent arcs?

29. *(11)* Determine the midpoint M of the line segment $\overline{CD}$ with endpoints $C(0, 6)$ and $D(5, 2)$.



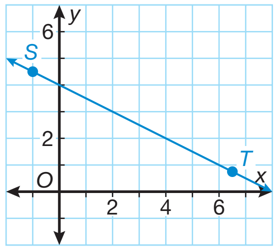
30. *(23)* What is the circumference of a circle with a radius measure of $7x$? Express your answer in terms of π.

LESSON
34 Properties of Parallelograms

Warm Up

1. **Vocabulary** The __________ of a line divides a segment into two congruent segments. *(2)*

2. Find the equation of line $\overline{ST}$. *(16)*



3. What is the name for an equiangular and equilateral quadrilateral? *(19)*

   A parallelogram **B** rhombus
   C square **D** trapezium

New Concepts

Squares, rhombuses, and rectangles are all types of parallelograms. All of them share some basic properties of parallelograms.

Exploration: Exploring Diagonals of Parallelograms

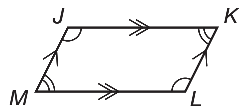
1. Draw a parallelogram *ABCD*. Trace it into a piece of patty paper and label the new figure *QRST*. Notice that $ABCD \cong QRST$.
2. Lay *QRST* over *ABCD* so that $\overline{ST}$ overlays $\overline{AB}$. What do you notice about their lengths? What do you suppose is the relationship between $\overline{AB}$ and $\overline{CD}$? What does this suggest about $\overline{AD}$ and $\overline{BC}$?
3. Lay *QRST* over *ABCD* so that $\angle S$ overlays $\angle A$. What do you notice about their measures? What do you suppose is the relationship between $\angle A$ and $\angle C$? What does this suggest about $\angle B$ and $\angle D$?
4. Draw diagonals $\overline{AC}$ and $\overline{BD}$. Fold *ABCD* so that *A* overlays *C*, making a crease which represents points that are equidistant from *A* and *C*. Unfold the paper and fold it again so that *B* overlays *D*, making another crease. What do you notice about the creases? What can you conclude about the diagonals?

Math Reasoning

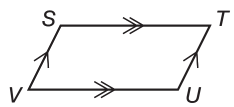
Generalize In what kind of parallelogram are adjacent sides congruent? … adjacent angles?

Properties of Parallelograms

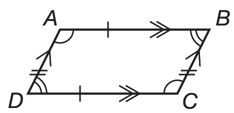
1. If a quadrilateral is a parallelogram, then its opposite angles are congruent.
   $\angle J \cong \angle L$ $\angle M \cong \angle K$



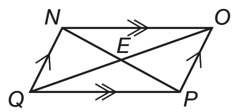
2. If a quadrilateral is a parallelogram, then its consecutive angles are supplementary.
   $m\angle S + m\angle T = 180°$ $m\angle T + m\angle U = 180°$
   $m\angle U + m\angle V = 180°$ $m\angle V + m\angle S = 180°$



3. If a quadrilateral is a parallelogram, then its opposite sides are congruent.
   $\overline{AD} \cong \overline{BC}$ $\overline{AB} \cong \overline{DC}$



4. If a quadrilateral is a parallelogram, then its diagonals bisect each other.
   $NE = EP$ $QE = EO$



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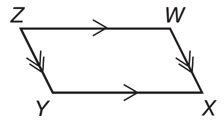
Hint
Lines 6 and 8 contain the final statements for this proof. This proof might make a good flowchart proof since the concluding statements would be presented at the same time.

Example 1 Proving Opposite Angles of a Parallelogram Are Congruent

Given: $WXYZ$ is a parallelogram.
Prove: $m\angle W = m\angle Y$, $m\angle X = m\angle Z$

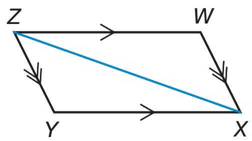


SOLUTION

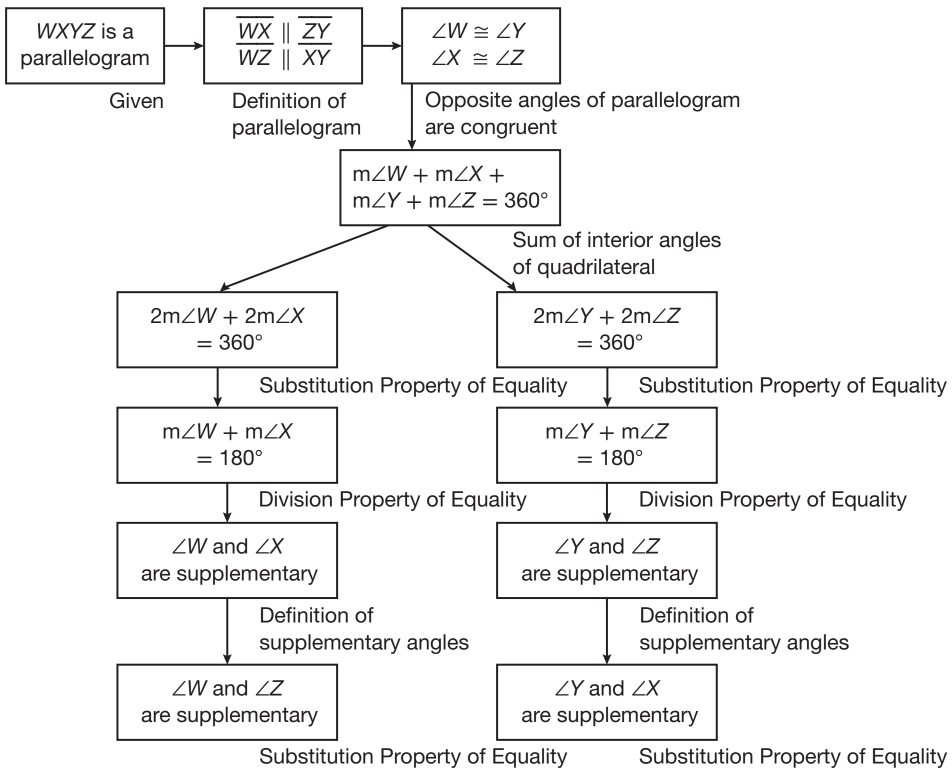
Statements	Reasons
1. $WXYZ$ is a parallelogram	1. Given
2. $\overline{WX} \parallel \overline{ZY}$ and $\overline{WZ} \parallel \overline{XY}$	2. Definition of parallelograms
3. $m\angle W + m\angle X = 180°$ $m\angle Z + m\angle Y = 180°$	3. Same-side interior angles are supplementary
4. $m\angle W + m\angle Z = 180°$ $m\angle X + m\angle Y = 180°$	4. Same-side interior angles are supplementary
5. $m\angle X + m\angle Y = m\angle W + m\angle X$	5. Substitution Property of Equality
6. $m\angle Y = m\angle W$	6. Subtraction Property of Equality
7. $m\angle Z + m\angle Y = m\angle X + m\angle Y$	7. Substitution Property of Equality
8. $m\angle Z = m\angle X$	8. Subtraction Property of Equality

Example 2 Proving Consecutive Angles of a Parallelogram Are Supplementary

Given: $WXYZ$ is a parallelogram.
Prove: $\angle W$ and $\angle X$, $\angle Z$ and $\angle Y$, $\angle W$ and $\angle Z$, and $\angle X$ and $\angle Y$ are supplementary.



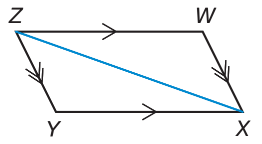
SOLUTION



Caution

When two or more angles have a common vertex, three letters must be used to name each angle.

Example 3 Proving Opposite Sides of a Parallelogram Congruent



Given: $WXYZ$ is a parallelogram.
Prove: $\overline{WZ} \cong \overline{XY}$, $\overline{WX} \cong \overline{ZY}$

SOLUTION

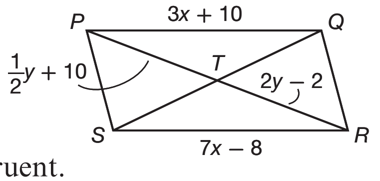
$WXYZ$ is a parallelogram, so $\overline{WZ} \parallel \overline{XY}$ and $\overline{WX} \parallel \overline{ZY}$. The diagonal $\overline{ZX}$ is congruent to itself by the Reflexive Property. Since $\overline{ZX}$ is a transversal for both sets of parallel lines, $\angle WXZ \cong \angle XZY$ and $\angle ZXY \cong \angle WZX$. Since two angles and the included side of $\triangle WZX$ are congruent to two angles and their included side in $\triangle YXZ$, $\triangle WZX \cong \triangle YXZ$ by ASA Triangle Congruence. By CPCTC, $\overline{WZ} \cong \overline{XY}$ and $\overline{WX} \cong \overline{ZY}$.

Math Reasoning

Model Draw a parallelogram, a rhombus, a square, and a rectangle. Which of these parallelograms, if any, appear to have congruent diagonals?

Example 4 Finding Unknown Measures of a Parallelogram

$PQRS$ is a parallelogram.



a. Find the value of x.

SOLUTION

Opposite sides of parallelograms are congruent.
Therefore, $PQ = SR$.

$3x + 10 = 7x - 8$
$x = 4.5$

b. Find the value of y.

SOLUTION

Diagonals of a parallelogram bisect each other. Therefore, $PT = TR$.

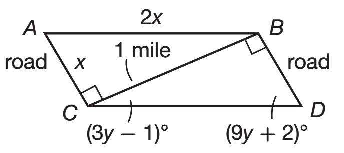
$\frac{1}{2}y + 10 = 2y - 2$
$y = 8$

Hint

When finding unknown values in a parallelogram, first think about each of the four properties of parallelograms and decide which one could apply to the problem.

Example 5 Application: Farming

In Kansas, the Highway Department builds gravel roads to evenly divide parcels of land. If a farmer has a plot of land shaped like a parallelogram with a diagonal of 1 mile, as shown, calculate the values of x and y to the nearest hundredth.



SOLUTION

Use the Pythagorean Theorem to solve for x.

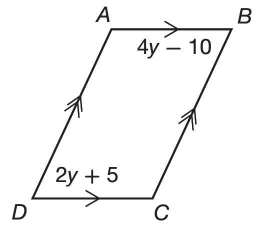
$x^2 + 1^2 = (2x)^2$
$x^2 + 1^2 = 4x^2$
$0.58 \approx x$

Consecutive angles of a parallelogram are supplementary. Therefore, $(m\angle ACB + m\angle BCD) + m\angle CDB = 180°$.

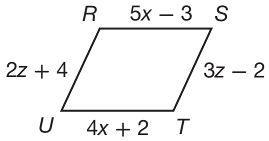
$90 + (3y - 1) + (9y + 2) = 180$
$91 + 12y = 180$
$12y = 89$
$y \approx 7.42$

Lesson Practice

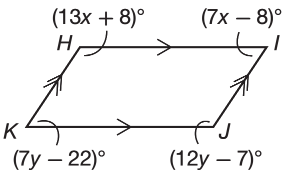
a. Find the value of y in parallelogram $ABCD$.
(Ex 1, 4)



b. Find the values of x and z in parallelogram $RSTU$.
(Ex 2, 4)



c. Find the values of x and y in parallelogram $HIJK$.
(Ex 3, 4)

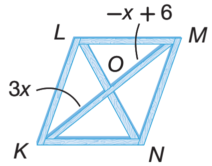


d. Prove Property 4 of parallelograms: if a quadrilateral is a parallelogram, then its diagonals bisect each other. Justify your reasoning in a paragraph proof, and draw an example.
(Ex 3, 4)

e. An old fence gate, shown here, is starting to lean. Find each measure.
(Ex 5)

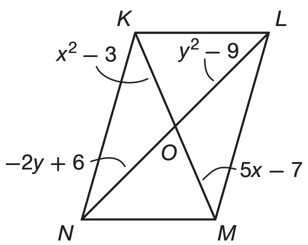
1. KM

2. KO



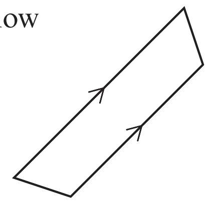
Practice Distributed and Integrated

* **1.** **Multi-Step** Figure $KLMN$ is a parallelogram. Find the length of the diagonals.
(34)



2. If $\triangle ABC \cong \triangle DEF$, $m\angle ACB = 9x - 77°$, and $m\angle DFE = 4x + 33°$, what is the value of x?
(28)

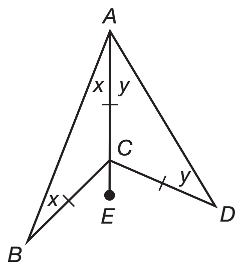
3. **Analyze** Classify this quadrilateral, and explain how you know.
(19)



4. Draw a valid conclusion from these conditional statements:
(21)

If Molly does her homework, then she learns the material.
If she learns the material, then she will do well on the test.
If she does well on the test, then she will get a good grade in the course.

5. **Justify** In the diagram, $AC = BC$, and $DC = BC$. Prove that $m\angle BCD = 2x + 2y$.
(27)



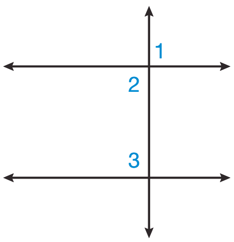
* **6.** (22) **Construction** In order to place a square toolshed in his backyard, Landon measures an area with a diagonal length of 12 feet. What area of the yard will the shed occupy? *Hint: Use the Pythagorean Theorem to find the length of one side of the shed.*

7. (17) **Multiple Choice** Consider the following statements.

If Martha wears jeans, then it is the weekend.
If it is the weekend, then Martha wears jeans.

The second statement is the __________ of the first.

A inverse **B** negation
C contrapositive **D** converse



8. (31) Write a flowchart proof.
Given: $\angle 2$ and $\angle 3$ are supplementary.
Prove: $\angle 1$ and $\angle 3$ are supplementary.

9. (Inv 3) What is the sum of all the interior, exterior, and central angles in a convex hexagon?

10. (18) **Write** Explain why the measure of an exterior angle of a triangle is equal to the sum of its two remote interior angles.

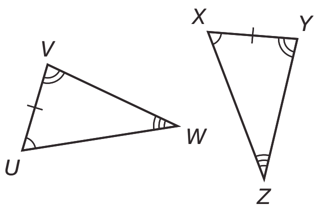
***11.** (34) Briefly state the four properties of parallelograms.

12. (22) What is the area of a rectangle with a length of 4 and a diagonal length of 5?

13. (17) **Error Analysis** For the contrapositive of the statement, "If a bus is green, then it is a downtown bus," Keisha has written, "If a bus is not green, then it is not a downtown bus." Identify any errors Keisha has made.

14. (30) **Multiple Choice** Which of these statements describes these triangles?

A $\triangle VWU \cong \triangle ZYX$ **B** $\triangle XYZ \cong \triangle WVU$
C $\triangle WUV \cong \triangle ZXY$ **D** $\triangle YZX \cong \triangle UWV$

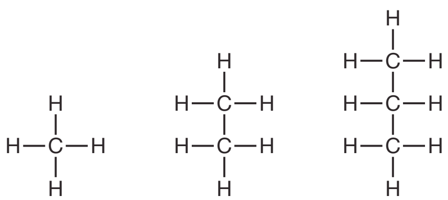


15. (23) **Irrigation** In order to water plants efficiently, many farmers use a long metal pole which sweeps over a circular area, spraying water on the crops below. If the radius of one such device is 150 feet, what is the area of the circle that gets watered, to the nearest ten square feet?

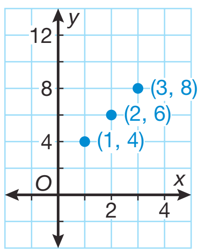
***16.** (33) **Multiple Choice** Which of the following sets of numbers represents the leg lengths of an acute triangle?

A 7, 4, 8 **B** 4, $2\sqrt{2}$, 5 **C** 9, 7, $2\sqrt{3}$ **D** 14, 15, 4

17. (16) **Chemistry** Linear alkanes are a class of organic molecules with only hydrogen (H) and carbon (C) atoms. This graph plots the number of carbon atoms of some linear alkanes on the *x*-axis and the number of hydrogen atoms on the *y*-axis. What is the equation of the line connecting them?

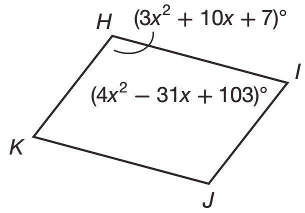


methane ethane propane



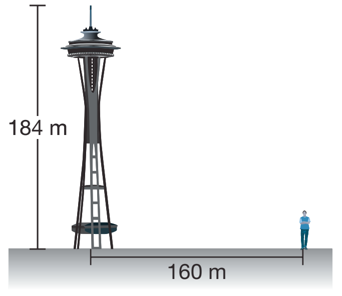
18. (19) **Write** Quadrilateral *WXYZ* has two pairs of parallel sides. Describe the steps needed to sketch *WXYZ*, and then sketch it.

***19.** (34) *HIJK* is a parallelogram. Find the measure of each angle.



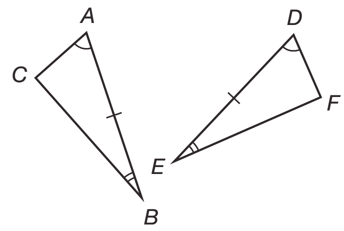
20. (28) Name the included angle between the sides $\overline{RS}$ and $\overline{TR}$ of $\triangle RST$.

***21.** (29, 33) **Architecture** The Seattle Space Needle is 184 meters tall. If a person is standing 160 meters away from the base, how far is the person from the top, to the nearest meter?



***22.** (31) **Generalize** Are there any proofs that can be done as a flowchart proof but cannot be proved with a paragraph proof? If so, give an example. If not, explain why.

23. (30) **Algebra** In $\triangle ABC$, $m\angle BAC = 67°$, $AB = (5x - 7)$, and $m\angle ABC = 23°$. In $\triangle DEF$, $m\angle EDF = 67°$, $EF = (2x + 4)$, $DE = (3x + 1)$, and $m\angle DEF = 23°$. What is the length of $\overline{EF}$?

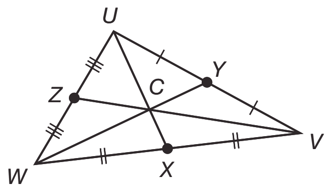


24. (29) **Algebra** A right triangle has a hypotenuse of 15 inches, with one leg twice the length of the other leg. What is length of each leg, to the nearest tenth of an inch?

25. (24) **Justify** Solve the equation $3(x - 2) = 4(x + 1)$ with a justification for each step.

***26.** (34) Three vertices of parallelogram *ABCD* are $A(-1, 5)$, $B(4, 5)$ and $C(3, 2)$. Find the coordinates of *D*.

Use the diagram to find the following measures.



27. (32) Given that $CX = 18$, determine the length of $\overline{UX}$.

28. (32) Given that $ZV = 72$, determine the length of $\overline{ZC}$.

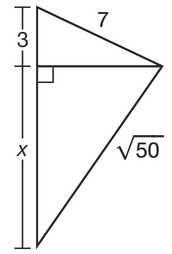
29. (20) **Algebra** Consider the following statement.

If two positive numbers are less than 1, then their product is positive and less than 1.

a. Rewrite the statement using inequalities.

b. State the converse and biconditional of your statement from part **a**. Is the biconditional true? Explain why or why not.

***30.** (33) **Algebra** Determine the value of x. Write your answer in simplified radical form.



LESSON
35 Finding Arc Lengths and Areas of Sectors

Warm Up

1. (23) **Vocabulary** A segment that has both endpoints on a circle and passes through the center of that circle is the ________ of the circle.
2. (23) Find the area and circumference of a circle with a 5-centimeter radius, to the nearest hundredth.
3. (23) What is the radius of a circle with an area of 40 square units, to the nearest hundredth?
4. (23) What is the area of a semicircle with a diameter of 10 inches, to the nearest hundredth?

New Concepts

Recall from Lesson 26 that minor and major arcs of a circle are parts of the circle's circumference. The length of these arc segments can be determined if the circle's radius and the arc's degree measure are known. The **arc length** of a circle is the distance along an arc measured in linear units.

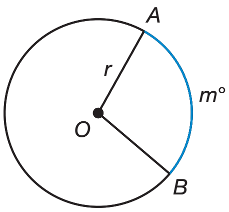
Math Reasoning

Formulate What parts of the arc length formula do you recognize? What does $\left(\frac{m°}{360°}\right)$ represent?

Arc Length

To find the length of an arc, use this formula, where m is the degree measure of the arc.

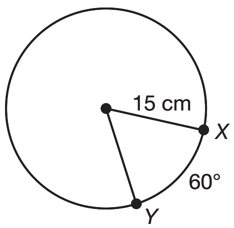
$$L = 2\pi r\left(\frac{m°}{360°}\right)$$



Example 1 Finding Arc Length

Find each arc length. Give your answer in terms of π.

a. Find the length of $\widehat{XY}$.



SOLUTION

$$L = 2\pi r\left(\frac{m°}{360°}\right)$$

$$L = 2\pi(15)\left(\frac{60°}{360°}\right)$$

$$L = 5\pi \text{ cm}$$

b. Find the length of an arc with a measure of 75° in a circle with a radius of 4 feet.

SOLUTION

$$L = 2\pi r\left(\frac{m°}{360°}\right)$$

$$L = 2\pi(4)\left(\frac{75°}{360°}\right)$$

$$L = \frac{5}{3}\pi \text{ ft}$$

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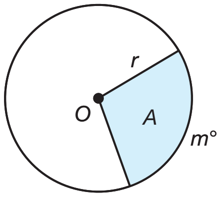
Every arc on a circle encompasses a portion of the circle's interior. The region inside a circle bounded by two radii of the circle and their intercepted arc is known as a **sector of a circle**. Finding the area of a sector is similar to finding arc length.

Area of a Sector
To find the area of a sector (A), use the following formula, where r is the circle's radius and m is the central angle measure: $$A = \pi r^2\left(\frac{m^\circ}{360^\circ}\right)$$



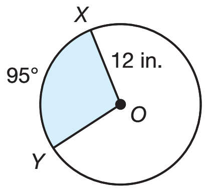
Math Reasoning

Formulate What part of the formula for area of a sector do you recognize?

Example 2 Finding the Area of a Sector

Find the area of each sector. Give your answer in terms of π.

a. Find the area of sector XOY.



SOLUTION

$$A = \pi r^2\left(\frac{m^\circ}{360^\circ}\right)$$

$$A = \pi(12)^2\left(\frac{95^\circ}{360^\circ}\right)$$

$$A = 38\pi \text{ in}^2$$

b. Find the area of a sector with an arc that measures 174° in a circle with a radius of 13 meters.

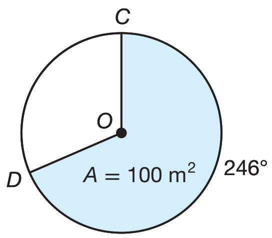
$$A = \pi r^2\left(\frac{m^\circ}{360^\circ}\right)$$

$$A = \pi(13)^2\frac{174^\circ}{360^\circ}$$

$$A = \frac{4901}{60}\pi \text{ m}^2$$

Example 3 Solving for Unknown Radius

Find the radius of the circle to the nearest hundredth of a meter.



SOLUTION

Substitute the known measures into the formula for the area of a sector, then solve for r.

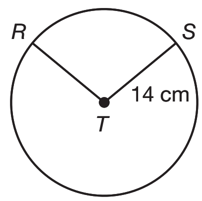
$$A = \pi r^2\left(\frac{m^\circ}{360^\circ}\right)$$

$$100 = \pi r^2\left(\frac{246^\circ}{360^\circ}\right)$$

$$r \approx 6.83 \text{ m}$$

Example 4 Solving for Unknown Central Angle

Find the central angle measure of $\overset{\frown}{RS}$ to the nearest hundredth of a degree, if the length of the arc is 12 centimeters.



SOLUTION

$$L = 2\pi r\left(\frac{m^\circ}{360^\circ}\right)$$

$$12 = 2\pi(14)\left(\frac{m^\circ}{360^\circ}\right)$$

$$m^\circ \approx 49.11^\circ$$

Example 5 Application: Farming

A spray irrigation system has a radius of 150 feet. If it rotates through a 175° central angle, what is the area that the system covers? Round your answer to the nearest square foot.

Hint

When you are finding the area of a sector, do not forget to express the answer in square units.

SOLUTION

$$A = \pi r^2\left(\frac{m^\circ}{360^\circ}\right)$$

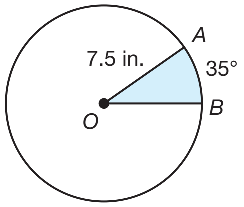
$$A = \pi(150)^2\left(\frac{175^\circ}{360^\circ}\right)$$

$$A \approx 34{,}361 \text{ ft}^2$$

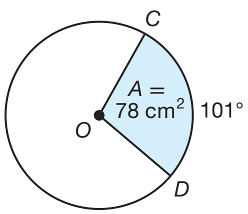
Lesson Practice

a. Find the length of an arc with a measure of 125° in a circle and 12-mile radius. Round to the nearest hundredth of a mile. *(Ex 1)*

b. Find the area of the sector to the nearest hundredth square inch. *(Ex 2)*



c. Find the radius to the nearest hundredth of a centimeter. *(Ex 3)*



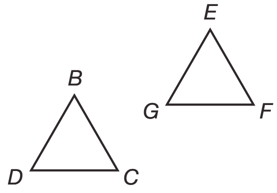
d. If a farmer wants his irrigation system to cover an area of 2 square miles, and his sprinkler rotates through 50°, what is the diameter of his circular field to the nearest hundredth of a mile? *(Ex 4)*

Practice Distributed and Integrated

* **1.** (35) **Write** Explain how the formula for arc length is similar to the formula for the area of a circle.

* **2.** (32) **Model** Find the centroid of $\triangle HIJ$ with vertices at $H(-6, 10)$, $I(-4, 2)$, and $J(-14, 6)$.

3. (25) Given that $\triangle BCD \cong \triangle EFG$, write the six congruence statements for the triangles.

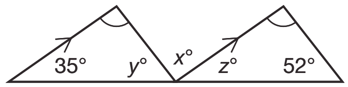


4. (21) **Analyze** What conclusion can be drawn from these statements?

If my computer is not working properly, then I will restart it.
My computer is not working properly.

* **5.** (34) Three vertices of parallelogram *HIJK* are $H(0, 10)$, $I(2, 10)$, and $K(-2, 0)$. Find the coordinates of J.

6. (18) **Visual Arts** An artist is making a motif design using the two triangles shown. The measurements must be exact in order to make the repeating pattern of the motif. What are the measurements of x, y, and z?



7. (26) **Multiple Choice** A minor arc has a measure of $(5x + 20)°$. Its corresponding major arc has a measure of $(8x - 50)°$. What is the correct value of x?

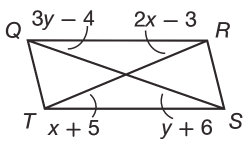
A 23.3 **B** 30
C 170 **D** 190

8. (20) **a.** State the converse of the following statement,

If a triangle is obtuse, then it has exactly two acute angles.

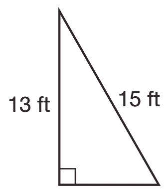
b. Determine whether the statement is true, and whether its converse is true.

* **9.** (34) *QRST* is a parallelogram. Find the length of the diagonals $\overline{TR}$ and $\overline{QS}$.



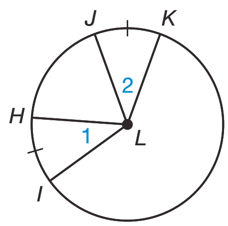
10. (Inv 3) **Carpentry** Margot is making a coffee table with a top in the shape of a regular hexagon. What measure should she make each angle of the table top?

11. (24) **Algebra** Solve the equation $x + 3 = \frac{4x + 5}{2}$, and justify each step.



12. (29) Find the unknown length in the triangle. Are these side lengths a Pythagorean triple?

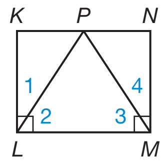
13. (26) Prove Theorem 26-1: In the same or congruent circles, congruent arcs have congruent central angles.
Given: $\widehat{JK} \cong \widehat{HI}$
Prove: $\angle 1 \cong \angle 2$



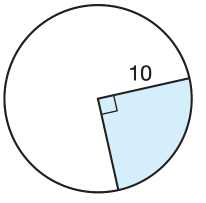
14. (33) The side lengths of a triangle are $4\sqrt{2}$, $4\sqrt{3}$, and 9. Is it a right triangle?

15. (22) **Calendars** A page of a calendar is 10 inches by $12\frac{1}{4}$ inches. The page is divided into 5 rows of 7 congruent boxes. What is the area of each of these boxes?

16. (27) Write a two-column proof.
Given: $\angle KLM$ and $\angle NML$ are right angles and $\angle 2 \cong \angle 3$.
Prove: $\angle 1 \cong \angle 4$



***17.** (35) **Error Analysis** Two students tried to find the area of the shaded region in the circle shown. Which solution is incorrect? Explain where the error was made.

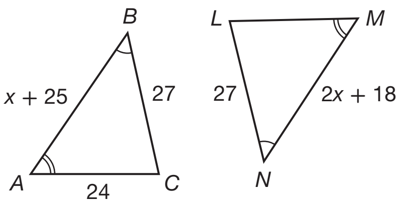


Marc	Stephanie
$A = 2\pi r\left(\frac{m°}{360°}\right)$	$A = \pi r^2\left(\frac{m°}{360°}\right)$
$A = 2\pi(10)\left(\frac{90°}{360°}\right)$	$A = \pi(10)^2\left(\frac{90°}{360°}\right)$
$A = 5\pi$	$A = 25\pi$
$A \approx 15.71$	$A \approx 78.54$

18. (21) What conclusion can be drawn from these statements?
If I lose my driver's license, then I need to go get a new one.
If I need to go get a new driver's license, then I will have to go to the DMV.

***19.** (35) **Games** D.J. is designing a spinner for a game. He wants the spinner to have a $\frac{3}{8}$ probability of landing on a particular sector. What size arc should he assign to this sector?

***20.** (30) These triangles are congruent. What is the perimeter of $\triangle LMN$?

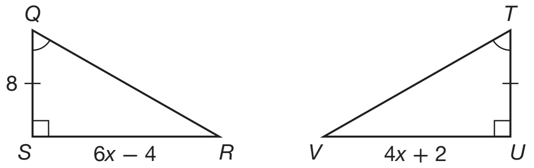


21. (20) State the disjunction of the statements, "A triangle is acute," and "A triangle has exactly two acute angles." Is the disjunction true or false?

***22.** (35) **Multi-Step** The major arc of a circle measures 240° and has a length of 18 centimeters. What is the length of the minor arc?

23. (14) **Justify** Alejandro made the conjecture, "For every integer x, $x^2 + 2x - 1$ is divisible by 2." Is the conjecture true of false? If it is false, provide a counterexample.

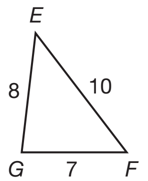
24. Given that $\triangle QSR \cong \triangle TUV$, find the area of each triangle.
(30)



25. Write the converse of the following statement.
(17)
If a triangle is a right triangle, then it has two acute angles.

xy^2 ***26.** **Algebra** Find the radius of a circle with an arc length of 40 centimeters and an arc measure of 60°.
(35)

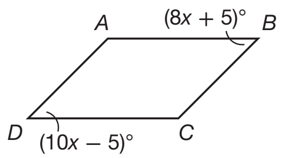
***27.** Is $\triangle EFG$ an obtuse, acute, or a right triangle?
(33)



28. **Verify** For a circle with a radius of 7 inches, Vanessa calculated the area to be about 43.98 square inches. Is Vanessa's calculation accurate? If not, what is the actual area of the circle to the nearest hundredth?
(23)

29. Give two Pythagorean triples that are related to the triple (5, 12, 13).
(29)

30. *ABCD* is a parallelogram. Find the measure of each angle.
(34)



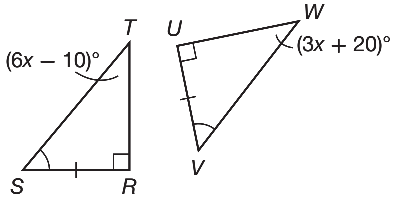
LESSON **36**

Right Triangle Congruence Theorems

Warm Up

1. (28) **Vocabulary** The ________________ Triangle Congruence Postulate states that if two sides and the included angle of one triangle are congruent to two corresponding sides and the included angle of another triangle, then the triangles are congruent.

2. (30) In $\triangle UVW$, what is the measure of $\angle W$?



3. (25) **Multiple Choice** If two triangles are congruent, then they have congruent ________.

A sides only **B** right angles
C sides and angles **D** angles only

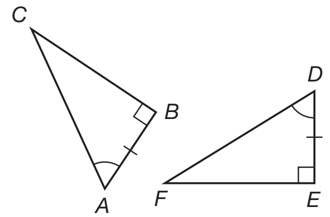
New Concepts

There are four ways to prove triangle congruence: by the SSS Postulate, SAS Postulate, ASA Postulate, or by the AAS Theorem. If a triangle is a right triangle however, there are several other ways to prove congruency.

> **Hint**
>
> It is assumed in all right triangle congruence theorems that the measure of the right angle—90°—is already known, so it only takes two other congruent parts to prove congruency.

Theorem 36-1: Leg-Angle (LA) Right Triangle Congruence Theorem

If a leg and an acute angle of one right triangle are congruent to a leg and an acute angle of another right triangle, then the triangles are congruent.

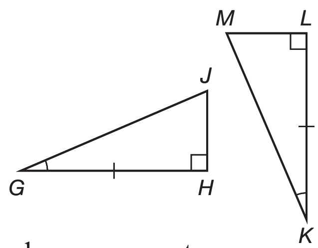


In the diagram, $\triangle ABC \cong \triangle DEF$.

The Leg-Angle Right Triangle Congruence Theorem follows from the ASA Postulate and the AAS Theorem. Notice that in the diagram, marking the right angle shows that the triangles are also congruent by the ASA Postulate.

Example 1 Using the Leg-Angle Triangle Congruence Theorem

a. Use the LA Congruence Theorem to prove that $\triangle GHJ$ and $\triangle KLM$ are congruent.



SOLUTION

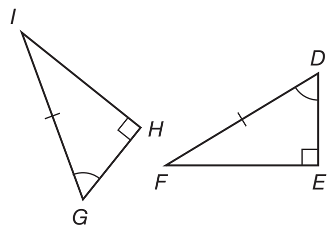
$\triangle GHJ$ and $\triangle KLM$ are both right triangles, so the LA Right Triangle Congruence Theorem can be used. The legs $\overline{GH}$ and $\overline{KL}$ are congruent as given. Acute angles $\angle G$ and $\angle K$ are also congruent. Therefore, by the LA Congruence Theorem, $\triangle GHJ \cong \triangle KLM$.

Online Connection
www.SaxonMathResources.com

Theorem 36-2: Hypotenuse-Angle (HA) Right Triangle Congruence Theorem

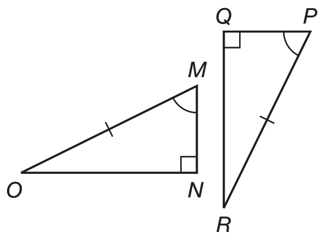
If the hypotenuse and an acute angle of one right triangle are congruent to the hypotenuse and an acute angle of another right triangle, then the triangles are congruent.

In the diagram, $\triangle GHI \cong \triangle DEF$.



Example 2 Using and Proving the Hypotenuse-Angle Triangle Congruence Theorem

a. Use the HA Congruence Theorem to prove that $\triangle MNO \cong \triangle PQR$.



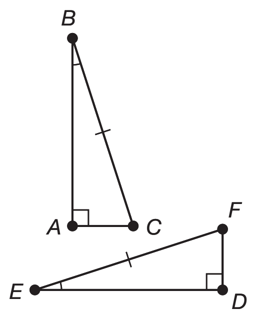
SOLUTION

The diagram tells us that both triangles are right triangles. In addition, they have congruent hypotenuses and acute $\angle M$ is congruent to acute $\angle P$. Therefore, by the HA Triangle Congruence Theorem, $\triangle MNO \cong \triangle PQR$.

Hint

Unless the question specifies, choose whichever proof method seems easiest. This paragraph proof is compact, but a two-column proof might be easier to follow.

b. Use a paragraph proof to prove the HA Triangle Congruence Theorem.



SOLUTION

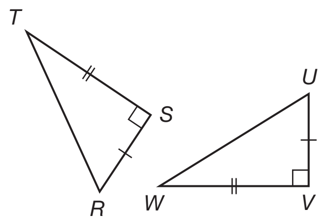
Sketch a diagram like the one shown.

By the Right Angle Congruence Theorem, $\angle D \cong \angle A$. It is given that $\overline{BC} \cong \overline{EF}$ and $\angle E \cong \angle B$. From the diagram, two angles and a non-included side of $\triangle ABC$ are congruent to two angles and a non-included side of $\triangle DEF$. Therefore, by the AAS Triangle Congruence Theorem, $\triangle ABC \cong \triangle DEF$.

Theorem 36-3: Leg-Leg (LL) Right Triangle Congruence Theorem

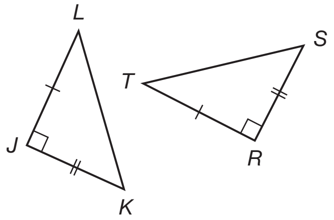
If the two legs of one right triangle are congruent to the two legs of another right triangle, then the triangles are congruent.

In the diagram, $\triangle RST \cong \triangle UVW$.



Example 3 Using the Leg-Leg Triangle Congruence Theorem

Use the LL Congruence Theorem to prove that $\triangle JKL \cong \triangle RST$.



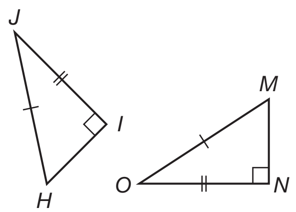
SOLUTION

Statements	Reasons
1. $\triangle JKL$ and $\triangle RST$ are right triangles	1. Given
2. $\overline{JL} \cong \overline{RT}$ and $\overline{JK} \cong \overline{RS}$	2. Given
3. $\triangle JKL \cong \triangle RST$	3. LL Triangle Congruence Theorem

Theorem 36-4: Hypotenuse-Leg (HL) Right Triangle Congruence Theorem

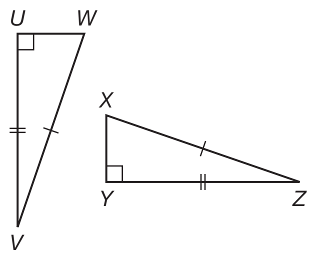
If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and a leg of another right triangle, then the triangles are congruent.

In the diagram, $\triangle HIJ \cong \triangle MNO$.



Example 4 Using the Hypotenuse-Leg Congruence Theorem

a. In $\triangle UVW$ and $\triangle YZX$, $\angle U$ and $\angle Y$ are right angles. Use the HL Congruence Theorem to prove that $\triangle UVW \cong \triangle YZX$.



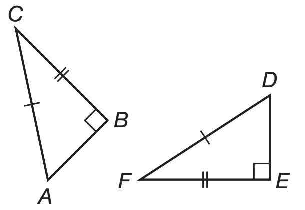
SOLUTION

Statements	Reasons
1. $\triangle UVW$ and $\triangle YZX$ are right triangles	1. Given
2. $\overline{VW} \cong \overline{ZX}$ and $\overline{UV} \cong \overline{YZ}$	2. Given
3. $\triangle UVW \cong \triangle YZX$	3. HL Triangle Congruence Theorem

Math Reasoning

Analyze In order to prove right triangle congruence by the SAS, ASA, AAS, or SSS theorems, the right angle is considered part of the proof. How is the HL Theorem different?

b. Prove the Hypotenuse-Leg Triangle Congruence Theorem.



Given: $\triangle ABC$ and $\triangle DEF$ are right triangles.
$\overline{AC} \cong \overline{FD}$ and $\overline{BC} \cong \overline{EF}$.

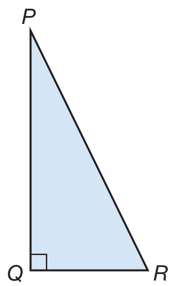
Prove: $\triangle ABC \cong \triangle DEF$

SOLUTION

Statements	Reasons
1. $\triangle ABC$ and $\triangle DEF$ are right triangles	1. Given
2. $\overline{AC} \cong \overline{DF}$ and $\overline{BC} \cong \overline{EF}$	2. Given
3. $AC = DF$ and $BC = EF$	3. Definition of congruent line segments
4. $AC^2 = AB^2 + BC^2$ $DF^2 = DE^2 + EF^2$	4. Pythagorean Theorem
5. $AB = \sqrt{AC^2 - BC^2}$ $DE = \sqrt{DF^2 - EF^2}$	5. Solve for AB and DE
6. $DE = \sqrt{AC^2 - BC^2}$	6. Substitution Property of Equality
7. $DE = AB$	7. Transitive Property
8. $\triangle ABC \cong \triangle DEF$	8. SSS Triangle Congruence Postulate

Example 5 Application: Engineering

Rachel must design a plastic cover to fit exactly over the metal plate shown below. The cover will contain a right angle. Rachel knows that she only needs to pick two other dimensions to make sure that the cover is congruent to the plate. List all the pairs of dimensions Rachel could use to ensure the cover is exactly the same size and shape as the metal plate. For each pair of dimensions, write which right triangle congruence theorem applies.

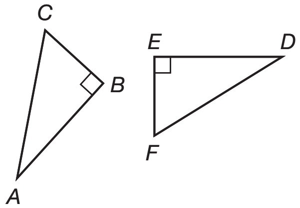


SOLUTION

QR and $m\angle P$	(LA)	PQ and $m\angle R$	(LA)
PQ and $m\angle P$	(LA)	QR and $m\angle R$	(LA)
PR and $m\angle P$	(HA)	PR and $m\angle R$	(HA)
PQ and QR	(LL)	PR and PQ	(HL)
PR and QR	(HL)		

Lesson Practice

Use the diagram to answer problems a through d.



a. Suppose $\overline{AB} \cong \overline{DE}$ and $\angle A \cong \angle D$. Use the LA Triangle Congruence Theorem to prove that $\triangle ABC \cong \triangle DEF$.
(Ex 1)

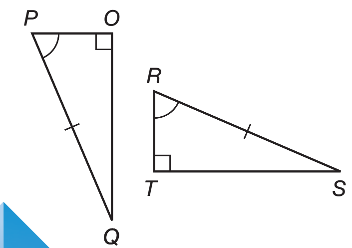
b. Suppose $\overline{AC} \cong \overline{DF}$ and $\angle A \cong \angle D$. Use the HA Triangle Congruence Theorem to prove that $\triangle ABC \cong \triangle DEF$.
(Ex 2)

c. Suppose $\overline{AB} \cong \overline{DE}$ and $\overline{BC} \cong \overline{EF}$. Use the LL Triangle Congruence Theorem to prove that $\triangle ABC \cong \triangle DEF$.
(Ex 3)

d. Suppose $\overline{AC} \cong \overline{DF}$ and $\overline{BC} \cong \overline{EF}$. Use the HL Congruence Theorem to prove that $\triangle ABC \cong \triangle DEF$.
(Ex 4)

e. **Engineering** Refer to Example 5. Suppose Rachel provides $PR = 14.2$ centimeters and $QR = 8.9$ centimeters as dimensions for the plastic cover. In this case, which theorem proves that the cover will fit the metal plate?
(Ex 5)

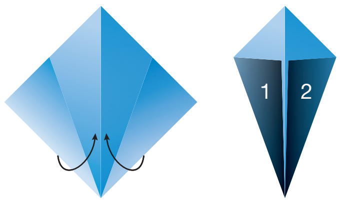
Practice Distributed and Integrated



* **1.** Using the diagram, prove that $\triangle OPQ \cong \triangle TRS$. Name any theorems used.
(36)

2. **Generalize** If $\overset{\frown}{AB}$, $\overset{\frown}{BC}$, and $\overset{\frown}{CD}$ are non-overlapping adjacent arcs, what is an expression for the measure of $\overset{\frown}{AD}$?
(26)

3. **Arts and Crafts** The diagram shows a square piece of paper that is folded to make a kite. The kite is composed of four triangles. Prove that the triangles labeled 1 and 2 are congruent.
(19, 36)



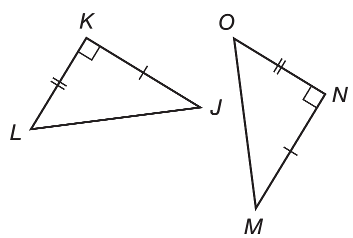
* **4.** **Error Analysis** Greta is calculating the area of a sector of a circle with a diameter of 5. The sector covers 24°. What error has she made?
(35)

$$\left(\frac{24^\circ}{360^\circ}\right)\pi(5)^2 = \left(\frac{1}{15}\right)(\pi)(25)$$

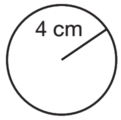
$$\frac{5}{3}\pi \approx 5.24$$

5. In a right triangle, the length of one leg is 1.2 yards and the length of the hypotenuse is 1.3 yards. What is the length of the third side?
(29)

* **6.** Use the Leg-Leg Congruence Theorem to prove that $\triangle JKL \cong \triangle MNO$.
(36)

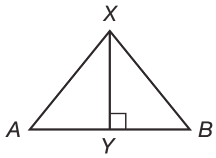


7. Calculate the circumference of the circle to the nearest centimeter.
(26)

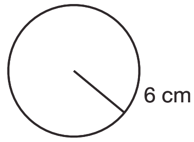


* **8. a.** Write the formula for the area of a trapezoid.
(22)
b. Transform the formula to solve for height.

9. *(31)* Write a paragraph proof showing that if point X is equidistant from points A and B in the triangle shown, then it lies along the perpendicular bisector of $\overline{AB}$.
Hint: *Draw a line through* X *that is perpendicular to* $\overline{\text{AB}}$ *at* Y.



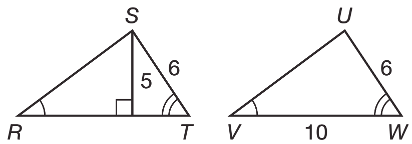
10. *(23)* Calculate the area of the circle shown to the nearest hundredth.



***11.** *(35)* **Catering** The catering service is dividing a small pizza, with a diameter of 8 inches, into four slices. What is the area of each slice, to the nearest tenth?

12. *(26)* **Algebra** If $m\widehat{AB} = (5x + 11)°$ and $m\widehat{CD} = (7x - 9)°$, what is the value of x if $\widehat{AB}$ and $\widehat{CD}$ are congruent?

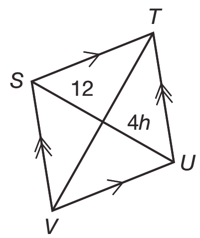
13. *(30)* Using information given in the diagram, determine the area of each triangle.



14. *(18)* **Multiple Choice** Which statement about an equiangular triangle is not true?

A Each pair of interior angles is complementary.
B The sum of the interior angle measures is 180°.
C Each exterior angle measures 120°.
D Each interior angle measures 60°.

15. *(34)* Find the value of h in parallelogram $STUV$.

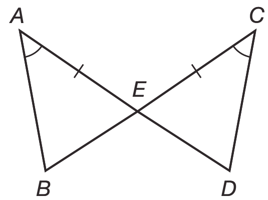


***16.** *(32)* In the triangle formed by the points (2, 2), (4, 0), and the origin, what is the equation of the line containing the median passing through the origin?

17. *(25)* In $\triangle ABC$, $AB = 15$, $BC = 20$, and $AC = 25$. In $\triangle DEF$, $DE = 25$, $EF = 15$, and $DF = 20$. Write the congruency statement for the triangles.

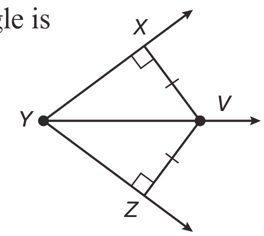
***18.** *(35)* What is the arc measure of one-sixth of a circle? What is the area of one-sixth of a circle that has a radius of 12, in terms of π?

19. *(30)* In the diagram given, prove that $\triangle ABE \cong \triangle CDE$.



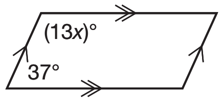
20. *(25)* **Write** If in $\triangle JKL$, $JK = 9$, $KL = 11$, and $JL = 12$, and in $\triangle PQR$, $PQ = 9$, $QR = 11$, and $PR = 11$, explain why $\triangle JKL$ and $\triangle PQR$ are not congruent.

***21.** *(27)* Complete the following proof showing that if a point in the interior of an angle is equidistant from the sides of the angle, then it is on the bisector of the angle.
Given: $\overline{VX} \perp \overline{YX}$, $\overline{VZ} \perp \overline{YZ}$, $VX = VZ$
Prove: V is on the bisector of $\angle XYZ$.



Statements	Reasons
1. $\overline{VX} \perp \overline{YX}$, $\overline{VZ} \perp \overline{YZ}$, $VX = VZ$	1. Given
2.	2. Definition of perpendicular
3.	3. Reflexive Property
4. $\triangle YXV \cong \triangle YZV$	4.
5. $\angle XYV \cong \angle ZYV$	5.
6. $\overrightarrow{YV}$ bisects $\angle XYZ$	6. Definition of angle bisector

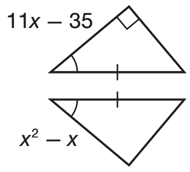
22. (34) Find the value of x in the parallelogram.



23. (33) **Gardening** A gardener tried to plant a garden in the shape of a right triangle. The longest side was 14 feet long and the two shorter sides were 5 feet and 13 feet long, respectively. Did the gardener succeed in making the garden a right triangle? If not, what kind of triangle describes the shape of the garden?

24. (28) **Write** Use the SAS Congruence Theorem to show that a diagonal of a square divides the square into two congruent triangles.

25. (28) For what values of x are these two triangles congruent?

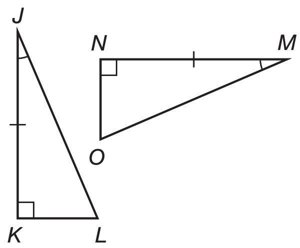


26. (20) **Biology** Consider the following statements.

An organism is an animal.
An organism has leaves.

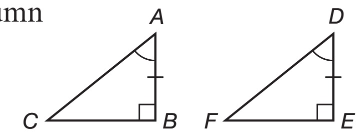
a. Make a truth table for the disjunction of these statements.
b. Explain why the disjunction is false.

***27.** (36) Use the Leg-Angle Congruence Theorem to prove that $\triangle JKL \cong \triangle MNO$.



28. (33) **Verify** Naomi measures the sides of a triangle as 5, 8, and 9 units, respectively, and declares the triangle to be acute. Is she correct? How do you know?

***29.** (27, 36) **Analyze** Copy and complete this two-column proof of the LA Congruence Theorem.



Statements	Reasons
1. $\triangle ABC$ and $\triangle DEF$ are right triangles	1. Given
2.	2. Given
3. $\angle A \cong \angle D$	3.
4. $\angle B$ and $\angle E$ are right angles	4.
5.	5.
6. $\triangle ABC \cong \triangle DEF$	6.

30. (32) **Write** Explain why only two altitudes of a triangle need to be drawn to find the orthocenter of the triangle.

LESSON
37 Writing Equations of Parallel and Perpendicular Lines

Warm Up

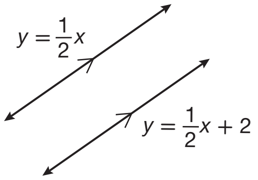
1. **Vocabulary** The _______ of a line is calculated by finding the rise/run.
(16)
2. Convert this equation from the point-slope form to slope-intercept form.
(SB 19)
$y - 4 = \frac{1}{2}(x + 4)$
3. **Multiple Choice** Which equation is a linear equation?
(16)
A $y = 12x + 4$ **B** $a^2 + b^2 = c^2$
C $y = x^2 - 4$ **D** $a = \pi r^2$

New Concepts

The coordinate plane provides a connection between algebra and geometry. Postulates 17 and 18 establish a simple way to find lines that are parallel or perpendicular on the coordinate plane.

Postulate 17: Parallel Lines Postulate

If two lines are parallel, then they have the same slope. All vertical lines are parallel to each other.



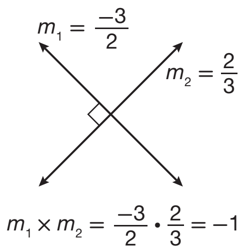
Math Reasoning

Justify What is the slope of a vertical line? Why do both Postulates 17 and 18 talk separately about vertical lines?

Perpendicular lines can also be found by looking at the slope.

Postulate 18: Perpendicular Lines Postulate

If two nonvertical lines are perpendicular, then the product of their slopes is -1. Vertical and horizontal lines are perpendicular to each other.

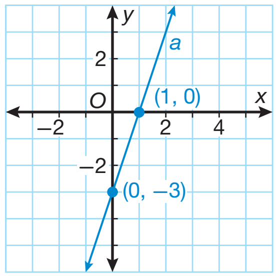


When the product of two numbers is -1, they are **opposite reciprocals**. The opposite reciprocal of a number is the reciprocal of that number with the sign reversed.

For example, the opposite reciprocal of $\frac{1}{2}$ is -2. The opposite reciprocal of -6 is $\frac{1}{6}$. Whenever two lines have slopes that are opposite reciprocals of each other, they are perpendicular lines.

Example 1 Finding the Slopes of Parallel and Perpendicular Lines

a. Find the slope of line *a*.



SOLUTION

Use the slope formula.

$$m = \frac{y_1 - y_2}{x_1 - x_2}$$

Choose any two points on the line, for example (1, 0) and (0, −3). Substitute the coordinates into the slope formula.

$$m = \frac{0 - (-3)}{1 - 0}$$

$$m = 3$$

b. Find the slope of a line parallel to line *a*.

SOLUTION

By the Parallel Lines Postulate, parallel lines have the same slope. The slope of a line that is parallel to line *a* is 3.

c. Find the slope of a line perpendicular to line *a*.

SOLUTION

By the Perpendicular Lines Postulate, the slopes of perpendicular lines are opposite reciprocals. The reciprocal of 3 is $\frac{1}{3}$. Changing the sign gives the opposite reciprocal, $-\frac{1}{3}$.

Reading Math

Recall that the symbol used in the slope-intercept form of a line for slope is *m*.

Example 2 Identifying Parallel and Perpendicular Lines

a. Are the lines $y = 2x + 4$ and $y = -3 + 2x$ parallel, perpendicular, or neither?

SOLUTION

By looking at the equations we can see that the slope of both lines is 2. Lines with the same slope are parallel, so these two lines are parallel to each other.

b. Are the lines $y = \frac{2}{3}x - 1$ and $y = \frac{3}{2}x$ parallel, perpendicular, or neither?

SOLUTION

The slope of the first line is $\frac{2}{3}$. The slope of the second line is $\frac{3}{2}$. These slopes are reciprocals of each other. They are not, however, opposite reciprocals, since both are positive. These lines are neither perpendicular nor parallel.

The point-slope formula for a line: $y - y_1 = m(x - x_1)$. Sometimes it is helpful to find a line passing through a given point that is parallel or perpendicular to another line. The point-slope formula can be used to solve problems like this, once you have discovered the slope of the parallel or perpendicular line.

Math Reasoning

Formulate How could you use the slope-intercept form of a linear equation to solve this problem, instead of the point-slope form?

Example 3 Graphing a Line Parallel to a Given Line

a. Find a line that is parallel to $y = x + 2$ and passes through point (3, 8).

SOLUTION

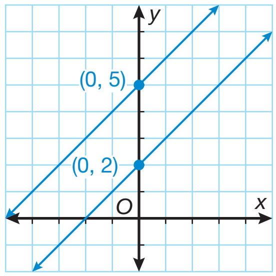
The slope of the given line is 1. Substitute the slope and the given point into the point-slope formula.

$y - y_1 = m(x - x_1)$	Point-slope formula
$y - 8 = 1(x - 3)$	Substitute.
$y = x + 5$	Solve.

b. Graph the parallel lines from part **a**.

SOLUTION

Both lines are now in slope-intercept form. The diagram shows both lines.



Example 4 Graphing a Line Perpendicular to a Given Line

a. Find a line that is perpendicular to $y = \frac{2}{3}x$ and passes through the point (2, 4).

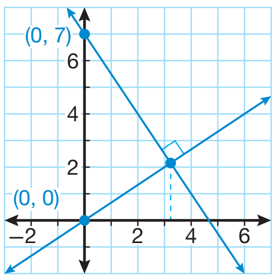
SOLUTION

The slope of the given line is $\frac{2}{3}$. A perpendicular line will have a slope that is the opposite reciprocal, or $-\frac{3}{2}$. Substitute this slope and the given point into the point-slope formula.

$y - y_1 = m(x - x_1)$	Point-slope formula
$y - 4 = -\frac{3}{2}(x - 2)$	Substitute.
$y = -\frac{3}{2}x + 7$	Solve.

b. Graph the perpendicular lines from part **a**.

SOLUTION



Example 5 Application: Swimming

In a race, one swimmer is swimming at a rate of 21 meters per second. Another swimmer gets a 5-meter head start, and also swims at 21 meters per second. What is the equation that will model the distance, y, that each swimmer has gone after x seconds? Will the first swimmer ever catch up to the second?

SOLUTION

Understand There are two swimmers, both swimming at a speed of 21 meters per second. One of them starts 5 meters ahead of the other. The problem asks us to write two linear equations representing this situation, and determine whether the first swimmer can ever catch up to the swimmer with the head start.

Plan The speed of each swimmer is a rate of change, or a slope. The head start of the second swimmer changes the graph's starting point, or its y-intercept. To determine whether or not the first swimmer can ever catch the second, we need to determine if the lines ever intersect.

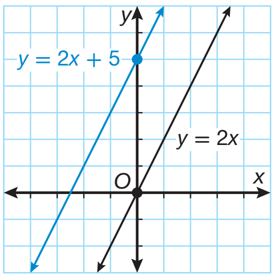
> **Math Reasoning**
>
> **Write** Explain in words why the first swimmer in this example will never be able to catch the second swimmer.

Solve Since we know that the slope of each line is 2, and the second swimmer has a y-intercept of 5, the equations for each line are given below.

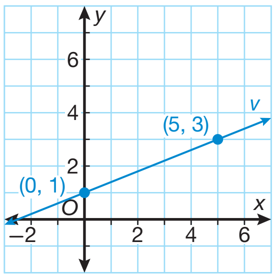
1st swimmer: $y = 2x$
2nd swimmer: $y = 2x + 5$

These lines both have a slope of 2, so they will never intersect. This tells us that the first swimmer will never catch up to the second swimmer.

Check Graph the lines $y = 2x$ and $y = 2x + 5$. They appear parallel.



Lesson Practice



a. *(Ex 1)* Find the slopes of lines that are parallel and perpendicular to line v.

b. *(Ex 2)* Are the lines $y = 3x - 2$ and $3y + x = 6$ parallel, perpendicular, or neither?

c. *(Ex 2)* Are the lines $4y = 2x + 3$ and $y + 2x = 9$ parallel, perpendicular, or neither?

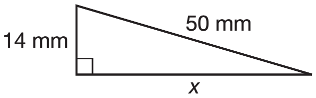
d. *(Ex 3)* Find and graph a line that is parallel to $y = -2x + 7$ and passes through the origin.

e. *(Ex 4)* Find and graph a line that is perpendicular to $y = -\frac{4}{3}x + 3$ and passes through the point (2, 3).

f. (Ex 5) **Sports** Two rowers are canoeing parallel to one another in a stream. The first rower is traveling at a rate of 2 meters per second. The second rower is traveling at a rate of 2.1 meters per second. The first rower is 4 meters ahead of the second rower. Write an equation for both rowers. Will the second rower catch up to the first? Why or why not?

Practice Distributed and Integrated

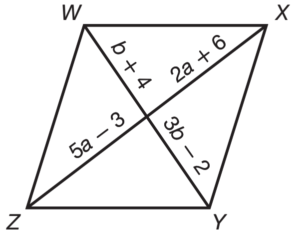
1. (29) **Analyze** Find the unknown length in the triangle shown. Do the side lengths form a Pythagorean triple?



2. (35) What is the area of a 30° sector of a circle with a radius of 9 miles? Express your answer in terms of π.

3. (21) Kyle says that if Marcy goes to the party, then he will go to the party. Marcy said she goes to every party. Assuming both statements are true, will Kyle go to the party?

4. (34) Find the value of a and b in the rhombus $WXYZ$ at right.

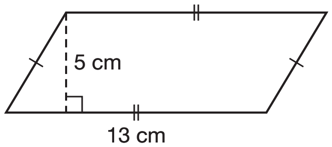


***5.** (37) Write the equation of the line through (3, 7) that is perpendicular to $y = 3x - 4$.

6. (Inv 3) **Generalize** Can the sum of the central angles for any convex polygon be determined? Explain.

7. (23) **Basketball** The rim of the basket in a basketball court encloses a circle with an area of about 0.16 square meters. What is the diameter of the basket, to the nearest hundredth of a meter?

8. (22) Find the area of the parallelogram at right.



9. (32) Find the orthocenter of a triangle with vertices at (–3, 0), (3, 0), and (–1, 4).

***10.** (37) **Algebra** The equations of two parallel lines are $y = \frac{3}{k}x + 14$ and $y = \frac{h}{2}x + 3$. Find the values of k and h.

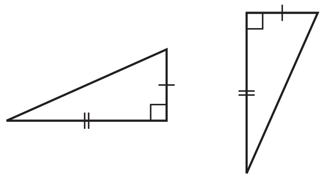
11. (14) Consider the following conjecture.

If a, b, *and* c *are lines in a plane, then they divide the plane into seven regions.*

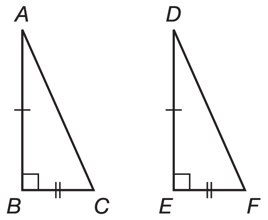
a. What is the hypothesis of the conjecture? What is its conclusion?

b. Find a counterexample to the conjecture.

***12.** (36) Which congruence theorem applies to these triangles?



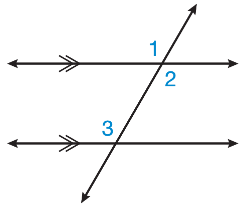
***13.** *(27, 36)* Complete this two-column proof of the LL Right Triangle Congruence Theorem: If two legs of a right triangle are congruent to the two legs of another right triangle, then the triangles are congruent.
Given: $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, $\triangle ABC$ and $\triangle DEF$ are right triangles.
Prove: $\triangle ABC \cong \triangle DEF$



Statements	Reasons
1. $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$	1. Given
2.	2. Definition of right triangle
3. $\angle B \cong \angle E$	3.
4. $\triangle ABC \cong \triangle DEF$	4.

Complete the following statements:

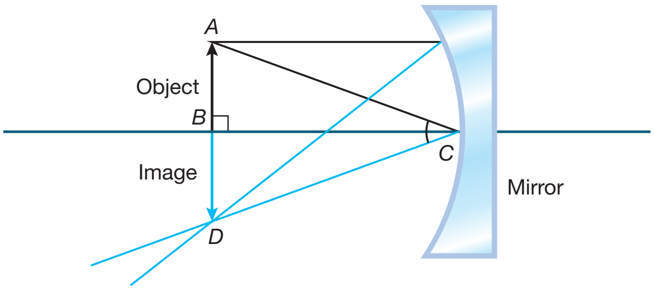
14. *(Inv 1)* Angle 1 is congruent to $\angle 2$ by the ____________________.



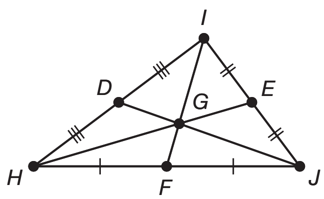
15. *(Inv 1)* Since $\angle 1$ is congruent to $\angle 2$, and $\angle 2$ is congruent to $\angle 3$ by the Alternate Interior Angles Theorem, by the ______________________________, $\angle 1$ is congruent to $\angle 3$.

***16.** *(37)* **Error Analysis** A student says the equation of a line perpendicular to $y = \frac{12}{13}x + \frac{47}{13}$ is $y = \frac{13}{12}x + \frac{47}{13}$. Explain the student's error.

***17.** *(36)* **Optics** In this optical diagram, $\triangle ABC$ and $\triangle DBC$ are congruent. Explain why.



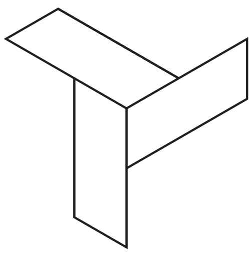
18. *(32)* In $\triangle HIJ$, $HE = 12$ and $GI = 4.6$. Find GF and GE.



19. *(29)* **Multiple Choice** Which of the following is a Pythagorean triple?
A (6, 9, 10) **B** (7, 24, 25)
C (3, 6, 9) **D** (1, 2, 3)

20. *(35)* The radius of $\odot J$ measures 8 inches. Points K and L lie on $\odot J$. If the length of $\widehat{KL}$ is 1.2 inches, what is the measure of $\widehat{KL}$ to the nearest tenth?

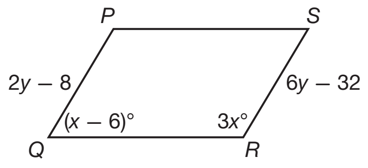
21. *(19)* **Design** A wind power energy company's logo is made up of three parallelograms as shown. Copy the logo and indicate with tick marks all pairs or sets of parallel sides.



22. *(33)* Parallelogram *EFGH* has adjacent side lengths of 13 and 17 units. One of its diagonals is 20 units long. Could the parallelogram be a rectangle?

23. *(26)* **Generalize** If $\overset{\frown}{AC}$ is a minor arc, and $\overset{\frown}{AB}$ and $\overset{\frown}{BC}$ are non-overlapping adjacent arcs, what is an expression for the measure of the major arc associated with $\overset{\frown}{AC}$ in terms of $\overset{\frown}{AB}$ and $\overset{\frown}{BC}$?

24. *(34)* In the diagram, *PQRS* is a parallelogram. Find the value of x and y.

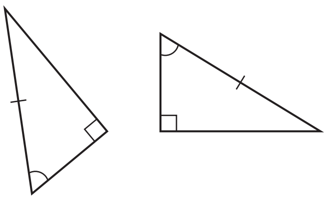


25. *(Inv 3)* **Design** Shen is drawing a Canadian one-dollar coin, which is a regular 11-sided polygon. Approximately what size should he make each angle?

***26.** *(37)* **Highways** The interstate system consists of divided highways with parallel lanes of traffic. Suppose the path of the eastbound lanes is modeled by $y = \frac{4}{3}x$, and the highway is intersected by a bridge that is perpendicular to the highway. Write equations to model both the path of the westbound lanes and the path of the bridge.

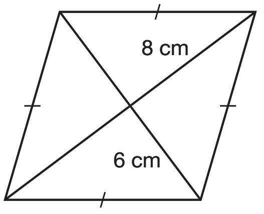
27. *(34)* **Coordinate Geometry** Parallelogram *JKLM* on the coordinate plane has vertices at $J(-1, 2)$, $K(5, 2)$ and $L(2, 6)$. Where is the fourth vertex of *JKLM* located?

***28.** *(36)* Which congruence theorem applies to these triangles?



29. *(33)* **Formulate** In any isosceles right triangle, what is the ratio of the length of the hypotenuse to the length of a leg? Would a higher ratio indicate that the triangle is acute or obtuse?

30. *(22)* Find the area of the figure at right.



LESSON

38 Perpendicular and Angle Bisectors of Triangles

Warm Up

1. (32) **Vocabulary** A segment whose endpoints are a vertex of the triangle and the midpoint of the side opposite the vertex is called the _______. (***median, altitude, base***)

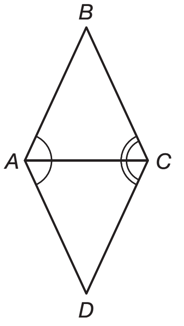
2. (30) The proof below shows that $\triangle ABC \cong \triangle ADC$. Match each step of the proof with the correct justification. A justification may be used more than once.

a. $\angle BAC \cong \angle DAC$ — **A** ASA Postulate

b. $\angle BCA \cong \angle DCA$ — **B** Reflexive Property

c. $\overline{AC} \cong \overline{AC}$ — **C** Given

d. $\triangle ABC \cong \triangle ADC$



3. (3) The measure of $\angle SRU$ is 78°. The angle bisector is $\overrightarrow{RT}$. What is $m\angle SRT$?

New Concepts

An angle bisector divides an angle into two congruent angles. When all three angles of a triangle are bisected, the point of concurrency is called the **incenter of the triangle**. The incenter of the triangle is equidistant from all of the sides of the triangle.

> **Math Language**
>
> The **point of concurrency** is the point where things intersect. In this case, the point of concurrency of all three angle bisectors is the incenter of the triangle.

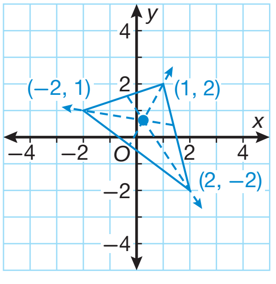
> **Hint**
>
> Refer to Construction Lab 3 to see how to construct an angle bisector.

Example 1 Finding an Incenter in the Coordinate Plane

Use a compass and a straightedge to find the incenter of a triangle whose vertices are at (–2, 1), (1, 2), and (2, –2) in a coordinate plane.

SOLUTION

To find the incenter, bisect each of the triangle's angles using the methods described in Construction Lab 3. Once all three angle bisectors have been drawn, the central point where they intersect is the incenter.



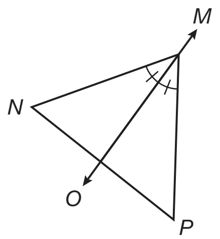
In addition to finding the incenter of a triangle, angle bisectors can also be used to find the lengths of segments in the triangle. When an angle bisector intersects the side of a triangle, it makes a proportional relationship, given by Theorem 38-1.

Online Connection
www.SaxonMathResources.com

Theorem 38-1: Triangle Angle Bisector Theorem

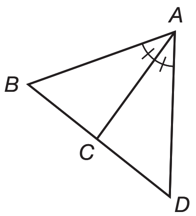
If a line bisects an angle of a triangle, then it divides the opposite side proportionally to the other two sides of the triangle.

In the diagram, $\frac{PM}{PO} = \frac{NM}{NO}$.



Example 2 Using the Triangle Angle Bisector Theorem

Using the diagram at the right, find BC if $AD = 15$, $DC = 8$, and $AB = 20$.



SOLUTION

$$\frac{AD}{DC} = \frac{AB}{BC}$$

$$\frac{15}{8} = \frac{20}{x}$$

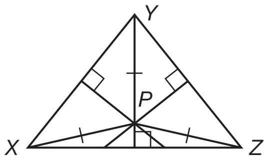
$$15x = 160$$

$$x = 10.\overline{6}$$

> **Math Language**
>
> The circumcenter, orthocenter, and centroid of a triangle will always be collinear. The line that all three points lie on is known as the **Euler Line**.

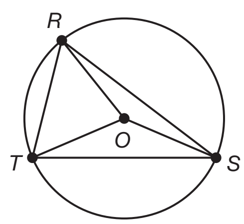
A perpendicular bisector divides a line segment into two congruent segments and is perpendicular to the segment. If perpendicular bisectors are drawn for every side of a triangle, the point of concurrency is the **circumcenter of the triangle**. The circumcenter of a triangle is equidistant from every vertex in the triangle.

In the diagram below, point P is the circumcenter of the triangle, so $PX = PY = PZ$.



The circumcenter is not always inside a triangle. A right triangle's circumcenter lies on the hypotenuse, and an obtuse triangle's circumcenter is outside the triangle.

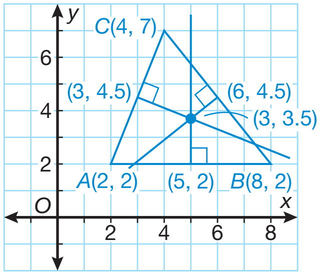
The circumcenter lies at the center of the circle that contains the three vertices of the triangle. Any circle that contains all the vertices of a polygon is called a **circumscribed circle**. Any polygon with each vertex on a circle is an **inscribed polygon**.



> **Math Reasoning**
>
> **Justify** Even though all three perpendicular bisectors of a triangle's sides go through the circumcenter, only two need to be known in order to find the circumcenter. Why?

Example 3 Finding a Circumcenter in the Coordinate Plane

Find the circumcenter of a triangle with vertices at $A(2, 2)$, $B(8, 2)$, and $C(4, 7)$.



SOLUTION The perpendicular bisector of each segment needs to be found. The midpoint of $\overline{AB}$ is (5, 2) and the midpoint of $\overline{AC}$ is (3, 4.5).

For each of these midpoints, a line needs to be found that is perpendicular to the segment on which it lies. Find the slope between the segment's two endpoints, and then use linear equations to find the line.

$\overline{AB}$ lies on the line $y = 2$, and $\overline{AC}$ lies on $y = \frac{5}{2}x - 3$.

Perpendicular lines can be found through the midpoints using the method learned in Lesson 37.

The line perpendicular to $\overline{AB}$ through (5, 2) is $x = 5$.

The line perpendicular to $\overline{AC}$ through (3, 4.5) is $y = -\frac{2}{5}x + 5.7$.

To find the circumcenter, solve the system of equations.

$x = 5$

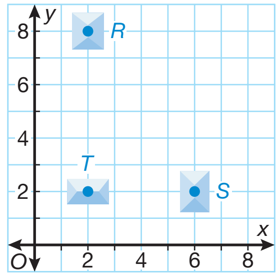
$y = -\frac{2}{5}x + 5.7$

$y = 3.7$

Finally, substitute this value of y into one of the equations above to find x. The coordinates of the orthocenter are $(5, 3.1\overline{6})$.

Example 4 Application: City Planning

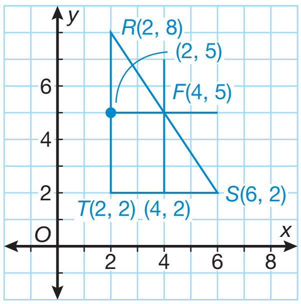
A gas company has three gas stations located at points $R(2, 8)$, $S(6, 2)$, and $T(2, 2)$, as shown. The storage facility is equidistant from the three gas stations. Find the location of the storage facility.



SOLUTION

Since the storage facility is equidistant from the gas stations, it is at the circumcenter of $\triangle RTS$.

By looking at the graph, you can see that the equation for $\overline{TS}$ is $y = 2$, and the equation for $\overline{RT}$ is $x = 2$.

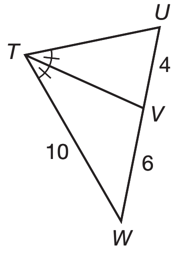


The midpoint of $\overline{TS}$ is (4, 2) and the midpoint of $\overline{RT}$ is (2, 5). The horizontal line $y = 5$ is perpendicular to $\overline{RT}$. The vertical line $x = 4$ is perpendicular to $\overline{TS}$. These two lines intersect on the hypotenuse at (4, 5), so the storage facility is located at (4, 5).

Lesson Practice

a. Use a compass and a straightedge to find the incenter of a triangle whose vertices are at (−3, 1), (3, −2), and (2, 4) in a coordinate plane. *(Ex 1)*

b. Using the diagram at the right, find the length of $\overline{TU}$ if $UV = 4$, $TW = 10$, and $WV = 6$. *(Ex 2)*



c. Find the circumcenter of a triangle with vertices at (3, −1), (−1, −4), and (−3, −1) in a coordinate plane. *(Ex 3)*

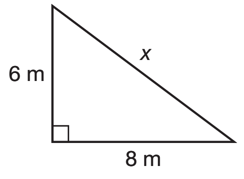
d. A restaurant owner wants to place his new restaurant equidistant from three nearby grocery stores that will supply him. They are located at $A(0, 0)$, $B(4, 0)$ and $C(0, 6)$. Where should he place his restaurant? *(Ex 4)*

Practice Distributed and Integrated

1. *(33)* Classify the following triangles as acute, obtuse, or right, based on the three side lengths given.
a. 5, 9, 12
b. 6, 17, 17.5
c. 9, 12, 15

* **2.** *(38)* **Fire Stations** Three towns want to build a fire station that can serve them all. Explain how the towns might go about finding the optimal location for the fire station.

3. *(29)* **Analyze** Find the unknown length in the triangle. Do the side lengths form a Pythagorean triple?



4. *(7)* **Algebra** Test the conjecture that the sum of the first n squares is given by the expression $\frac{1}{6}n(n + 1)(2n + 1)$.

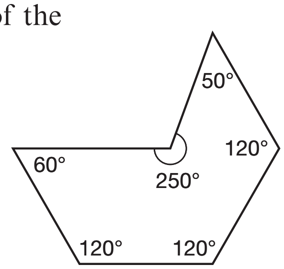
5. *(24)* Letty is solving the equation $3x - 4 = 21$. Which property of equality should she use first?

6. *(28)* What is the included side of $\angle QRS$ and $\angle QRT$?

* **7.** *(37)* **Predict** If line x is parallel to line y and line y is perpendicular to line z, describe the relationship between lines x and z.

8. *(23)* **Driving** A traffic circle is a round intersection. The first one built in the United States is the Columbus Circle in New York City, and has a circular monument in the center covering approximately 148,000 square feet. What is the radius of the monument, to the nearest hundredth?

* **9.** *(Inv 3)* Determine the sum of the measures of the angles in the polygon shown at right.

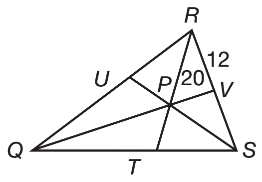


10. (25) **Generalize** If $\triangle LMN \cong \triangle DEF$, what can you determine about the heights of the triangles as measured from side $\overline{LM}$ and from side $\overline{DE}$, respectively?

11. (21) **Verify** Hamid reasoned, "All safety glasses protect your eyes. Sunglasses protect your eyes. Therefore, all sunglasses are safety glasses." What is wrong with Hamid's conclusion?

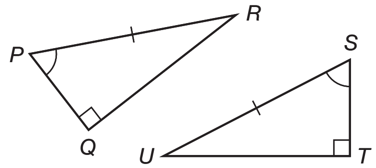
***12.** (37) Find the line parallel to $y = -\frac{1}{2}x - 2$ that passes through the origin.

***13.** (38) In the diagram at right, P is the incenter of $\triangle QRS$. Find PT.



14. (35) **Baking** Louis is baking a cake for a party that celebrates the birthdays of three different people. He wants to use three different kinds of frosting, making three equal sections on the top of the round cake, each with a person's name on it. If the cake has a radius of 5 inches, what is the surface area of each wedge of the cake that Louis needs to cover with frosting, to the nearest hundredth?

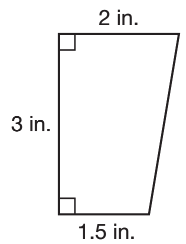
***15.** (36) **Error Analysis** Mira states that she can use the Leg-Angle Congruence Theorem to prove these triangles congruent. Is she correct about the triangles being congruent? Is her justification correct? Explain.



***16.** (38) **Construction** A contractor wants to cut a circular piece of stone out of a scrap piece that is shaped like a right triangle. She wants the circle to be as large as possible. How should she determine how to cut the piece of stone in order to get the largest possible circle out of it? Explain your reasoning and support your answer with a drawing.

17. (32) **Coordinate Geometry** Find the centroid of a triangle with vertices at (1, 4), (5, 6), and (5, 0).

18. (22) What is the area of the trapezoid in the diagram?



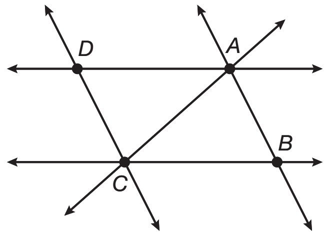
19. (24) **Analyze** Helen wrote the following proof:

Statements	Reasons
1. $x = y$	1. Given
2. $x^2 = xy$	2. Multiplication Property of Equality
3. $x^2 - y^2 = xy - y^2$	3. Subtraction Property of Equality
4. $(x - y)(x + y) = y(x - y)$	4. Factor.
5. $\frac{(x-y)(x+y)}{(x-y)} = \frac{y(x-y)}{(x-y)}$	5. Division Property of Equality
6. $x + y = y$	6. Simplify.
7. $x + y - y = y - y$	7. Subtraction Property of Equality
8. $x = 0$	8. Simplify.

Is there something wrong with Helen's proof? If so, which step is wrong and why?

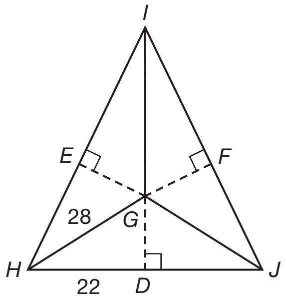
20. (34) **Multiple Choice** Which statement is *not* true of a rhombus?
A Consecutive angles are supplementary. **B** The diagonals bisect each other.
C Opposite angles are complementary. **D** All sides are congruent.

21. (31) In the diagram, $\overline{AD} \parallel \overline{BC}$ and $\overline{DC} \parallel \overline{AB}$. Prove that $\triangle ACB \cong \triangle CAD$ using a flowchart or paragraph proof.



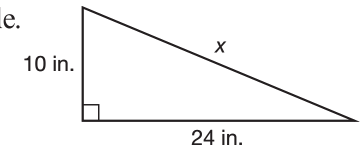
22. (29) **Home Maintenance** A window washer needs the top of his ladder to rest against the bottom of a window that is 25 feet above the ground. However, the shrubs against the house require him to place the ladder 10 feet away from the house. Assuming that the ground and house meet at right angles, what length of ladder will he need, to the nearest foot?

***23.** (38) Using the diagram find FG if G is the incenter of $\triangle HIJ$.



24. (20) In the city of Logica, downtown buses are blue and suburban buses are green. State the conjunction of the two statements, "A bus is a downtown bus," and "A bus is green." Is the conjunction true or false?

25. (29) Find the unknown length of the side in the triangle.



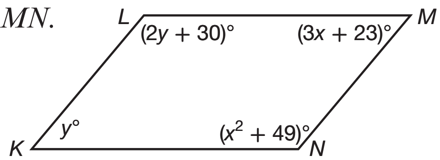
26. (35) In $\odot C$, $\overset{\frown}{AB}$ measures 44° and the radius AC is 5 centimeters. What is the area of sector ACB, to the nearest hundredth?

***27.** (37) Find the line parallel to $y = \frac{3}{2}x + \frac{1}{2}$ that passes through the point (3, 4).

***28.** (36) In $\triangle JKL$ and $\triangle PQR$, $\angle J$ and $\angle P$ are right angles, $\overline{KL} \cong \overline{QR}$, and $\angle Q \cong \angle K$. Use the Hypotenuse-Angle Congruence Theorem to prove that $\triangle JKL \cong \triangle PQR$.

29. (30) Prove that if $m\angle ABC = m\angle ACB$, then $AB = AC$. *Hint: Prove* $\triangle$ABC $\cong$ $\triangle$ACB.

30. (34) Find the value of x and y in parallelogram $KLMN$.



LAB

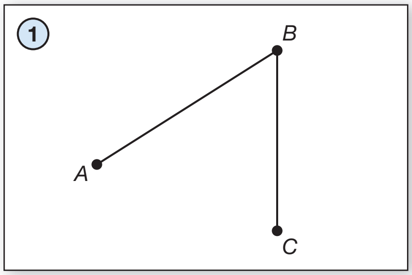
6

Circle Through Three Noncollinear Points

Construction Lab 6 *(Use with Lesson 38)*

In Lesson 38, you learned about perpendicular bisectors of a triangle. This lab shows you how to construct a circle through three noncollinear points using perpendicular bisectors.

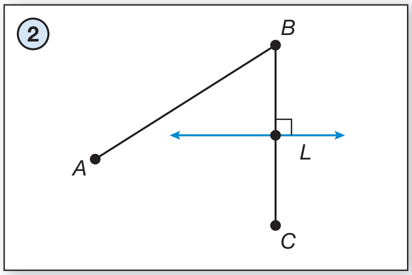
1. Begin with three noncollinear points *A*, *B*, and *C*. Draw $\overline{AB}$ and $\overline{BC}$.



Hint

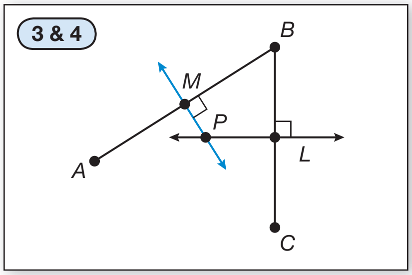
To construct the perpendicular bisector, use the method you learned in Construction Lab **3**.

2. Construct the perpendicular bisector of $\overline{BC}$ and label the midpoint *L*.

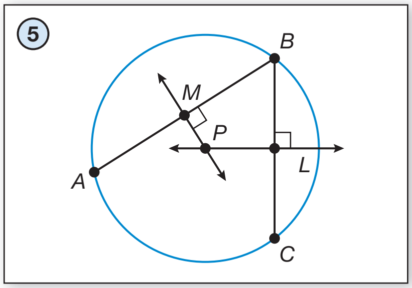


3. Construct the perpendicular bisector of $\overline{AB}$ and label the midpoint *M*.

4. The point of intersection of the two perpendicular bisectors is the center of the circle. Label it *P*.



5. Center the compass on *P* and draw the circle with radius *PA*. As you draw the circle with this radius, the points *A*, *B*, and *C* will lie on the circle.



Lab Practice

Practice constructing circles through these sets of given points:

a. (2, 5), (3, 4), and (3, 2)

b. (0, 2), (2, 0), and (−1, 1)

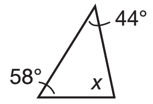
LESSON 39

Inequalities in a Triangle

Warm Up

1. (15) **Vocabulary** The angle formed by one side of a polygon and the extension of an adjacent side is called the __________ angle. (***exterior, corresponding, interior***)

2. (13) Determine the value of x in the diagram.



3. (13) **Multiple Choice** Classify the triangle in the diagram.
   A isosceles **B** obtuse
   C acute **D** right

New Concepts

The lengths of each side of a triangle are related to the measures of each angle in the triangle according to Theorems 39-1 and 39-2.

Theorem 39-1
If one side of a triangle is longer than another side, then the angle opposite the first side is larger than the angle opposite the second side.

Theorem 39-2
If one angle of a triangle is larger than another angle, then the side opposite the first angle is longer than the side opposite the second angle.

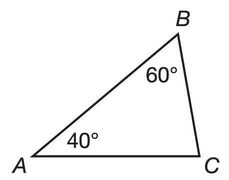
Math Reasoning

Predict Using Theorems 39-1 and 39-2, what can you say about the angle measures of an isosceles triangle?... of an equilateral triangle?

In other words, a triangle's largest side is always opposite its largest angle, and it smallest side is always opposite its smallest angle.

Example 1 Ordering Triangle Side Lengths and Angle Measures

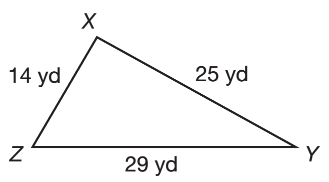
a. Order the side lengths in $\triangle ABC$ from least to greatest.



SOLUTION

The Triangle Angle Sum Theorem shows that the missing angle is 80°. Therefore, the side with the greatest length is $\overline{AB}$ because it is opposite the largest angle. The shortest side is $\overline{BC}$, as it is opposite the smallest angle. The final order of sides, from least to greatest length, is $\overline{BC}$, $\overline{AC}$, $\overline{AB}$.

b. Order the measures of the angles in $\triangle XYZ$ from least to greatest.



SOLUTION

The shortest side of the triangle is $\overline{XZ}$, therefore the measure of the opposite angle, $\angle Y$, is the least of the three angles. The longest side is $\overline{YZ}$, so it is opposite the angle with the greatest measure, $\angle X$. Therefore the order of angles is $\angle Y$, $\angle Z$, $\angle X$.

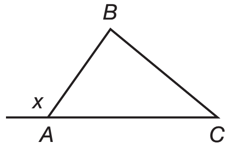
> **Math Language**
>
> **Remote interior angles** are the two angles of a triangle that are not adjacent to an identified exterior angle.

Recall from Lesson 18 that the measure of the exterior angle of a triangle is equal to the sum of the two remote interior angles. This result leads to the Exterior Angle Inequality Theorem.

Theorem 39-3: Exterior Angle Inequality Theorem
The measure of an exterior angle is greater than the measure of either remote interior angle.

Example 2 Proving The External Angle Inequality Theorem

In the given triangle, the exterior angle is labeled as x. Prove that x is greater than the measures of $\angle B$ or $\angle C$.



SOLUTION

By the Exterior Angle Theorem, we know that $m\angle B + m\angle C = x$. We also know, from the definition of an angle, that both $\angle B$ and $\angle C$ have a measure greater than 0°. Rearranging the Exterior Angle Theorem, $m\angle B = x - m\angle C$. Since $m\angle C$ is greater than 0, $m\angle B$ must be less than x.

> **Math Reasoning**
>
> **Analyze** Imagine three line segments, with the sum of two of the segments equal to the third. How would a "triangle" made from these three segments appear?

It is not true that any three line segments can make a triangle. Only line segments of certain lengths can form the three sides needed for a triangle. The requirements are given in the Triangle Inequality Theorem.

Theorem 39-4: Triangle Inequality Theorem
The sum of the lengths of any two sides of a triangle must be greater than the length of the third side.

For example, a triangle could not have side lengths of 3, 5, and 9 because the sum of 3 and 5 is less than 9.

Example 3 Applying the Triangle Inequality Theorem

a. Decide if each set of side lengths could form a valid triangle: (3, 4, 5), (5, 11, 6), and (1, 9, 5).

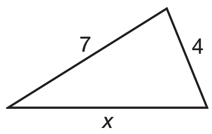
SOLUTION

For the first set, no combination of two sides can be found that will sum to less than the length of the other side, so since $3 + 4 = 7$, and $7 > 5$, these side lengths could represent a triangle.

For the second set, the sum of the two short sides, 5 and 6, equals exactly the length of the third side, 11. Since the sum is not greater than the third side, but only equal to it, this set cannot represent a triangle.

For the third set, the sum of the two short sides, 1 and 5, sum to less than the third side, 9, so this set also cannot represent a triangle.

b. Find the range of values for x in the given triangle.



SOLUTION

Using the Triangle Inequality Theorem, the following three statements are true:

$x + 4 > 7$ $\quad$ $x + 7 > 4$ $\quad$ $7 + 4 > x$

$x > 3$ $\quad$ $x > -3$ $\quad$ $11 > x$

The second inequality is invalid, since a side length cannot be negative. Combining the two valid statements, we obtain the solution $3 < x < 11$.

Hint

See Skills Bank 15 for more information about using inequalities.

Example 4 Application: Planning a Trip

Simone took a flight from Atlanta to London (a distance of 4281 miles), then flew from London to New York City (a distance of 3470 miles), and then took a flight back to Atlanta. Assuming that all three trips are straight lines, determine the range of distances (from least to greatest) she could have traveled altogether.

SOLUTION

Let x represent the distance Simone traveled back to Atlanta from New York City. Using the Triangle Inequality Theorem, the following three statements are true:

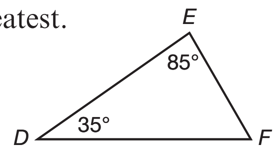
$x + 3470 > 4281$ $\quad$ $x + 4281 > 3470$ $\quad$ $3470 + 4281 > x$

$x > 811$ $\quad$ $x > -811$ $\quad$ $7751 > x$

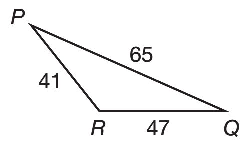
Combining the two valid statements, $811 < x < 7751$. Therefore, the shortest distance Simone could have traveled is $3470 + 4281 + 811 = 8562$ miles. The farthest she could have traveled is $3470 + 4281 + 7751 = 15{,}502$ miles.

Lesson Practice

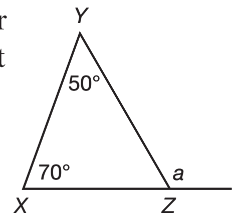
a. Order the side lengths in $\triangle DEF$ from least to greatest.
(Ex 1)



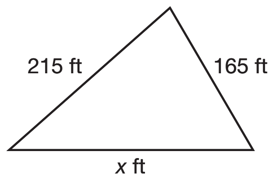
b. Order the measures of the angles in $\triangle PQR$ from least to greatest.
(Ex 1)



c. Show that in the triangle, the measure of the exterior angle at vertex Z is greater than the angle measure at vertex X or at vertex Y.
(Ex 2)



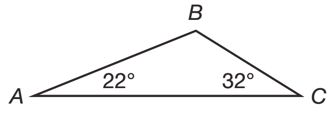
d. Find the range of values for x in the given triangle.
(Ex 3)



Practice Distributed and Integrated

1. (36) In $\triangle GHJ$ and $\triangle KLM$, it is given that $\angle G$ and $\angle K$ are right angles, $\overline{GH} \cong \overline{KL}$, and $\angle H \cong \angle L$. Prove that $\triangle GHJ \cong \triangle KLM$.

* 2. (39) Write the side lengths of $\triangle ABC$ in order from least to greatest.



3. (29) **Algebra** An isosceles right triangle has a hypotenuse of 10 inches. What is the length of each leg, to the nearest tenth of an inch?

4. (26) **Security** To protect a circular area of a park, cameras are placed on a lamppost in the center of the area. If each camera can observe an arc measure of 75°, how many cameras are needed to observe the entire area around the lamppost?

5. (32) **Model** Draw a right triangle and sketch the altitude of each vertex. Where is the orthocenter of the triangle located? Will the orthocenter be located in the same place on every right triangle?

* 6. (39) **Multiple Choice** Which group of line segment lengths can be used to form a triangle?

A 4, 4, 9 **B** 3, 7, 12
C 1, 5, 7 **D** 3, 4, 6

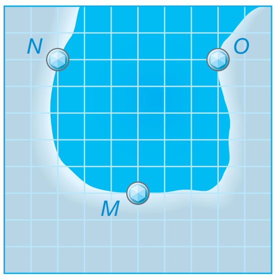
* 7. (39) **Analyze** Explain whether or not it is possible to construct a triangle with side lengths of 34 inches, 14 inches, and 51 inches.

8. (34) If rectangle $STUV$ has an area of 27 square units, and $ST = 3(TU)$, find the length of each side of $STUV$.

9. (31) **Error Analysis** On $\overline{AD}$, B is the midpoint of $\overline{AC}$ and C is the midpoint of $\overline{BD}$. The paragraph proof below was written to prove that $AB = CD$. Identify the mistake in the proof.

Since B *is the midpoint of* $\overline{\text{AC}}$, AB = BC *by the definition of midpoints. Since* C *is the midpoint of* $\overline{\text{BD}}$, BC = CD *by the definition of midpoints. By the Reflexive Property,* AB = CD.

*10. (38) **Nautical Maps** Three lighthouses surround a harbor, as shown in the drawing. The lighthouses are equidistant from a buoy in the harbor. Copy the diagram and draw the location of the buoy based on the locations of the lighthouses.



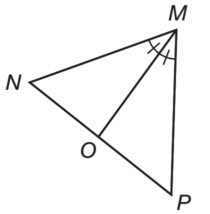
11. (36) **Engineering** A bridge uses several girder sections that are right triangles. Each section is made so that its legs are 10 feet and 12.5 feet long, respectively. Explain why the sections are congruent to each other.

12. (22) **Decorating** Kirsten is buying carpet for a rectangular room that is 11 feet by 13 feet. How many square feet of carpet does she need?

13. (32) Find the centroid of a triangle with vertices at (0, −2), (0, −6), and (2, −4).

14. What is the included angle of $\overrightarrow{TS}$ and $\overrightarrow{TR}$?
(28)

***15.** **Algebra** Using the diagram at right, find MN in terms of x if $NO = 2$, $OP = 4x$, and $MP = x + 7$.
(38)

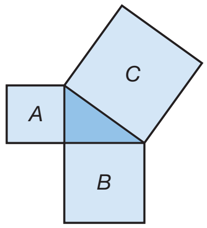


***16.** **Multiple Choice** Which expression represents two parallel lines?
(37)

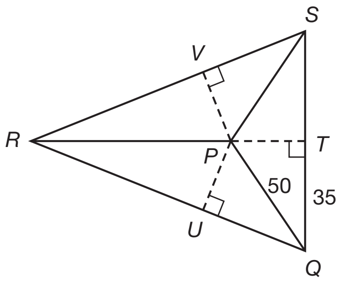
A $y = 2x + 3, y = -2x + 3$ **B** $y = -2x + 4, y = -6 - 2x$

C $y = \frac{1}{2}x + 3, y = -2x - 3$ **D** $y = 3x + 7, \frac{1}{3}y = -x - 7$

17. **Urban Development** A development company plans to build three square single-story buildings that completely surround a triangular park. Buildings A and B will be situated at right angles to each other and have an area of 1296 square feet and 2500 square feet, respectively. What will be the area of building C? Explain.
(Inv 2)



***18.** Using the diagram, P is the incenter of $\triangle QRS$. Find PU.
(38)



19. **Write** Explain why the size of each central angle in a regular polygon decreases as the number of sides increases.
(Inv 3)

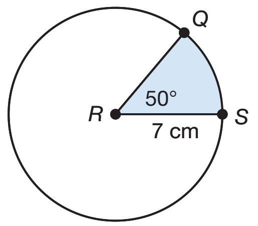
20. **Justify** Provide a justification for each step of the proof. The first one is done for you.
(24)

Statements	Reasons
1. $16 = 4(3x - 8)$	1. Given
2. $\frac{16}{4} = \frac{4(3x - 8)}{4}$	2.
3. $4 = 3x - 8$	3.
4. $4 + 8 = 3x - 8 + 8$	4.
5. $12 = 3x$	5.
6. $\frac{12}{3} = \frac{3x}{3}$	6.
7. $4 = x$	7.
8. $x = 4$	8.

***21.** Find the line parallel to $y = 2x + 3$ that passes through the point $(-1, -1)$.
(37)

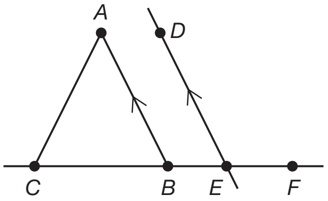
22. Use an inequality to indicate the minimum length of the longest side, x, of $\triangle ABC$ if the two shorter sides of the triangle measure 8 and 7 and $\triangle ABC$ is obtuse.
(33)

23. Calculate the area of sector QRS in the diagram to the nearest hundredth of a centimeter.
(35)



24. Davindra, who is 15 years old, is one year older than twice the age of her younger sister, Preetha. Find the age of Preetha and justify each step.
(24)

25. In the diagram, $\overline{AB} \parallel \overline{DE}$. Write a paragraph to prove that $m\angle CAB + m\angle ACB = m\angle DEF$.
(31)



26. A circle with a radius of 6 meters has an arc that measures 35°. If this arc and its associated sector are completely removed from the circle, what is the length of the major arc that remains, to the nearest tenth of a meter?
(35)

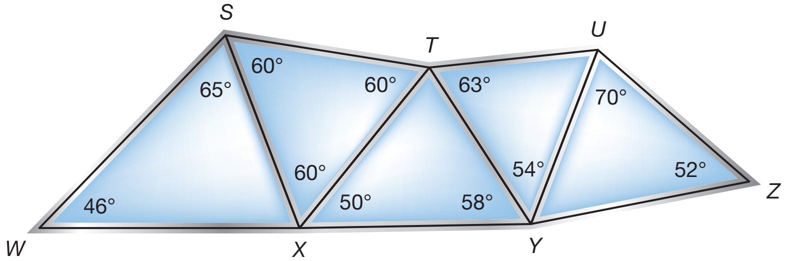
27. Classify the following triangles as acute, obtuse, or right, based on the three side lengths given.
(33)

a. 4, 4, $\sqrt{34}$

b. 6, 5, 8

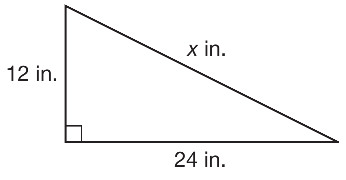
c. 10, 24, 26

***28.** **Construction** An important part of many structures is the structural support truss, a series of interlocking metal rods that form triangles, which gives the structure its rigidity. The diagram below shows a section of a support truss for a geodesic dome. What is the shortest length of metal used in this section of a geodesic dome structural support truss?
(39)



***29.** **Predict** If line a is perpendicular to line b and line c is perpendicular to line b, describe the relationship between lines a and c.
(37)

30. Find the value of x in the figure shown. Give your answer in simplified radical form.
(29)



LESSON 40

Finding Perimeters and Areas of Composite Figures

Warm Up

1. (8) **Vocabulary** The size of the region bounded by a closed geometric figure is that figure's _____.

2. (13) Determine the perimeter of a right triangle with a 25-mm base and a 60-mm height.

3. (22) Which is the correct formula for the area of a trapezoid?

   A $A = \frac{1}{2}bh$ **B** $A = \frac{1}{2}(b_1 + b_2)h$

   C $A = bh$ **D** $A = (b_1 + b_2)h$

New Concepts

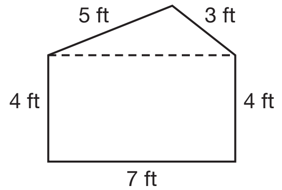
A **composite figure** is a plane figure made up of simple shapes or a three-dimensional figure made up of simple three-dimensional figures. The perimeter of a plane composite figure is the sum of the lengths of its sides.

Math Reasoning

Generalize Explain why the perimeter of a composite figure cannot be found by adding the perimeters of the shapes that compose it.

Example 1 Finding Perimeters of Composite Figures

Find the perimeter of the composite figure.



SOLUTION

Start at a vertex and move around the figure, adding the side lengths in order.

$P = 3 + 5 + 4 + 7 + 4$

$= 23$ ft

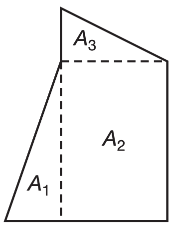
The perimeter is 23 feet.

Postulate 19: Area Congruence Postulate
If two polygons are congruent, then they have the same area.

Postulate 20: Area Addition Postulate

The area of a region is equal to the sum of the areas of its nonoverlapping parts.

In the diagram, $A = A_1 + A_2 + A_3$.



Online Connection
www.SaxonMathResources.com

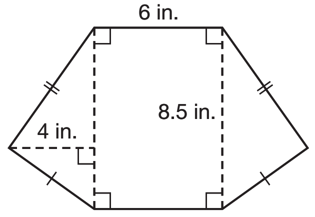
Hint

Sometimes a figure will need to be divided into parts in order to find the area. Look for parts of the figure that appear to be rectangles or triangles and draw dotted lines to indicate how the figure should be divided.

The Area Congruence Postulate and the Area Addition Postulate make it possible to find the area of complex composite figures by breaking them down into simpler shapes and finding the area of each shape.

Example 2 Finding Areas of Composite Figures

Find the area of this composite figure.



SOLUTION

Left triangle:

$A_1 = \frac{1}{2}bh$

$A_1 = \frac{1}{2}(8.5)(4) = 17 \text{ in}^2$

Therefore, the area of the left triangle is 17 in^2.

Rectangle:

$A_2 = bh$

$A_2 = (8.5)(6) = 51 \text{ in}^2$

Therefore, the area of the rectangle is 51 in^2.

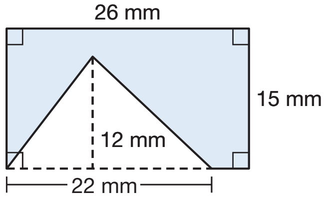
By the SSS Postulate, the two triangles in the figure are congruent. Therefore, by Postulate 19, the right triangle has the same area as the left triangle, 17 square inches.

By the Area Addition Postulate, the area of the composite figure is

$A = 17 + 51 + 17 = 85$ square inches.

Example 3 Finding Areas of Composite Figures by Subtracting

Find the area of the shaded region.



SOLUTION

Outer rectangle:

$A_1 = bh$

$A_1 = (26)(15) = 390 \text{ mm}^2$

Therefore, the area of the outer rectangle is 390 mm^2.

"Missing" triangle:

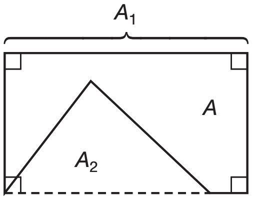
$A_2 = \frac{1}{2}bh$

$A_2 = \frac{1}{2}(22)(12) = 132 \text{ mm}^2$

Therefore, the area of the triangle is 132 mm^2.

The Area Addition Postulate implies that the area of the rectangle is the area of the region we want to find plus the area of the triangle:

$A_1 = A + A_2$.



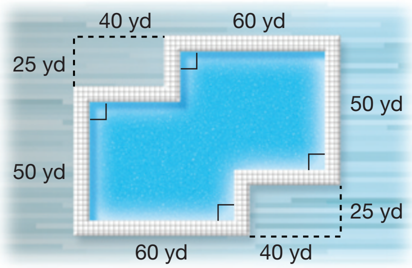
Therefore,

$$390 = A + 132$$
$$A = 258 \text{ mm}^2$$

The area of the shaded region is 258 mm^2.

Example 4 Application: Architecture

The diagram shows the plan for a reflecting pool that will form part of a new downtown plaza.



a. The edge of the pool needs to be tiled. What is the perimeter of the pool that will need tiling?

SOLUTION

$P = 25 + 40 + 50 + 60 + 25 + 40 + 50 + 60$

$= 350$ yd

Therefore, the perimeter of the pool is 350 yards.

b. What area of concrete will be needed for the bottom of the pool?

SOLUTION

Outer rectangle:

$A_1 = bh$

$= (40 + 60)(25 + 50)$

$A_1 = (100)(75) = 7500 \text{ yd}^2$

Therefore, the area of the outer rectangle is 7500 yd^2.

Each "missing" rectangle:

$A_2 = bh$

$A_2 = (40)(25) = 1000 \text{ yd}^2$

Therefore, the area of each "missing" rectangle is 1000 yd^2.

Let A be the area of the pool bottom. By the Area Addition Postulate,

$$A_1 = A + A_2 + A_2$$
$$7500 = A + 1000 + 1000$$
$$7500 - 2000 = A$$
$$A = 5500 \text{ yd}^2$$

5500 square yards of concrete will be needed.

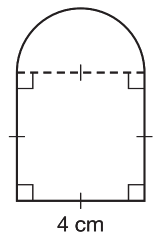
Math Reasoning

Formulate A semicircle has radius r. Give a formula to find the area and the perimeter of the semicircle.

Lesson Practice

a. *(Ex 1)* A composite figure made up of a rectangle and a triangle has a 3-inch horizontal side, two 4.5-inch vertical sides, and two 4-inch sloping sides. What is its perimeter?

Use this composite figure to answer problems b and c.



b. *(Ex 2)* Determine the area of the figure. Express your answer in terms of π.

c. *(Ex 3)* A rectangle with dimensions 3-by-1.5 centimeters is removed from the bottom left corner of the figure. Determine the area of the new figure. Express your answer in terms of π.

d. *(Ex 4)* A new office building has one side in the shape of a right triangle on top of a rectangle. The rectangle is 420 feet tall and 120 feet wide. The triangle's base is 120 feet long and its vertical leg is 160 feet long. What is the perimeter of this side of the building?

e. *(Ex 4)* How much glass is needed to cover the side of the building in part **d**?

Practice Distributed and Integrated

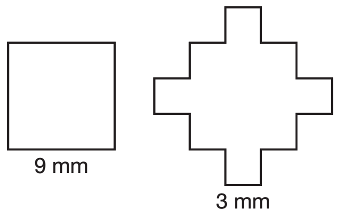
1. (37) **Multi-Step** Find the slope of the line that passes through the points $X(2, -3)$ and $Y(4, 1)$. Then, find a perpendicular line that passes through point $Z(-2, 2)$.

2. (29) **Error Analysis** Isabelle has determined that the legs in a right triangle are 21 centimeters and 75 centimeters long, and the hypotenuse is 72 centimeters long. What error has she made?

3. (33) If a triangle has side lengths of $2x$, $3x$, and $4x$, is it an acute, obtuse, or a right triangle?

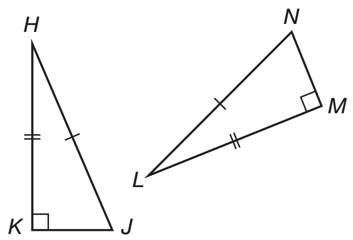
4. (37) Find the line perpendicular to $y = \frac{1}{4}x + 7$ that passes through the origin.

* **5.** (40) These figures are the first two stages of an abstract design. Determine the perimeter and area of each figure.



6. (21) **Write** For two statements p and q, what can be determined about the truth value of p when "if p, then q" is a true statement and q is false? Why?

7. (36) What congruence theorem is used to prove that $\triangle HJK \cong \triangle LNM$? Explain.

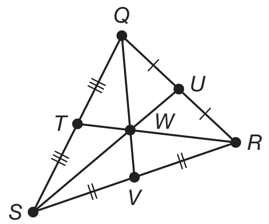


8. (23) The U.S. dime has a radius of 8.96 millimeters. Estimate the area of one side of a dime to the nearest whole square millimeter.

* **9.** (38) Use a compass and a straightedge to find the incenter of a triangle whose vertices are at (0, 0), (5, −3), and (−1, −4) in a coordinate plane.

10. (29) Antoine solved for the length of a hypotenuse in a right triangle and found that $c^2 = 490$. What is the value of c, in simplified radical form?

11. (32) In the diagram, $QW = 14$ and $TR = 9$. Find QV and TW.

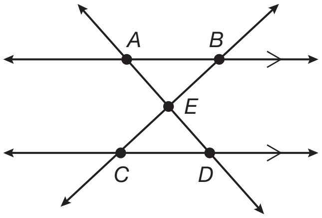


12. (26) **Track and Field** Tara is running along a circular track and Petra is standing at the center of the circle. Petra observes Tara running an arc measure of 56° clockwise, then a measure of 34° counterclockwise, and then a measure of 67° clockwise. What is the measure of the arc from her starting point to where Tara stands now?

***13.** *(39)* If a triangle were constructed with the smallest third side possible, which angle would measure the greatest, if the two given sides measure 14 yards and 31 yards?

14. *(35)* In $\odot C$, $\overset{\frown}{AB}$ measures 65°, and the area of sector ABC is 9 square units. What is the radius of $\odot C$, to the nearest hundredth unit?

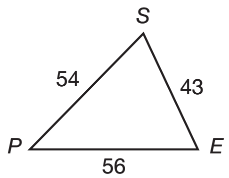
15. *(31)* **Error Analysis** Using the diagram, Lily decided that $\triangle ABE \cong \triangle CDE$. She wrote a paragraph proof, which is given below. What mistake did Lily make?



The diagram shows that $\overline{AB} \parallel \overline{CD}$. *Therefore, because alternate interior angles are congruent,* $\angle BAE \cong \angle EDC$ *and* $\angle ABE \cong \angle ECD$. *Since* $\angle AEB$ *and* $\angle CED$ *are vertical angles,* $\angle AEB \cong \angle CED$. *Since the corresponding angles of the triangles are all congruent,* $\triangle ABE \cong \triangle DCE$.

***16.** *(40)* A composite figure is made up of a rectangle that is 4 inches wide by 10 inches long and two semicircles that fit exactly on two adjacent sides of the rectangle. Determine the figure's perimeter and area. Express your answers in terms of π.

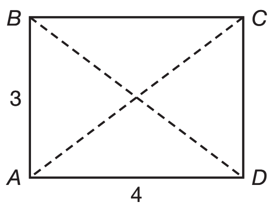
***17.** *(39)* Write the angles of $\triangle SEP$ in order from least to greatest measure.



18. *(36)* In triangles UVW and XYZ, $\angle V$ and $\angle Y$ are right angles, $\overline{UW} \cong \overline{XZ}$, and $\angle U \cong \angle X$. Use the Hypotenuse-Angle Congruence Theorem to prove that $\triangle UVW \cong \triangle XYZ$.

19. *(26)* **Security** A camera covers an arc measure of 45°. What fraction of a full rotation does the camera cover? How many cameras would be needed to be able to look in all directions?

20. *(34)* In the rectangle $ABCD$, $BD = 2x - 3$. What is the value of x? What is the length of $\overline{AC}$?



21. *(23)* **Forestry** A tree's circular trunk has a circumference of 4 feet 9 inches. To the nearest inch, how wide is its trunk?

22. *(37)* **Multiple Choice** Find the pair of perpendicular lines.

A $2x - 3y = 4$, $y = -\frac{2}{3}x - 3$

B $y = \frac{1}{2}x + 7$, $y = 2x - 5$

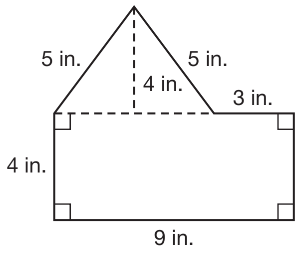
C $-3y + 2x + 7 = 0$, $-3x + 2y - 4 = 0$

D $2x + y = 8$, $x - 2y = 1$

23. (Inv 3) **Write** Explain why the size of each exterior angle in a regular polygon decreases as the number of sides increases.

***24.** (39) **Justify** Can a triangle have sides with the lengths of 8 feet, 8 feet, and 16 feet?

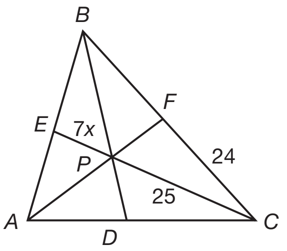
Use the diagram to complete the problems.



***25.** (40) Determine the perimeter of the figure.

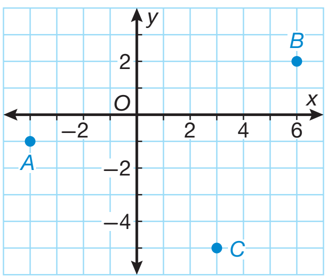
***26.** (40) Determine the area of the figure.

***27.** (38) **Algebra** Find the value of x that makes P the incenter of the triangle shown.



28. (29) **Driving** A car drives 20 miles east and then 45 miles south. To the nearest hundredth of a mile, how far is the car from its starting point?

Use the given diagram. Round to the nearest hundredth.



29. (9) Find the length of $\overline{AB}$.

30. (9) Find the length of $\overline{BC}$.

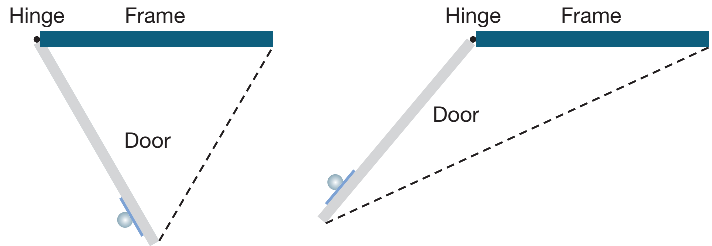
INVESTIGATION

4

Inequalities in Two Triangles

In Lesson 39, you learned to verify whether three segments of given lengths could be used to form a triangle. This involved an inequality in one triangle. In this investigation, you will explore inequalities in two triangles.

1. Consider a door hinge. As shown in the diagram, the doorframe, door, and the distance between them make a triangle. The doorframe and the door itself are always the same length, so two sides of this triangle are fixed. What happens to the third side as the door is opened?



2. **Model** Use a protractor to construct two triangles. Your triangles should have two pairs of congruent sides, and a third side that changes length depending on how much you open the "hinge," which is the included angle of the fixed sides. The hinge angles on your two triangles should have different measures.

3. **Write** Consider the door hinge example from step 1. If two triangles have two pairs of congruent sides, but their included angles are not congruent, what conclusion can you make about the third side?

Math Language

An **included angle** is an angle that is formed by two adjacent sides of a polygon. A hinge, like the one on a door, is an included angle of the doorframe and the door itself.

Theorem 40-1: Hinge Theorem
If two sides of one triangle are congruent to two sides of another triangle and the included angle of the first triangle is greater than the included angle of the second triangle, then the third side of the first triangle is longer than the third side of the second triangle.

The Hinge Theorem is useful when comparing two triangles with two pairs of congruent sides. Is the converse of the Hinge Theorem also true? Follow the steps below.

4. Draw an angle with side segments that are 4 and 5 inches long. Connect their endpoints to make a triangle. Label the vertex across from the 4-inch side A, the vertex across from the 5-inch side B, and the last vertex C. Measure $\overline{AB}$ and label its length on your diagram.

5. Draw a second angle, with a different measure from the first, that also has side segments that are 4 and 5 inches long. Connect their endpoints to make a triangle. Label the vertex across from the 4-inch side D, the vertex across from the 5-inch side E, and the last vertex F. Measure $\overline{DE}$ and label its length.

6. Measure the angles at vertices *C* and *F* using a protractor. What do you notice about these angles in comparison to each other and to the lengths of the sides opposite them?

7. The two triangles you drew illustrate the converse of the Hinge Theorem. State the converse of the Hinge Theorem, and determine whether it is true for your triangles.

Math Reasoning

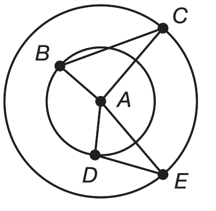
Analyze The Hinge Theorem is a more complex conditional statement than you have seen, taking the form "If (*p* and *q*), then *r*." Write out the statements *p*, *q*, and *r*.

Theorem 40-2: Converse of the Hinge Theorem

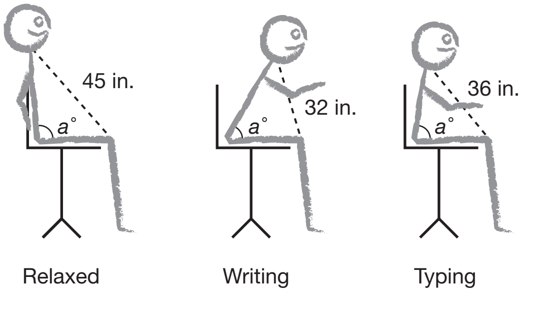
If two sides of one triangle are congruent to two sides of another triangle and the third side of the first triangle is longer than the third side of the second triangle, then the measure of the angle opposite the third side of the first triangle is greater than the measure of the angle opposite the third side of the second triangle.

The Hinge Theorem and the Converse of the Hinge Theorem can be used to compare two triangles.

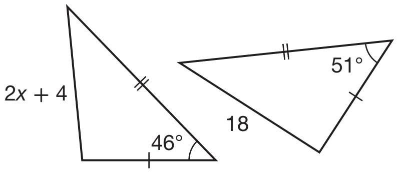
8. **Multi-Step** Concentric circles are circles that have the same center but different radii measures. The circles illustrated here are concentric. The measure of $\angle BAC$ is 93°, and the measure of $\angle DAE$ is 60°. Explain why BC must be greater than DE.



9. **Ergonomics** Depending on the task a person is performing, the angle that a person's torso makes with their legs changes. The diagrams below show a student in three different sitting positions: relaxed, writing, and typing. In which position is the angle measure at the hip the greatest? … the least? Explain how you know.

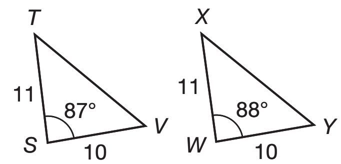


10. Write an inequality that gives the possible values of *x* in the diagram.

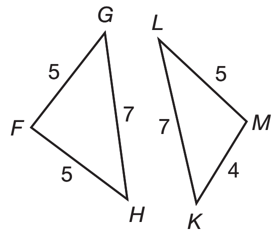


Investigation Practice

a. Use an inequality to compare the lengths of TV and XY.



b. Use an inequality to compare the measures of $\angle G$ and $\angle L$.

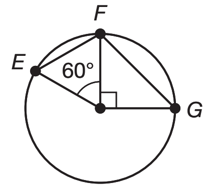


c. **Write** Describe how the hood of a car illustrates the Hinge Theorem.

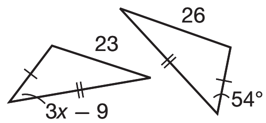
d. **Multiple Choice** Choose the most correct answer for the given diagram.

A $EF = FG$ **B** $EF < FG$

C $EF > FG$ **D** not enough information



e. **Algebra** Find the range of values for x.



f. **Door Hinges** To prevent a door from opening too far and hitting a wall, a doorstop can be placed on the hinge of the door. A door in a small closet swings through a straight-line distance of 40 inches, while another door in a washroom swings through a straight-line distance of 48 inches. Which doorstop needs to be set to open to a larger angle? Explain.

g. **Playground Equipment** Kelvin and his friend Theo are swinging on a swing set at the local park. Both swings are the same length. Kelvin swings through an angle of 47° and Theo swings through an angle of 44°. Which of the two friends is swinging through the greatest distance? Explain.