LESSON
41 Ratios, Proportions, and Similarity

Warm Up

1. (SB) **Vocabulary** A statement showing that two ratios are equal is called a(n) __________. (*proportion, inequality, direct variation*)

2. (25) Is this statement true or false?
If all three sides of one triangle are congruent to all three sides of another triangle, the two triangles are congruent.

3. (25) Given $\triangle ABC \cong \triangle DEF$, tell which sides are congruent.

4. (30) If in $\triangle XYZ$ and $\triangle CAM$, $\angle X \cong \angle C$, $\overline{XY} \cong \overline{CA}$, and $\angle Y \cong \angle A$, which postulate or theorem can prove the triangles congruent?

New Concepts

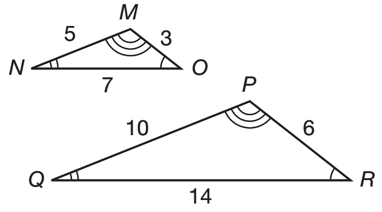
A **ratio** is a comparison of two values by division. The ratio of two quantities, a and b, can be written in three ways: a to b, a:b, or $\frac{a}{b}$ (where $b \neq 0$). A statement that two ratios are equal is called a **proportion**.

Math Reasoning

Write Explain the difference between a ratio and a proportion.

Example 1 Writing Ratios and Proportions

Consider $\triangle MNO$ and $\triangle PQR$.



a. Write a ratio comparing the lengths of segments $\overline{MN}$ to $\overline{NO}$ to $\overline{OM}$.

SOLUTION

The quantity that is mentioned first in a ratio is written first.

$MN = 5$, $NO = 7$, $OM = 3$. Therefore $MN:NO:OM = 5:7:3$.

b. Write a ratio comparing MN to PQ in three ways.

SOLUTION

$MN = 5$ and $PQ = 10$. The ratio of MN to PQ can be written as 5 to 10, 5:10, or $\frac{5}{10}$. Ratios can be reduced just like fractions. Reducing this ratio results in $\frac{1}{2}$.

c. Write a proportion to show that $MN:PQ = NO:QR$

SOLUTION

$MN:PQ = NO:QR$ can be written as $\frac{5}{10} = \frac{7}{14}$. Notice that the proportion is true since both sides of the proportion reduce to $\frac{1}{2}$.

In the proportion $\frac{a}{b} = \frac{c}{d}$, a and d are the **extremes**, and b and c are the **means**. One way to solve a proportion to find a missing value is to use cross products. The **cross product** is the product of the means and the product of the extremes. In other words, if $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$.

Online Connection
www.SaxonMathResources.com

Example 2 Solving Proportions with Cross Products

Solve the proportion $\frac{3}{15} = \frac{x}{50}$ to find the value of x.

SOLUTION

Find the cross products to solve the proportion.

$$\frac{3}{15} = \frac{x}{50}$$

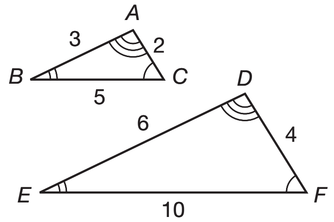
$$3 \times 50 = 15x$$

$$x = 10$$

Reading Math

The symbol ∼ shows that two figures are similar, and should be read "is similar to."

Two figures that have the same shape, but not necessarily the same size, are **similar**. In the diagram, $\triangle ABC$ is similar to $\triangle DEF$. All congruent figures are also similar figures, but the converse is not always true.



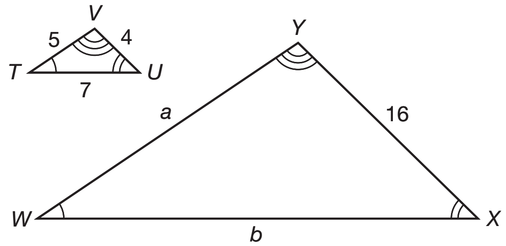
In **similar polygons**, the corresponding angles are congruent and the corresponding sides are proportional. In the diagram, $\triangle ABC$ and $\triangle DEF$ have congruent angles, and each pair of their corresponding sides has the same proportional relationship. A **similarity ratio** is the ratio of two corresponding linear measurements in a pair of similar figures. The following similarity ratios can be written for $\triangle ABC$ and $\triangle DEF$.

$$\frac{DE}{AB} = \frac{6}{3} = 2 \qquad \frac{EF}{BC} = \frac{10}{5} = 2 \qquad \frac{FD}{CA} = \frac{4}{2} = 2$$

Like congruence, similarity is a transitive relation. The Transitive Property of Similarity states that if $a \sim b$, and $b \sim c$, then $a \sim c$.

Example 3 Using Proportion to Find Missing Lengths

Find the unknown side lengths in the two similar triangles.



SOLUTION

The triangles are similar so corresponding sides are proportional: $\frac{VT}{YW} = \frac{TU}{WX} = \frac{UV}{XY}$.

Therefore, $\frac{5}{a} = \frac{7}{b} = \frac{4}{16}$.

Hint

Using the Symmetric Property, the cross product $ad = bc$ may also be written as $bc = ad$.

Solve each proportion using a known ratio and a ratio with an unknown.

$$\frac{5}{a} = \frac{4}{16} \qquad \frac{7}{b} = \frac{4}{16}$$

$$4a = 5 \times 16 \qquad 4b = 7 \times 16$$

$$a = 20 \qquad b = 28$$

Therefore, $WX = 28$ and $YW = 20$.

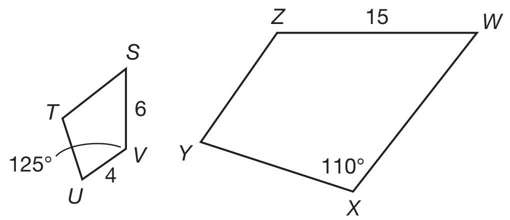
A **similarity statement** is a statement indicating that two polygons are similar by listing their vertices in order of correspondence. Much like writing a congruence statement, corresponding angles have to be named in the same order.

Hint

When setting up proportions, all your ratio's numerators should come from the same figure, and the denominators should all come from the same figure as well. It does not matter which figure's sides you use for the numerator or the denominator.

Example 4 Finding Missing Measures of Similar Polygons

In the diagram, $STUV \sim WXYZ$.



a. What are the measures of $\angle T$ amd $\angle Z$?

SOLUTION

The quadrilaterals are named in order of corresponding vertices, and corresponding angles of similar figures are congruent. Therefore, $\angle S \cong \angle W$, $\angle T \cong \angle X$, $\angle U \cong \angle Y$, and $\angle V \cong \angle Z$.

$m\angle T = m\angle X = 110°$, and $m\angle Z = m\angle V = 125°$.

b. What is the length of $\overline{YZ}$?

SOLUTION

In the quadrilaterals, $\overline{SV}$ and $\overline{WZ}$ are corresponding sides and $\overline{UV}$ and $\overline{YZ}$ are corresponding sides. Therefore, $\frac{SV}{WZ} = \frac{UV}{YZ}$. Substitute the values:

$$\frac{6}{15} = \frac{4}{YZ}$$

$$6 \times YZ = 15 \times 4$$

$$YZ = 10$$

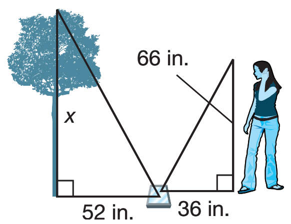
The length of $\overline{YZ}$ is 10 units.

Math Reasoning

Generalize Are all right triangles similar? Explain.

Example 5 Application: Optics

Siobhan is using a mirror and similar triangles to determine the height of a small tree. She places the mirror at a distance where she can see the top of the tree in the mirror. According to the measures in Siobhan's triangles, what is the height of the tree to the nearest inch?



SOLUTION

The two triangles are similar, so corresponding sides are proportional.

$$\frac{x}{66} = \frac{52}{36}$$

$$36x = 66 \times 52$$

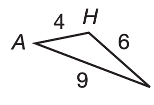
$$x \approx 95$$

The tree is about 95 inches tall.

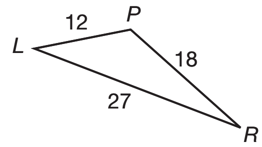
Lesson Practice

Use the two similar triangles to answer *a* through a through *c*.

a. *(Ex 1)* Write a ratio comparing the lengths of segments $\overline{HA}$ to $\overline{AM}$ to $\overline{MH}$.



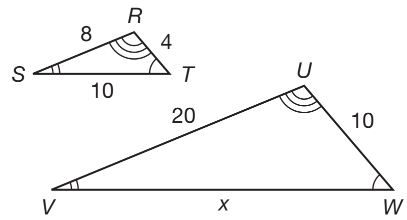
b. *(Ex 1)* Write a ratio comparing AM to LR in three ways, in simplest form.



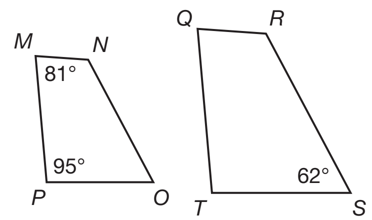
c. *(Ex 1)* Write a proportion to show that $HM{:}PR = AM{:}LR$.

d. *(Ex 2)* What is the value of x in the proportion $\frac{8}{7} = \frac{x}{21}$?

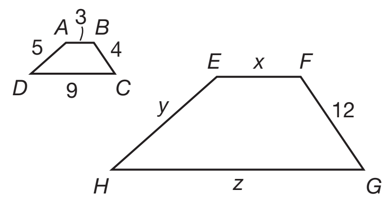
e. *(Ex 3)* If $\triangle RST \sim \triangle UVW$, find the missing length in $\triangle UVW$.



f. *(Ex 4)* If the polygons $MNOP$ and $QRST$ are similar, what are the measures of $\angle O$ and $\angle R$?



g. *(Ex 4)* If the polygons $ABCD$ and $EFGH$ are similar, what are the values of x, y, and z?

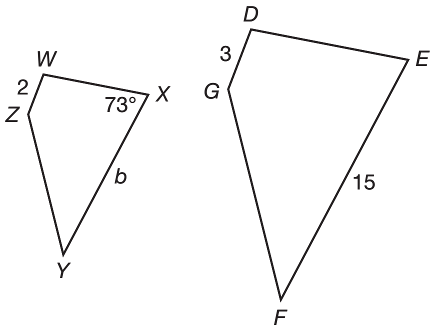


h. *(Ex 5)* Cree uses a 21-foot ladder and a 12-foot ladder while painting the exterior of a house. Each ladder forms the same angle with the ground. If the longer ladder reaches 18 feet up the wall, how high does the other ladder reach, to the nearest foot?

Practice Distributed and Integrated

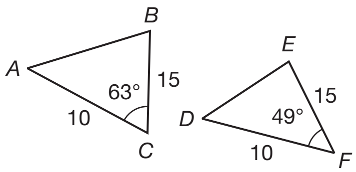
1. **Error Analysis** Alexander made a convex pentagon and a convex quadrilateral. He measured and then added the measures of all the exterior angles, finding a sum of 900°. Is this possible? Explain.
(Inv 3)

* 2. If the quadrilaterals at right are similar, what is the measure of $\angle E$? What is the length of $\overline{XY}$?
(41)



3. **Reunions** Three friends who live far away from each other want to meet for a reunion. Explain how the friends might go about finding a location for the reunion that is equidistant from each friend.
(38)

4. Compare the lengths of $\overline{AB}$ and $\overline{DE}$.
(Inv 4)

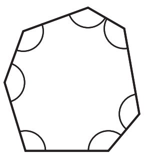


5. **Predict** If line l is perpendicular to line m and line n is parallel to line m, describe the relationship between lines l and n.
(37)

* 6. What is the value of x in the proportion $\frac{9}{x+2} = \frac{27}{9}$?
(41)

7. If $x = 5$ and $y = x$, which property of equality can be used to conclude that $y = 5$?
(24)

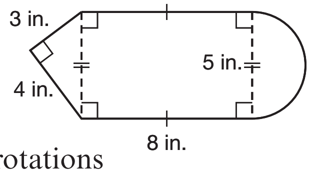
8. Name this polygon. Is it equiangular? Is it equilateral? Is it regular or irregular?
(15)



9. **Generalize** Given that the sum of all the angle measures in a triangle is 180°, prove that the interior angles in convex quadrilateral $ABCD$ sum to 360°.
(27)

10. **Verify** Use the formula for the sum of the interior angles of a convex polygon to verify the Triangle Sum Theorem.
(Inv 3)

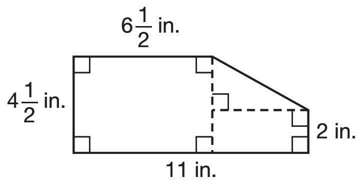
*11. **Multi-Step** Determine the area of this figure. Express your answer in terms of π.
(40)



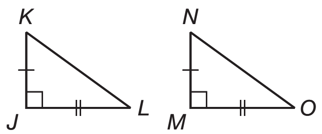
12. **Wheels** A car's wheels are 18 inches across. To the nearest hundredth of an inch, how far will the car move with four complete rotations of its wheels?
(23)

13. Give a Pythagorean triple that is proportional to the triple (7, 24, 25).
(29)

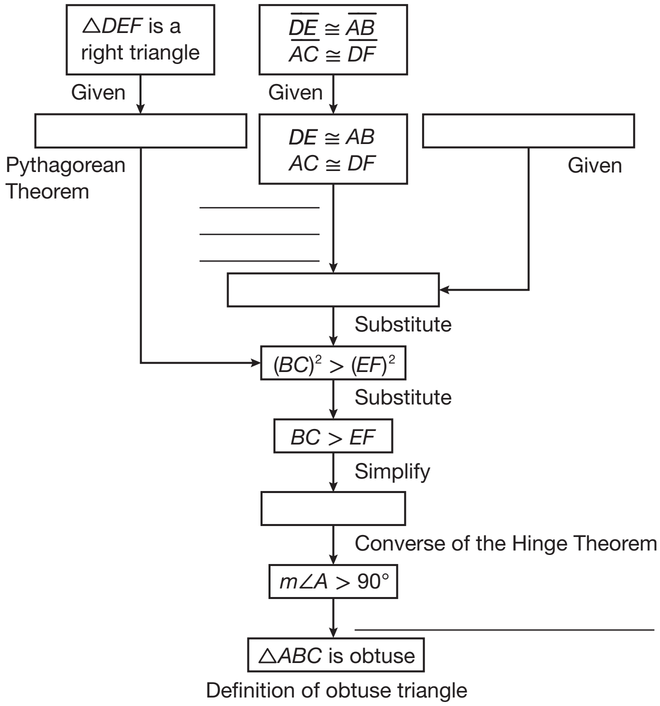
14. **Engineering** Determine the area of this metal plate.
(40)



15. Triangle JKL is congruent to $\triangle MNO$. What triangle is $\triangle OMN$ congruent to?
(25)

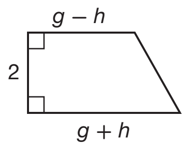


***16.** Complete the flowchart to prove the Pythagorean Inequality Theorem.
(31)
Given: $(BC)^2 > (AB)^2 + (AC)^2$, $\triangle DEF$ is a right triangle, $\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$
Prove: $\triangle ABC$ is an obtuse triangle.



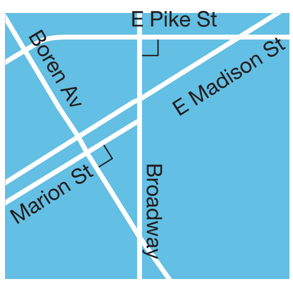
17. In a triangle with vertex D, $\overline{DM}$ is a median, and C is the centroid of the triangle. What is the length of $\overline{DC}$ if $\overline{DM}$ measures 70.2 centimeters? What theorem or postulate justifies your answer?
(32)

18. What is an expression for the area of the trapezoid?
(22)



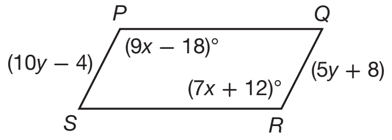
19. In $\triangle MNO$, $m\angle MNO = 37°$, $m\angle NOM = 53°$, $NO = 50$ feet, $MO = 40$ feet, and $MN = 30$ feet. In $\triangle RST$, $m\angle RST = 53°$, $m\angle TRS = 37°$, and $RS = 50$ feet. Determine all other lengths.
(30)

20. **Geography** Two right triangles have been marked on a map of Seattle, WA. The distance from Pike Street to Broadway along E. Madison Street is equal to the distance from Marion Street to Boren Avenue along Broadway. Also, Marion Street and E. Madison Street run parallel. Prove that the triangles are congruent.
(36)



***21.** **Photography** A rectangular 8-by-10-inch photograph has to be reduced to $\frac{1}{4}$ its original dimensions to be placed in a magazine. What will be the dimensions of the reduced photograph?
(41)

Use the figure to find the unknown measures. Quadrilateral *PQRS* is a parallelogram.



22. (34) Solve for x, then find the measure of each interior angle of the parallelogram.

23. (34) Solve for y, then find the perimeter if $PQ = 2PS$.

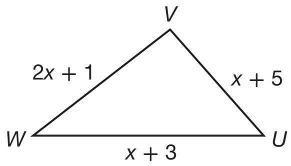
24. (Inv 4) Fill in the blanks with the correct words.

If two sides of one ________ are congruent to two sides of another triangle and the included _______ of the first triangle is greater than the included angle of the second triangle, then the third side of the first triangle is longer than the third _____ of the second triangle.

25. (39) **Write** Can a triangle have a side length that is the sum of the other two side lengths? Explain.

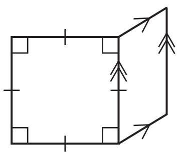
26. (30) If in $\triangle ABC$, $AB = 10$, $m\angle ABC = 5°$, and $m\angle CAB = 60°$, and in $\triangle DEF$, $m\angle EFD = 5°$, $m\angle DEF = 60°$, and $FE = 10$, write a congruency statement for these triangles.

27. (39) **Algebra** If $x = 1$, which angle would have the least measure in the triangle?



28. (23) For two circles, the first circle has a radius of 0.5 units, and the second radius is the circumference of the first. Using 3.14 for π, what is the area of the second circle?

29. (40) **Multiple Choice** Which two formulas are used to calculate the area of this composite figure?



A $A_1 = 4s,\ A_2 = \frac{1}{2}bh$

B $A_1 = \frac{1}{2}\ell w,\ A_2 = bh$

C $A_1 = s^2,\ A_2 = bh$

D $A_1 = \ell w,\ A_2 = \frac{1}{2}bh$

30. (22) **Write** Explain why you can use the formula, $A = \frac{1}{2}d_1 d_2$, for both the area of rhombuses and the area of squares.

LESSON 42

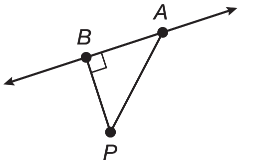
Finding Distance from a Point to a Line

Warm Up

1. (5) **Vocabulary** Two lines that meet at 90° are ____________.
2. (8) State the formula for the Pythagorean Theorem: __________.
3. (9) The formula to find the distance between two points is: ______________________.
4. (16) What is the slope of the line $y = 3x + 1$?
5. (37) The slope of the line perpendicular to $y = -2x + 1$ is __.

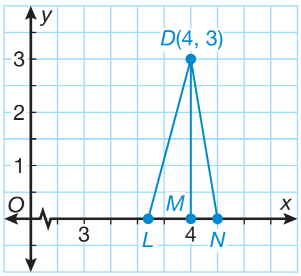
New Concepts

Given a line $\overleftrightarrow{AB}$ and a point P, what is the shortest distance between P and $\overleftrightarrow{AB}$? Notice that $\triangle ABP$ is a right triangle, and $\overline{AP}$ is the hypotenuse. The hypotenuse is always the longest side of a right triangle, so $\overline{AP}$ must be longer than $\overline{PB}$.



Example 1 Choosing the Closest Point

Which point on the line $y = 0$ is closest to point D—$L(3.6, 0)$, $M(4, 0)$, or $N(4.25, 0)$?



SOLUTION

Use the distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$DL = \sqrt{(3.6 - 4)^2 + (0 - 3)^2} \approx 3.03$$

$$DM = \sqrt{(4 - 4)^2 + (0 - 3)^2} = 3$$

$$DN = \sqrt{(4.25 - 4)^2 + (0 - 3)^2} \approx 3.01$$

M is the closest point to D.

Math Reasoning

Verify Consider the point (0, 3). The perpendicular line from this point to the x-axis is 3 units long. Is there another segment from this point to the x-axis that is shorter? Try using the distance formula to find the distance from (0, 3) to (1, 0) or (−1, 0). Is the distance greater or less?

Theorem 42-1
Through a line and a point not on the line, there exists exactly one perpendicular line to the given line.

Theorem 42-2
The perpendicular segment from a point to a line is the shortest segment from the point to the line.

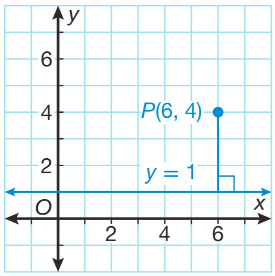
Theorem 42-1 indicates that there is only one such segment. The length of a perpendicular segment from a point to a line is referred to as the **distance from a point to a line**.

Example 2 Finding Distance to a Line

a. Find the distance from $P(6, 4)$ to the line $y = 1$.

SOLUTION

The perpendicular distance is the distance between the y-coordinates of the point and the line.

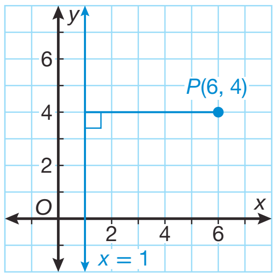


The distance between point P and the line is 3 units.

b. Find the distance from $P(6, 4)$ to the line $x = 1$.

SOLUTION

Point P is 5 units to the right of the line $x = 1$. The perpendicular distance is the difference between the x-coordinates. The distance is 5 units.



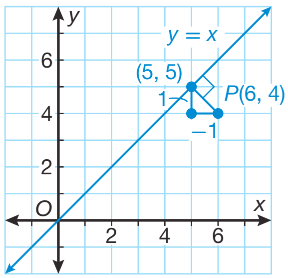
Math Reasoning

Write Describe what happens when you use the distance formula to find the distance between a point and a vertical line.

c. Find the distance from $P(6, 4)$ to the line $y = x$.

SOLUTION

Find the line perpendicular to the line $y = x$ that includes point $P(6, 4)$.



The slope of the perpendicular line is the negative reciprocal of the slope of the line $y = x$, which is -1.

Start at point $P(6, 4)$. Use the slope to find other points on the perpendicular line. Draw the line.

Notice that the lines intersect at (5, 5). Find the distance between (6, 4) and (5, 5) using the distance formula.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance formula

$d = \sqrt{(6 - 5)^2 + (4 - 5)^2}$ Substitute

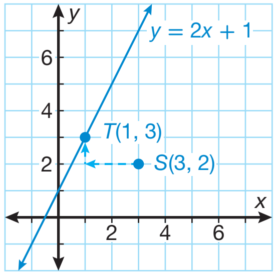
$d = \sqrt{2}$ Simplify

$d \approx 1.414$

So, the distance from the point to the line is about 1.41 units.

Example 3 Finding the Closest Point on a Line to a Point

Given the equation $y = 2x + 1$ and the point $S(3, 2)$, find the point on the line that is closest to S. Find the shortest distance from S to the line.



SOLUTION

Draw the line and the point on a coordinate grid.

Next, find the slope of the line. In this case, the slope is 2.

Find the slope of a line perpendicular to the given line. The negative reciprocal of 2 is $-\frac{1}{2}$.

Use the slope to find more points on the perpendicular line. Draw the line. The lines intersect at (1, 3).

Use the distance formula to find the distance between (3, 2) and (1, 3).

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Distance formula
$d = \sqrt{(1 - 3)^2 + (2 - 3)^2}$	Substitute.
$d = \sqrt{(-2)^2 + (1)^2}$	Simplify.
$d = \sqrt{5}$	Simplify.
$d \approx 2.24$	

So, the distance from the point to the line is about 2.24 units.

Hint

In this example, it is easy to see that the lines intersect at (1, 3). Sometimes, however, it may be necessary to solve the system of equations given by the original line and the perpendicular line to find the exact intersection.

Theorem 42-3
The perpendicular segment from a point to a plane is the shortest segment from the point to the plane.

Because parallel lines are always the same distance from one another, the distance from any point on a line to a line that is parallel is the same, regardless of which point you pick.

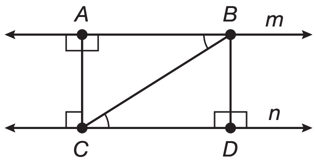
Theorem 42-4
If two lines are parallel, then all points on one line are equidistant from the other line.

Example 4 Proving All Points on Parallel Lines are Equidistant

Prove that if two lines are parallel, then all the points on one line are equidistant from the other line.

Given: $m \parallel n$

Prove: $\overline{AC} \cong \overline{DB}$



SOLUTION

Draw a diagram like the one above, with two parallel lines cut by a transversal. Draw two perpendicular segments from the endpoints of the transversal to the opposite parallel line, forming $\triangle ABC$ and $\triangle DCB$. To prove Theorem 42-4, we need to show that points A and C are the same distance apart as points B and D, or that $\overline{AC} \cong \overline{DB}$.

Statements	Reasons
1. $m \parallel n$	1. Given
2. $\angle CAB \cong \angle BDC$	2. All right angles are congruent
3. $\angle ABC \cong \angle DCB$	3. Alternate Interior Angles
4. $\overline{BC} \cong \overline{CB}$	4. Reflexive Property
5. $\triangle ABC \cong \triangle DCB$	5. AAS Congruence Theorem
6. $\overline{AC} \cong \overline{DB}$	6. CPCTC

Therefore, all points on one line are equidistant from the other line.

Example 5 Application: Delivery Routes

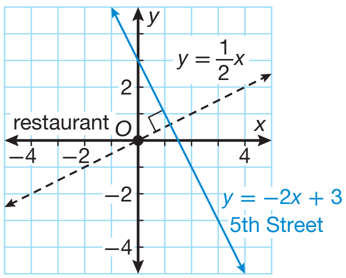
A pizza restaurant only delivers to customers who are less than 3 miles away. The restaurant, located at the origin, receives a call from a customer who lives at the closest point to the restaurant on 5th Street, which can be represented by the line $y = -2x + 3$. If each unit on a coordinate plane represents 1 mile, does this customer live close enough for delivery?

Math Reasoning

Analyze If the restaurant is within 3 miles of a point on 5th Street that is not the closest point, then the closest point will also be within 3 miles. Can you think of such a point on 5th Street?

SOLUTION

Draw the restaurant at the origin of a coordinate plane. Graph the line representing 5th Street, as shown in the diagram. To find the shortest distance the customer could be from the restaurant, find a line perpendicular to $y = -2x + 3$ through the origin.



The opposite reciprocal of 5th Street's slope is $\frac{1}{2}$. Use the point-slope formula to find an equation for the line.

$y - y_1 = m(x - x_1)$ Point-slope form

$y - 0 = \frac{1}{2}(x - 0)$ Substitute.

$y = \frac{1}{2}x$ Simplify.

To find the intersection of these lines, graph them or solve them as a system of equations, as shown below.

$$y = \frac{1}{2}x \qquad\qquad y = -2x + 3$$

$$\frac{1}{2}x = -2x + 3$$
$$x = -4x + 6$$
$$5x = 6$$
$$x = 1.2$$

Substituting this value into either equation will reveal that the y-coordinate for this point is 0.6. Use the distance formula to find the distance between (1.2, 0.6) and the origin.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance equation

$d = \sqrt{(1.2 - 0)^2 + (0.6 - 0)^2}$ Substitute.

$d = \sqrt{1.2^2 + 0.6^2}$ Simplify.

$d \approx 1.34$ mi Simplify.

The distance between these points is less than 3 miles, so the customer is within delivery distance.

Lesson Practice

a. Find the distance between the line $y = 6$ and $(-3, -5)$.
(Ex 2)

b. Find the distance between the line $x = 9$ and $(-3, -5)$.
(Ex 2)

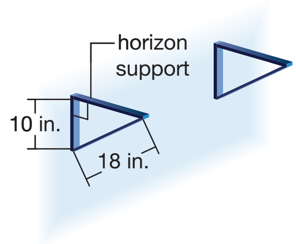
c. Find the distance between the line $y = 2x$ and (6, 2).
(Ex 2)

d. Find the closest point on the line $y = 3x - 1$ to (5, 4).
(Ex 3)

e. Mitchell lives on Woodland Avenue at the closest point to his school. The equation $y = 3x + 3$ can be used to represent Woodland Avenue. His school lies at the origin. The school bus will pick him up only if he lives farther than 2 miles from the school. If each unit on a coordinate plane represents 1 mile, find the distance from his house to the school to determine if he will be allowed to ride the school bus.
(Ex 5)

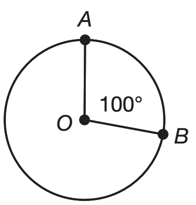
Practice Distributed and Integrated

1. *(36)* **Carpentry** Denzel needs to cut a replacement support for the roof of his house. The existing supports are 18 inches long and fit against the wall 10 inches below the roof. Explain how Denzel can make sure the triangles in this diagram are congruent.



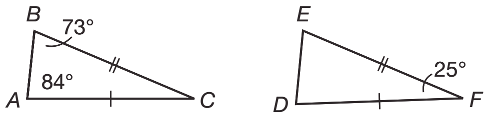
2. *(23)* The outer circumference of Stonehenge, a ring of standing stones in England, is about 328 feet. To the nearest tenth of a foot, what is the length of the radius?

3. *(26)* What is the measure of $\overset{\frown}{AB}$ in this circle?



4. *(33)* The side lengths of a triangle are 5, 12, and c, respectively. If $c = 14$, is the triangle obtuse, acute, or right? …if $c = 13$? …if $c = 12$?

*** 5.** *(Inv 4)* **Multi-Step** Which side is longer, $\overline{AB}$ or $\overline{DE}$? Explain.



*** 6.** *(27)* Complete this two-column proof.
Given: ∠1 and ∠2 are straight angles
Prove: ∠1 ≅ ∠2

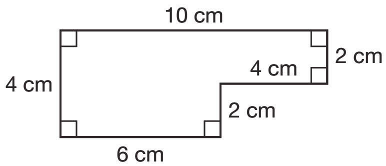
*** 7.** *(41)* **Multiple Choice** The sides of a triangle are 6, 9, and 12 inches long. If the shortest side of a similar triangle is 19.2 inches, what is the longest side of the triangle?
A 25.2 in. **B** 25.6 in.
C 28.8 in. **D** 38.4 in.

*** 8.** *(36)* In △*ABC* and △*DEF*, ∠*B* and ∠*E* are right angles, $\overline{BC} \cong \overline{EF}$, and ∠*C* ≅ ∠*F*. Use the Leg-Angle Congruence Theorem to prove that △*ABC* ≅ △*DEF*.

*** 9.** *(42)* Find the distance between (4, 5) and the line $y = -\frac{3}{2}x - 2$.

10. *(37)* **Algebra** Line *j* passes through points (4, 6) and (3, 2). Line *k* passes through points (x, −1) and (−3, 3). What value of x makes these lines parallel?

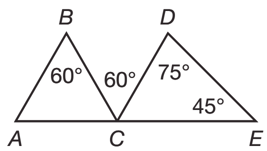
11. *(40)* Determine the area of this figure.



12. *(29)* **Framing** Justine has built a frame for her painting that measures 27 by 36 inches. In order for the corners of the frame to be at right angles, what length must the diagonal be?

13. *(29)* Give two Pythagorean triples that are proportional to the triple 5, 12, 13.

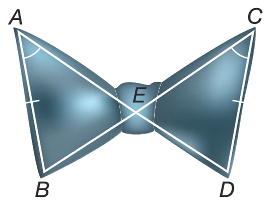
***14.** *(27)* **Analyze** In the diagram shown, prove that m∠*BAC* = m∠*ABC*.



***15.** *(42)* **Analyze** Find the distance between the line $y = 4x - 1$ and the point (4, 15).

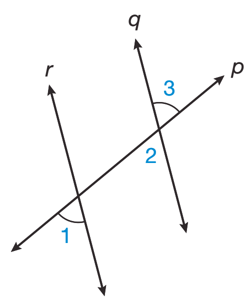
***16.** *(42)* Find the distance from (−2.15, 3.28) to the line $x = 11.3$.

17. *(30)* **Design** Dean wants to make a symmetric bowtie. As in the diagram, he knows that *AB* = *CD*, and m∠*BAE* = m∠*DCE*. Prove, using a two-column proof, that △*ABE* ≅ △*CDE*.



18. *(28)* What is the included side of △*RST* that is between ∠*TRS* and ∠*RST*?

19. *(12)* Use the Converse of the Corresponding Angles Postulate to prove that lines *q* and *r* in this figure are parallel.



20. *(39)* **Write** Three side lengths in a triangle are given by the expressions $2x$, $4x$, and $3x$. Which would be opposite to the angle with the greatest measure? Explain.

21. *(37)* **Multi-Step** Find the slope of the line that passes through the points (3, 4) and (−1, 3). Then find a parallel line that passes through point (4, 2).

22. (24) The steps of the algebraic proof below are correct. However, the justifications for each step are out of order. Determine the correct order for the justifications.

Statements	Reasons
1. $9x = 3(2x + 3)$	1. Simplify
2. $9x = 6x + 9$	2. Division Property of Equality
3. $9x - 6x = 6x + 9 - 6x$	3. Given
4. $3x = 9$	4. Simplify
5. $\frac{3x}{3} = \frac{9}{3}$	5. Subtraction Property of Equality
6. $x = 3$	6. Distributive Property

***23.** (42) Find the distance from $(-1.2, 1.7)$ to the line $x = 3$.

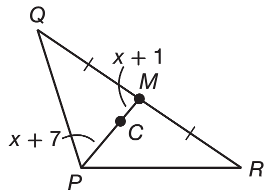
24. (20) State the converse of the following statement, and write the two statements as a biconditional.

If two numbers are positive, then their product is positive.

25. (24) A triangle has an area of 4, a base of $2x + 4$, and a height of $x - 1$. Solve for x and provide a justification of each step.

***26.** (41) **Algebra** If rectangles $KLMN$ and $QRST$ are similar, and $LM = 4$, $RS = 9$, and QR is 5 greater than KL, write and solve a proportion to find the measures of $\overline{KL}$ and $\overline{QR}$.

27. (32) In this diagram, find the value of x and the lengths of $\overline{PC}$, $\overline{CM}$, and $\overline{PM}$ if C is the centroid of the triangle.



***28.** (41) **Generalize** Explain why each statement is true or false.

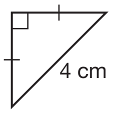
a. *Two squares are always similar.*

b. *A rectangle and a square are always similar.*

29. (33) **Justify** A triangle has side lengths that are 5, 8, and 11. Classify the triangle by its angles. Cite a theorem or postulate to justify your answer.

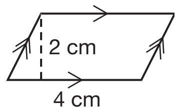
30. (40) **Games and Puzzles** Determine the area of each tangram piece.

a.

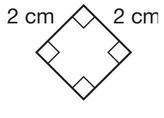


b.

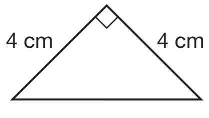
c.



d.



e.



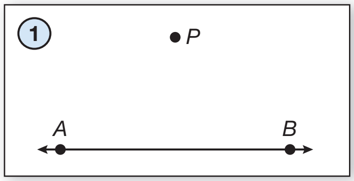
LAB 7

Perpendicular through a Point Not on a Line

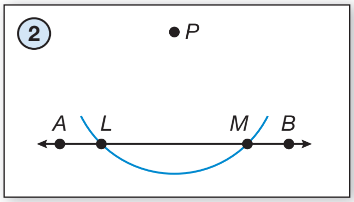
Construction Lab 7 *(Use with Lesson 42)*

Lesson 42 focuses on finding the distance from a given point to a line. This distance is defined by the perpendicular segment from the point to the line. This lab shows you how to construct a line that is perpendicular to a given line through a point not on that line.

1. Begin with line $\overleftrightarrow{AB}$ and a point P not on the line.



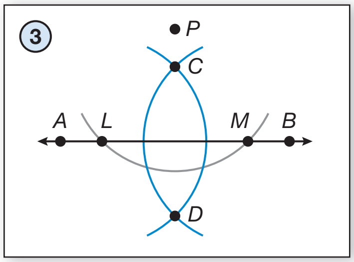
2. Use a compass to draw an arc centered at P that intersects $\overleftrightarrow{AB}$ at two points. Label the points of intersection L and M.



Hint

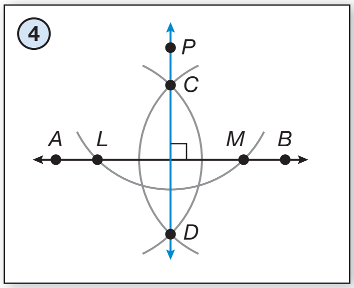
If you use the same compass setting, one of the points where the two arcs intersect will be P.

3. Now construct the perpendicular bisector of $\overline{LM}$ by drawing an arc centered at L that extends above and below $\overleftrightarrow{AB}$, and repeating this with an arc centered at M.



4. Label the points where the two arcs intersect C and D.

   Notice that $\overleftrightarrow{CD}$ passes through P and is perpendicular to $\overleftrightarrow{AB}$.



Lab Practice

For each given line and point, graph the line, then construct a line that is perpendicular to it, and which passes through the given point.

a. $y = 2x - 2$; $(0, 3)$

b. $y = \frac{1}{3}x + 1$; $(5, 6)$

LESSON 43

Chords, Secants, and Tangents

Warm Up

1. (23) **Vocabulary** The distance around a circle is the circle's ____________.

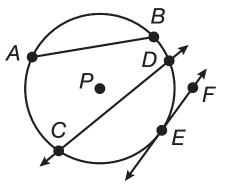
2. (23) A line segment with one endpoint at the center of the circle and the other endpoint on the circle is called a:
   A chord **B** secant
   C radius **D** minor arc

3. (26) The central angle of a circle divides the circle into two arcs. These arcs are called the:
   A major arc and minor arc **B** semicircle and minor arc
   C radius and chord **D** large arc and small arc

4. (23) What is the radius of a circle with an area of 125 cm^2, to the nearest hundredth of a centimeter?

New Concepts

The diameter and radius of a circle are two special segments that can be used to find properties of a circle. There are three more special segments common to every circle. They are chords, secants, and tangents.



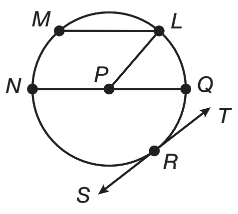
A **chord** is a line segment whose endpoints lie on a circle. In the diagram, $\overline{AB}$ and $\overline{CD}$ are chords of $\odot P$.

A **secant of a circle** is a line that intersects a circle at two points. In the diagram, $\overleftrightarrow{CD}$ is a secant of $\odot P$.

A **tangent of a circle** is a line in the same plane as the circle that intersects the circle at exactly one point, called a **point of tangency**. E is a point of tangency on $\odot P$, and $\overleftrightarrow{EF}$ is a tangent line.

Example 1 Identifying Lines and Segments that Intersect Circles

Use the figure at right to answer parts **a**, **b**, and **c**.



a. Identify three radii and a diameter.

SOLUTION Three radii are $\overline{PL}$, $\overline{PQ}$ and $\overline{PN}$. $\overline{PM}$ and $\overline{PR}$ are also radii. A diameter is $\overline{NQ}$.

Math Reasoning

Analyze Is a diameter also a chord? If so, why?

b. Identify two chords.

SOLUTION One chord is $\overline{ML}$. Another chord is the diameter, $\overline{NQ}$. Since any two endpoints on the circle can make a chord, you could also answer $\overline{MQ}$, $\overline{QR}$, $\overline{QL}$, $\overline{MR}$, $\overline{MN}$, $\overline{NR}$, $\overline{LN}$ and $\overline{LR}$ are also chords.

c. Name a tangent to the circle and identify the point of tangency.

SOLUTION The tangent is $\overleftrightarrow{ST}$. The point of tangency is R.

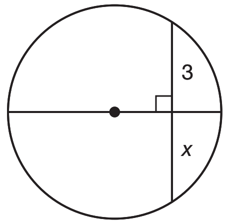
These special segments can be used to find unknown lengths in circles with the help of the theorems presented in this lesson.

Theorem 43-1
If a diameter is perpendicular to a chord, then it bisects the chord and its arcs.

Theorem 43-2
If a diameter bisects a chord other than another diameter, then it is perpendicular to the chord.

Example 2 Finding Unknowns Using Chord-Diameter Relationships

a. Find the length of x.

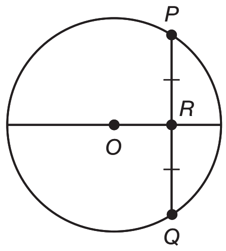


SOLUTION

Since the diameter is perpendicular to the chord, the chord is bisected by the diameter by Theorem 43-1.

So, $x = 3$.

b. Find $m\angle ORP$.

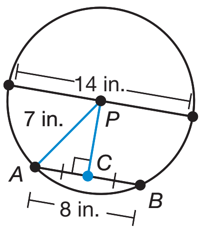


SOLUTION

Since the diameter bisects the chord, the chord must be perpendicular to the diameter by Theorem 43-2.

Thus, $m\angle ORP = 90^\circ$

c. The circle shown has a diameter of 14 inches. Chord $\overline{AB}$ is 8 inches long. How far is $\overline{AB}$ from the center of the circle, to the nearest hundredth of an inch?



SOLUTION

Construct $\overline{AP}$ and $\overline{CP}$ so C is the midpoint of $\overline{AB}$. Since C is the midpoint of $\overline{AB}$, $\overline{AC}$ is 4 inches long. $\overline{AP}$ is a radius of the circle, so it is 7 inches long.

Since $\triangle PAC$ is a right triangle, you can apply the Pythagorean Theorem to find the length of $\overline{CP}$.

$$7^2 = 4^2 + x^2$$
$$49 = 16 + x^2$$
$$33 = x^2$$
$$x \approx 5.74$$

The chord is about 5.74 inches from the center of the circle.

> **Hint**
>
> Finding the lengths of chords or other measurements in the circle will often require drawing an auxiliary line that makes a right triangle with the chord and the diameter of the circle.

Online Connection
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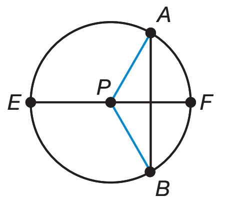
In fact, any segment that is a perpendicular bisector of a chord is also a diameter of the circle. This leads to Theorem 43-3.

Theorem 43-3
The perpendicular bisector of a chord contains the center of the circle.

Every diameter passes through the center of the circle, so another way of stating Theorem 43-3 is that the perpendicular bisector of a chord is also a diameter or a line containing the diameter.

Example 3 Proving the Perpendicular Bisector of a Chord Contains the Center

Given: $\overline{EF}$ is the perpendicular bisector of $\overline{AB}$.
Prove: $\overline{EF}$ passes through the center of $\odot P$.



SOLUTION

Statements	Reasons
1. $\overline{EF}$ is the perpendicular bisector of $\overline{AB}$.	1. Given
2. $\overline{PA}$ and $\overline{PB}$ are congruent.	2. Definition of a radius
3. Point P lies on the perpendicular bisector of $\overline{AB}$.	3. If a point is equidistant from the endpoints of a segment, then the point lies on the perpendicular bisector of the segment (Theorem 6-6).
4. $\overline{EF}$ passes through the center of $\odot P$.	4. $\overline{EF}$ is the perpendicular bisector of $\overline{AB}$.

Therefore, the center of the circle lies on the perpendicular bisector of a chord.

Math Reasoning

Write Write the proof of Theorem 43-3 as a paragraph proof.

One final property of chords is that all chords that lie the same distance from the center of the circle must be the same length, as stated in Theorem 43-4.

Theorem 43-4
In a circle or congruent circles: • Chords equidistant from the center are congruent. • Congruent chords are equidistant from the center of the circle.

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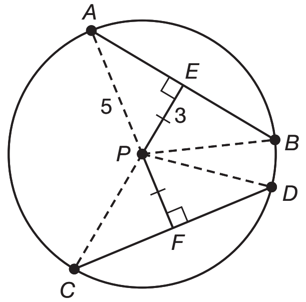
Math Reasoning

Write Combine the two statements in Theorem 43-4 to make a biconditional statement.

Example 4 Applying Properties of Congruent Chords

Find CD, if $AP = 5$ units and $PE = 3$ units.



SOLUTION

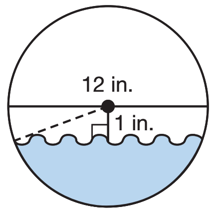
$\overline{AB}$ and $\overline{CD}$ are equidistant from the center of the circle, so they are congruent by Theorem 43-4. Use the Pythagorean Theorem to find the length of $\overline{AE}$.

$a^2 + b^2 = c^2$
$3^2 + b^2 = 5^2$
$b = 4$

By Theorem 43-1, E is the midpoint of $\overline{AB}$, so $AB = 8$. Since $\overline{AB} \cong \overline{CD}$, $CD = 8$ units.

Example 5 Application: Plumbing

Two identical circular pipes have diameters of 12 inches. Water is flowing 1 inch below the center of both pipes. What can be concluded about the width of the water surface in both pipes? Calculate the width.



SOLUTION

Taking a cross section of the pipe, the width of the water surface is a chord of the circle. Since it is given that the water surface is one inch below the center in both pipes, we can conclude that the width of the water surface is equal in both pipes.

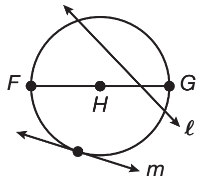
The radius that has been drawn into the diagram forms a right triangle with the surface of the water and the distance between the water and the center of the pipe. The radius of the pipe is 6 inches. Let x represent half the width of the water surface. Use the Pythagorean Theorem to solve for x.

$a^2 + b^2 = c^2$
$1^2 + x^2 = 6^2$
$x^2 = 35$
$x \approx 5.92$

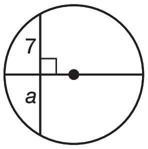
The width of the water surface is two times x, or approximately 11.84 inches.

Lesson Practice

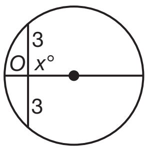
a. Identify each line or segment that intersects the circle.
(Ex 1)



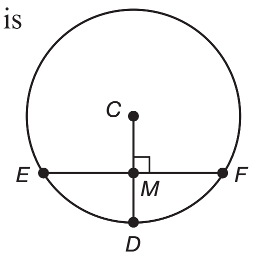
b. Determine the value of a.
(Ex 2)



c. Determine the value of x.
(Ex 2)



d. Prove the first part of Theorem 43-1: If a diameter is perpendicular to a chord, then it bisects the chord.
(Ex 3)
Given: $\overline{CD} \perp \overline{EF}$
Prove: $\overline{CD}$ bisects $\overline{EF}$



e. If a circle has a diameter of 9 inches and a chord that is 5 inches long, what is the distance from the chord to the center of the circle, to the nearest hundredth of an inch?
(Ex 4)

f. A pipe with an 16-inch diameter has water flowing through it. If the water makes a chord across the pipe that is 15 inches long, how close to the center of the pipe is the water?
(Ex 5)

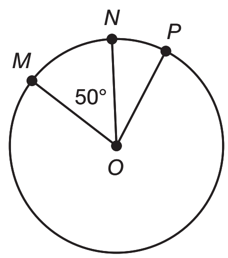
Practice Distributed and Integrated

1. Which of the following is not a parallelogram?
(34)
A rhombus **B** rectangle
C square **D** trapezoid

*** 2.** **Farming** A farmer has 264 feet of fencing to enclose a triangular field. If the farmer wants the sides of the triangle to be in the ratio 3:4:5, what is the length of each side of the field?
(41)

3. **Algebra** A convex polygon has exterior angles $(4x + 10)°$, $(5x + 2)°$, and $(x + 8)°$. What is the value of x?
(Inv 3)

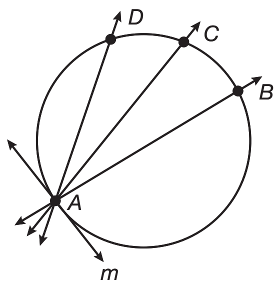
*** 4.** Find the point on the line $y = -0.25x + 9$ that is closest to $(1, -4)$.
(42)



5. In the diagram, $m\widehat{MN} = 50°$ and $m\widehat{MP} = 80°$. What is the measure of $\widehat{NP}$?
(26)

*** 6.** **Formulate** If ∠1 and ∠2 are adjacent central angles of a circle, show that the sum of their associated sector areas is equal to the sector area of their angle sum.
(35)

*** 7.** Identify each line or segment that intersects the circle at right. What type of intersecting line is each one?
(43)



8. **Multi-Step** Graph the line and find the slope of the line that passes through the points $D(4, 2)$ and $E(-3, 6)$. Then find the equation of a perpendicular line that passes through the point $F(-1, 2)$.
(37)

9. **Error Analysis** A composite figure is made up of a semicircle and a rectangle. The width of the rectangle is 3 centimeters, and the diameter of the semicircle is 5 centimeters. Derek calculated the perimeter of the figure like this:
(40)

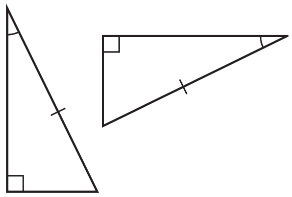
$P = 3 + 3 + 5 + \pi d$
$\quad = (11 + 5\pi)$ cm

Find and correct Derek's error.

10. *(29)* **Carpentry** A 25-meter-long diagonal piece of wood runs across a rectangular platform that has a length-to-width ratio of 4:3. What are the dimensions of the platform?

11. *(32)* When are all the medians and altitudes of a triangle congruent?

12. *(36)* **Multiple Choice** Which congruence theorem applies to these triangles?
A Leg-Leg Theorem **B** Leg-Angle Theorem
C Hypotenuse-Leg Theorem **D** Hypotenuse-Angle Theorem

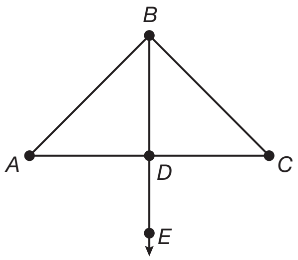


13. *(Inv 3)* **Algebra** Each central angle of a regular polygon is 1°. How many sides does the polygon have?

14. *(24)* Solve the equation $3(2x - 1) = 15$ with a justification for each step.

***15.** *(43)* A circle has a diameter of 22 centimeters. A chord in the circle is 11 centimeters long. What is the *exact* distance from the chord to the center of the circle?

16. *(38)* **Given:** $\overline{BE}$ is the perpendicular bisector of $\overline{AC}$ at point D.
Prove: $\triangle ABD \cong \triangle CBD$

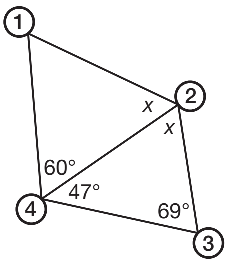


***17.** *(42)* **Algebra** Given the equation $y = 2x + 1$, and $R(4, 4)$, find the point on the line that is closest to R.

18. *(32)* **Multi-Step** Find the orthocenter of $\triangle MNP$ with vertices $M(2, 2)$, $N(2, 8)$, and $P(-6, 4)$.

19. *(26)* **Formulate** An arc measure of 180° forms a semicircle. Use this fact to derive a formula for the perimeter of a semicircle.

20. *(39)* **Disc Golf** In disc golf, a player tries to throw a flying disc into a metal basket target. Four of the disc golf targets are shown in the diagram. Which of the targets are closest together?



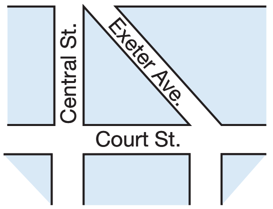
21. *(10)* Identify the hypothesis and conclusion of the following statement.

If two planes intersect, then they intersect on exactly one line.

***22.** *(43)* **Write** Explain the relationship between two chords that are equidistant from the center of a circle.

23. *(35)* The area of a circle is 40.6 square meters. If an arc's length is 2.8 meters, what is the arc's measure?

24. *(29)* **Driving** Esther normally takes Exeter Avenue to work, but it is closed for repairs. For the detour, she must travel south on Central Street for 2.2 miles and then east on Court Street for 3.4 miles. How much longer is the detour than her normal route, to the nearest tenth of a mile?



***25.** *(42)* Find the distance between the two parallel lines $y = 3x + 1$ and $y = 3x - 18$.

***26.** *(41)* **Model** Can the following be modeled with straws? If not, explain.

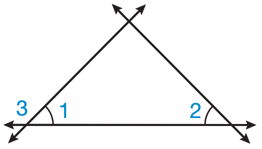
a. Two similar equilateral triangles.

b. Two equilateral triangles that are not similar.

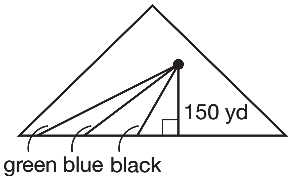
27. *(31)* Write a paragraph proof.

Given: Angle 1 is congruent to ∠2.

Prove: Angle 2 and ∠3 are supplementary.

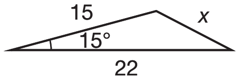


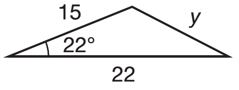
28. *(29, 33)* **Skiing** Developers are designing a new ski hill in the Rocky Mountains. They use the standard difficulty rating of ski runs as green (easy), blue (medium), and black (difficult). To determine what color each of the runs should be, developers use a slope formula. Green runs have a slope of $\frac{1}{3}$. Blue runs have a slope of $\frac{1}{2}$. Black runs have a slope of 1. If all the runs start at 150 yards high, what is the length of each run, to the nearest tenth of a yard?



***29.** *(43)* Find the distance from the center of a circle with a diameter of 2.6 inches to a chord that is 1.2 inches in length, to the nearest hundredth of an inch.

30. *(Inv 4)* **Justify** Which is the longer side, x or y? Explain.





LESSON
44 Applying Similarity

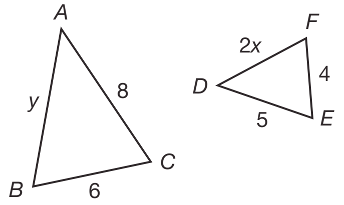
Warm Up

1. (41) **Vocabulary** In the proportion $2:5 = 7:x$, the values 2 and x are called the ________ of the proportion.

2. (41) The angles in a triangle have the ratio 1:2:3. Find the measures of the angles.

3. (41) Given the proportion $5:3:2 = 7:2x:4y$, find the values of x and y.

4. (41) In the diagram, $\triangle ABC \sim \triangle DEF$. Find the values of x and y to the nearest hundredth.



5. (41) **Multiple Choice** The ratio 2 to 4.1 is the same as:
   A 4 to 8.1
   B 3 to 6.1
   C 6 to 12.3
   D 5 to 10

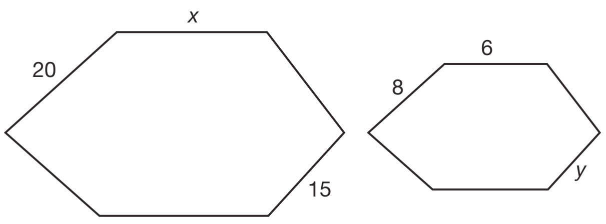
New Concepts

Recall from Lesson 41 that proportions can be used to find unknown measures in similar polygons. Any two regular polygons with the same number of sides are similar. Therefore, all regular polygons with the same number of sides are similar to each other.

Math Reasoning

Verify Are all isosceles triangles similar? Are all equiangular triangles similar? Why or why not?

Example 1 Using Similarity to Find Unknown Measures



The hexagons in the diagram are similar. Find the values of x and y.

SOLUTION

By looking at the corresponding segments with known lengths, a similarity ratio can be written: 20:8. Now, use a proportion to solve for x and y.

$$\frac{20}{8} = \frac{x}{6} \qquad \frac{20}{8} = \frac{15}{y}$$

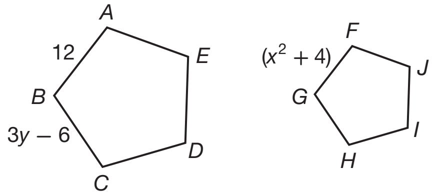
$$6 \times 20 = 8x \qquad 20y = 15 \times 8$$

$$120 = 8x \qquad 20y = 120$$

$$x = 15 \qquad y = 6$$

Online Connection
www.SaxonMathResources.com

Example 2 Applying Similarity to Solve for Unknowns



a. Pentagons *ABCDE* and *FGHIJ* are regular pentagons, and are similar to each other. The similarity ratio of *ABCDE* to *FGHIJ* is 3:2. Find the values of x and y.

SOLUTION

Set up a proportion using the similarity ratio and the ratio of *AB* to *FG*, then find the cross product and solve for x.

$\frac{3}{2} = \frac{12}{x^2 + 4}$	Set up a proportion.
$3(x^2 + 4) = 12 \times 2$	Cross multiply.
$3x^2 + 12 = 24$	Distribute and simplify.
$x = \pm 2$	Solve.

Since *ABCDE* is a regular pentagon, its sides are congruent. Therefore, $BC = AB$.

$3y - 6 = 12$	Substitute.
$3y = 18$	Add 6 to each side.
$y = 6$	Divide each side by 3.

b. What is the ratio of the perimeter of *ABCDE* to *FGHIJ*?

SOLUTION

Since both pentagons are regular, their sides are all congruent. *ABCDE* has five sides, each measuring 12 units, so its perimeter is 60.
Substituting $x = \pm 2$ to find the length of a side of *FGHIJ* shows that each side measures 8 units, so the perimeter of *FGHIJ* is 40.

Therefore, the ratio of the perimeters is 60:40, which reduces to 3:2.

Hint

Often, the negative answer to a root is disregarded in geometry, because a figure cannot have a negative measurement. In this case though, using the value $x = -2$ still results in a positive side length, so both answers are valid.

As you can see from Example 2, the perimeters of two similar figures share the same similarity ratio as their sides.

Theorem 44-1
If two polygons are similar, then the ratio of their perimeters is equal to the ratio of their corresponding sides.

Example 3 Proving the Relationship Between Perimeters of Similar Figures

> **Hint**
>
> A 2-column proof is good for proofs that use a lot of geometric symbols or algebra. A paragraph proof of this theorem would be difficult to read.

Given $\triangle PQR \sim \triangle STU$, prove that the ratio of their perimeters is 1:2 if the ratio of their corresponding sides is 1:2.

SOLUTION

Use a 2-column proof.

Statements	Reasons
1. $\triangle PQR \sim \triangle STU$	1. Given
2. $\frac{PQ}{ST} = \frac{QR}{TU} = \frac{RP}{US} = \frac{1}{2}$	2. Given
3. $2PQ = ST$	3. Cross multiply
4. $2QR = TU$	4. Cross multiply
5. $2RP = US$	5. Cross multiply
6. perimeter of $\triangle STU = ST + TU + US$	6. Definition of perimeter
7. perimeter of $\triangle STU = 2PQ + 2QR + 2RP$	7. Substitution Property of Equality
8. perimeter of $\triangle STU = 2(PQ + QR + RP)$	8. Simplify
9. perimeter of $\triangle PQR = PQ + QR + RP$	9. Definition of perimeter
10. perimeter of $\triangle STU = 2(\text{perimeter of } \triangle PQR)$	10. Substitution Property of Equality

Therefore, the ratio of the perimeter of $\triangle PQR$ to the perimeter of $\triangle STU$ is 1:2.

Example 4 Applying Similarity to Solve a Perimeter Problem

Figures *HIJK* and *LMNO* are similar polygons. Their corresponding sides have a ratio of 2:5. If the perimeter of figure *HIJK* is 27 inches, what is the perimeter of figure *LMNO*?

SOLUTION

Because *HIJK* and *LMNO* are similar polygons, the ratio of their perimeters is equal to the ratio of their corresponding sides.

Therefore, the ratio of *HIJK*'s perimeter to *LMNO*'s perimeter is 2:5.

Set up a proportion using this ratio to solve for the perimeter of *LMNO*.

$$\frac{27}{x} = \frac{2}{5}$$

$$2(x) = 5(27)$$

$$x = 67.5$$

The figure *LMNO* has a perimeter of 67.5 inches.

Example 5 Application: Map Scales

Foxx plans to jog 5000 meters a day in training for a race. The park where Foxx jogs is in the shape of a regular pentagon. The side length of the park is 5 centimeters long on a map with the scale $\frac{1\text{ cm}}{50\text{ m}}$. How many times does Foxx need to jog along the perimeter of the park to complete his daily training?

SOLUTION

First, find the perimeter of the park on the map.

Perimeter of a regular pentagon = 5s

$$P = 5(5\text{ cm})$$
$$P = 25\text{ cm}$$

The park on the map and the actual park are similar polygons. Therefore, the ratio of their perimeters is the same as the ratio of their corresponding sides.

So, the ratio of the park's perimeter on the map to the perimeter of the actual park is $\frac{1\text{ cm}}{50\text{ m}}$.

$$\frac{\text{perimeter on the map}}{\text{perimeter of the actual park}} = \frac{1\text{ cm}}{50\text{ m}}$$
$$\frac{25\text{ cm}}{\text{perimeter of the actual park}} = \frac{1\text{ cm}}{50\text{ m}}$$
$$\text{perimeter of the actual park} \times 1\text{ cm} = 50\text{ m} \times 25\text{ cm}$$
$$\text{perimeter of the actual park} = \frac{50\text{ m} \times 25\text{ cm}}{1\text{ cm}}$$
$$= 1250\text{ m}$$

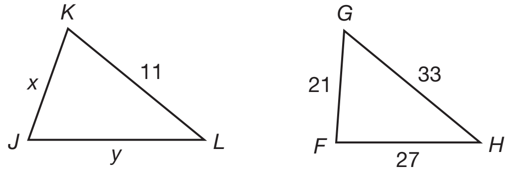
Since 1250 is $\frac{1}{4}$ of 5000, Foxx needs to jog along the perimeter of the park 4 times to complete his daily training.

Math Reasoning

Formulate Would Theorem 44-1 also apply to the circumference of a circle? Why or why not?

Lesson Practice

a. (Ex 1) $\triangle JKL$ and $\triangle FGH$ are similar triangles. Find the values of x and y.



b. (Ex 2) In $\triangle ABC$, $AB = x^2 - 7$, $BC = y + 4$, and $CA = 5$. In $\triangle DEF$, $DE = 6$, $EF = 12$, and $FD = 15$. $\triangle ABC \sim \triangle DEF$. Find the values of x and y. Then find the ratio of the perimeters of the two triangles.

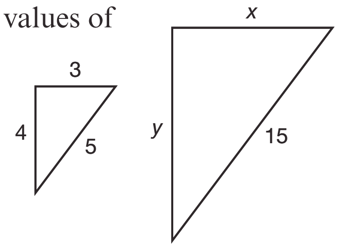
c. (Ex 4) Figures *ABCD* and *EFGH* are similar. The ratio of their corresponding sides is 3:5. If the perimeter of *EFGH* is 45 inches, what is the perimeter of figure *ABCD*?

d. (Ex 4) Pentagons *ABCDE* and *FGHIJ* are similar figures. The perimeter of *ABCDE* is 32 centimeters. The similarity ratio of *ABCDE* to *FGHIJ* is 2:9. What is the perimeter of *FGHIJ*?

e. (Ex 5) Jana and her brother Jacob are desiging their own tree house with two separate doors, one that is proportional to Jana's height and one that is proportional to Jacob's height. Jacob is 3 feet tall and Jana is 4 feet tall, so Jana decides that her door should be 5 feet tall by 2 feet wide. How tall should Jacob's door be, and what will its perimeter be?

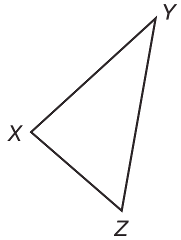
Practice Distributed and Integrated

1. (41) If the triangles shown are similar so that 3:4:5 = x:y:15, what are the values of x and y?



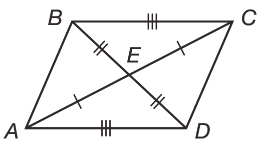
2. (28) **Design** Billy is making a kite $STUV$. He knows that $\overline{ST}$ and $\overline{TU}$ are the same length. Which angles does he need to know are equal to ensure that the two halves of the kite, $\triangle STV$ and $\triangle UTV$, are congruent?

3. (38) **Restaurants** A restaurant owner plans to add a triangular patio to her restaurant, as shown. She wants to position a fountain on the patio that is the same distance from each edge. Where should she position the fountain? Explain your reasoning. Support your answer with a diagram.



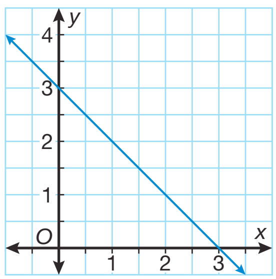
4. (29) **Write** Explain how the Pythagorean Theorem can be used to ensure that two pieces of metal in a frame are at right angles to each other.

5. (34) **Verify** Name two properties of parallelograms that can be used to justify the congruency marks on this diagram.



***6.** (44) Given $\triangle FGH \sim \triangle MNP$, with $FG = 8$ and $MN = 16$, what is the reduced ratio of their corresponding sides?

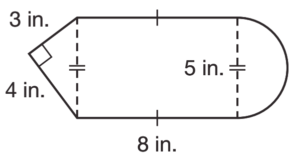
7. (16) **a.** Determine the slope and y-intercept of this graph of a line.
b. Write the equation of the line.



***8.** (44) Figure $RSTU$ is similar to figure $KLMN$. The ratio of their corresponding sides is 1:2. If the perimeter of $RSTU$ is 10 inches, what is the perimeter of $KLMN$?

9. (30) **Verify** Wesley has two triangles, $\triangle ABC$ and $\triangle XYZ$. He knows that $m\angle ABC = 67°$, $m\angle BAC = 23°$, $AB = 5$ units, $BC = 13$ units, $m\angle XYZ = 67°$, $m\angle YXZ = 23°$, and $ZY = 13$ units. He concludes that $XY = 5$ units. Verify that Wesley is correct.

***10.** (40) **Multi-Step** Determine the perimeter of this figure. Express your answer in terms of π.



***11.** (43) The radius of a circle is 15 centimeters. If a chord measuring 24 centimeters is perpendicular to the radius and cuts it into two parts, find the length of each part of the radius.

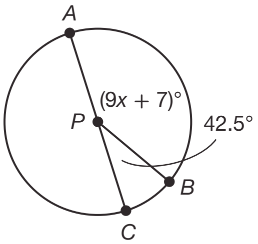
12. (37) **Write** When finding a perpendicular or parallel line to a given line, why is the y-intercept not used?

***13.** (44) Elijah wants to change the size of the tires on his bike. The old tires have a radius of 12 inches. The new tires have a radius of 15 inches. What is the ratio of the radius of the old tires to that of the new tires?

14. (39) **Justify** Explain how you know a triangle can have side lengths measuring 11, 15 and 21.

***15.** (42) Given the equation $y = \frac{2}{3}x - 2$ and $G(1, 3)$, find the point on the line that is closest to G.

Refer to this diagram to answer the next two questions.



16. (35) **a.** If $\overline{AC}$ is a diameter, what is the value of x?
b. If $PA = 10$ centimeters, what is the length of $\overset{\frown}{AB}$ to the nearest tenth of a centimeter? Of major arc $\overset{\frown}{ACB}$?

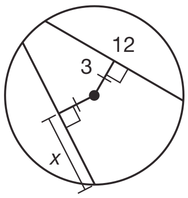
17. (35) If the radius of the circle is 25 inches, what is the area of sector BPC to the nearest tenth square inch?

***18.** (42) Find the distance from $(-1, 2)$ to the line $y = \frac{3}{2}x - 3$.

19. (Inv 3) **Error Analysis** Lionel calculated the sum of the interior angles of a convex polygon to be 1710°. Explain how you know he made an error.

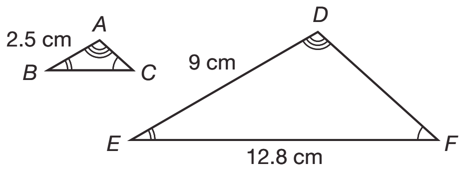
20. (33) Is a triangle with side lengths that measure 3.6, 4.8, and 6.2 a right, obtuse, or acute triangle?

***21.** (43) Using the diagram below, determine the value of x.



22. (38) **Emergency Alert** A town wants to place a weather emergency siren where the three schools can all hear the siren to alert them in case of an emergency. Explain how the town might go about finding the optimal location for the siren.

23. (41) **Multiple Choice** If the triangles are similar, which of these is the length of $\overline{BC}$?



A 1.3 centimeters
B 1.8 centimeters
C 3.2 centimeters
D 3.6 centimeters

24. (Inv 3) If the sum of the measures of the interior angles in a regular polygon is 1260°, how many sides does the polygon have?

25. (29) **Analyze** Jason is walking around an empty field, from the southwest corner to the northeast corner. The field is 140 yards long and 90 yards wide. How much farther will Jason have to walk if he walks around the edge of the field than if he cuts through the middle of the field, to the nearest yard?

***26.** (44) $\triangle CDE$ is an equilateral triangle. Its perimeter is 45 inches. If $\triangle CDE \sim \triangle VWX$, and the ratio of their corresponding sides is 3:1, what are the lengths of the sides of $\triangle VWX$?

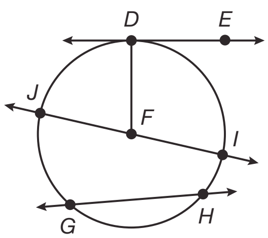
27. (40) A composite figure is made up of a right triangle and three squares. The triangle has a base length of 8 centimeters and a height of 15 centimeters. Each square fits exactly along one side of the triangle.

a. Determine the side length of the largest square.
b. Determine the perimeter of the figure.
c. Determine the area of the figure.

xy^2 **28.** (37) **Algebra** Line p passes through points (4, 7) and (4, 3). Line q passes through points (–2, 1) and (5, 1). Are the lines parallel, perpendicular, or neither?

29. (25) **Carpentry** Deklynn and Bonita are making two congruent triangles out of wood. Deklynn's triangle, $\triangle FGH$ has $m\angle G = 70°$ and $m\angle H = 80°$. If Bonita's triangle is $\triangle PQR$ and $\triangle FGH \cong \triangle PQR$, how large should she make $m\angle P$?

***30.** (43) Identify each line or segment that intersects the circle. What type of intersecting line is each one?



LESSON

45 Introduction to Coordinate Proofs

Warm Up

1. **Vocabulary** The formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ gives the ________ between two points.
(9)

2. Find the length of the line segment joining points $A(1, 3)$ and $B(4, 2)$, to the nearest tenth.
(9)

3. What is the midpoint of a line segment with endpoints $(3, -1)$ and $(5, 7)$?
(11)

4. Find the slope of the line joining points $(-1, 4)$ and $(2, -3)$.
(16)

5. Find the length of the line segment joining the origin and $(3, 4)$.
(9)

New Concepts

A **coordinate proof** is a style of proof that uses coordinate geometry and algebra. In a coordinate proof, a diagram is used that is placed on the coordinate plane. Figures can be placed anywhere on the plane, but it is usually easiest to place one side on an axis or to place one vertex at the origin.

Example 1 Positioning a Figure on the Coordinate Plane

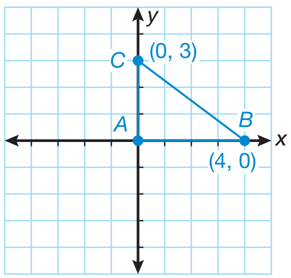
Triangle *ABC* has a base of 4 units and a height of 3 units. Angle *A* is a right angle. Position $\triangle ABC$ on the coordinate plane.

SOLUTION

There are various ways to position the triangle on the coordinate plane. A simple way is to use the origin (0, 0) as the vertex for *A*.

Place one of the legs of the triangle on the *x*-axis, and place the other leg on the *y*-axis.

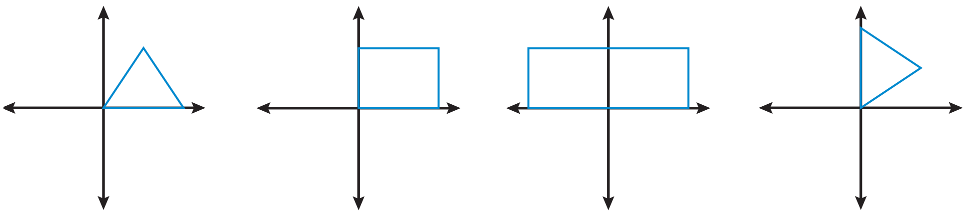
On the *x*-axis, label *B*(4, 0). On the *y*-axis, label *C*(0, 3). Draw the triangle.



Math Reasoning

Error Analysis In Example 1, if you were to place points *B* and *C* at (3, 0) and (0, 4) instead of the coordinates given in the solution, would the solution still be valid? Explain.

When a figure is placed in a convenient position on the coordinate plane, the equations and values used in a proof will be easier to work with. Below are examples of convenient placement for common figures.



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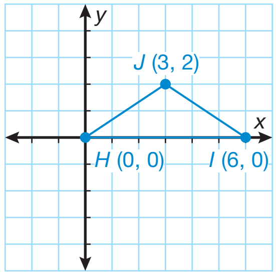
Example 2 Writing a Proof Using Coordinate Geometry

Use a coordinate proof to show that $\triangle HIJ$ is an isosceles triangle.



SOLUTION

If $\triangle HIJ$ is isosceles then, by definition, two of its sides must have equal length. Calculate each of the side lengths to verify that $\triangle HIJ$ is an isosceles triangle.

$$\begin{aligned} JI &= \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2} \\ &= \sqrt{(3-6)^2 + (2-0)^2} \\ &= \sqrt{(-3)^2 + 2^2} \\ &= \sqrt{9+4} \\ &= \sqrt{13} \end{aligned}$$

$$\begin{aligned} HJ &= \sqrt{(x_j - x_h)^2 + (y_j - y_h)^2} \\ &= \sqrt{(3-0)^2 + (2-0)^2} \\ &= \sqrt{3^2 + 2^2} \\ &= \sqrt{9+4} \\ &= \sqrt{13} \end{aligned}$$

$$\begin{aligned} HI &= \sqrt{(x_i - x_h)^2 + (y_i - y_h)^2} \\ &= \sqrt{(6-0)^2 + (0-0)^2} \\ &= \sqrt{6^2 + 0^2} \\ &= \sqrt{6^2} \\ &= 6 \end{aligned}$$

Since $\overline{JI}$ and $\overline{HJ}$ are the same length, $\triangle HIJ$ is an isosceles triangle.

Math Reasoning

Analysis Is it necessary to calculate the side length of all three segments of this triangle? Why or why not?

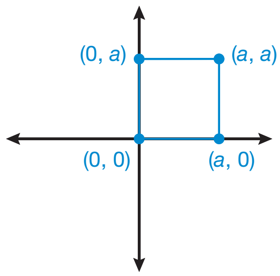
Sometimes a figure's dimensions might be unknown. When placing a figure with unknown dimensions on the coordinate plane, pick a convenient position and label the vertices of the figure using information that is given in the problem.

Example 3 Assigning Variable Coordinates to Vertices

a. A square has a side length, a. Place the square on the coordinate plane and label each vertex with an ordered pair.

SOLUTION

Place one vertex at the origin. Label the vertex at the origin (0,0). Because the vertex on the x-axis is a units away from the origin, its coordinates should be labeled $(a, 0)$. The vertex on the y-axis is a units up from the origin, so its coordinates are $(0, a)$. Finally, the fourth vertex is both a units to the right of the origin and a units up from the origin, at (a, a).

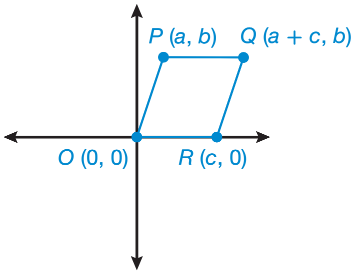


> **Hint**
>
> By placing a vertex of the parallelogram on the origin, one fewer variable can be used to diagram the parallelogram.

b. Given the parallelogram $OPQR$, with one side length labeled c, assign possible coordinates to the vertices.

SOLUTION

Place vertex O at (0, 0) and $\overline{OR}$ along the positive x-axis. Label vertices P, Q, and R. Assign $OR = c$, so the coordinates for R are $(c, 0)$.

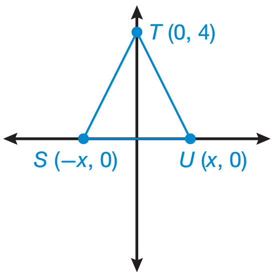


Give P the coordinates (a, b). Because $OPQR$ is a parallelogram, $PQ = OR$. Therefore, the x-coordinate of Q is the x-coordinate of P plus c units, or $a + c$. The coordinates of Q are $(a + c, b)$.

c. Assign coordinates to the vertices of isosceles $\triangle STU$ with a height of 4 from the vertex.

SOLUTION

Place vertex T on the y-axis so that its coordinates are (0, 4). If points S and U are placed such that they are equally distant from the y-axis, then they will form two right triangles with congruent hypotenuses. This ensures that the figure is an isosceles triangle. The coordinates of S and U are $(-x, 0)$ and $(x, 0)$, respectively.



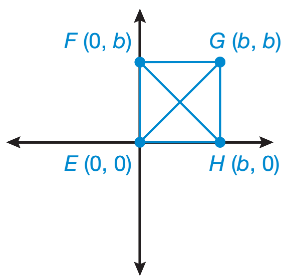
When you assign variable coordinates to a figure used in a proof, remember that the values you choose must apply to all cases. When the dimensions of a figure are not given, variables must be used to ensure the proof is valid for a figure of any size.

Example 4 Writing a Coordinate Proof

Prove that the diagonals of a square are perpendicular to one another.

SOLUTION

Assign square $EFGH$ a side length of b. Place E at (0, 0), F at $(0, b)$, G at (b, b), and H at $(b, 0)$. Draw the diagonals $\overline{FH}$ and $\overline{GE}$.



Calculate the slope of diagonals $\overline{FH}$ and $\overline{GE}$.

$$m_{FH} = \frac{0 - b}{b - 0} = \frac{-b}{b} = -1 \qquad m_{GE} = \frac{b - 0}{b - 0} = \frac{b}{b} = 1$$

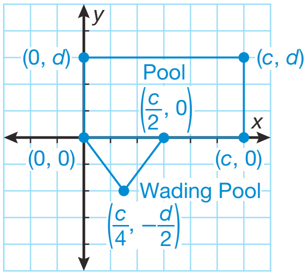
> **Hint**
>
> Recall that two nonvertical lines are perpendicular if and only if the product of their slopes is −1, and that the formula for slope m of a line is $m = \frac{y_2 - y_1}{x_2 - x_2}$.

Because the product of the two slopes is -1, $\overline{FH} \perp \overline{GE}$.

Notice that the slopes of the diagonals do not depend on the value of b. Therefore, for all squares, it is true that the diagonals of the square are perpendicular to each other.

Example 5 Application: Constructing a Swimming Pool

A contractor has been hired to build a swimming pool with a smaller wading pool beside it. The contractor draws a diagram of what he plans to build and overlays a coordinate grid on it, as shown. Show that the wading pool has a surface area that is one-eighth the size of the larger pool's surface area.



SOLUTION

The area of a rectangle is ℓw. The pool in the diagram has a length of c and a width of d, so its total area is cd. The wading pool is a triangle. The area of a triangle is $\frac{1}{2}bh$. The height of the wading pool is $\frac{d}{2}$ and the length of its base is $\frac{c}{2}$. Substitute these values into the formula for area of a triangle.

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}\left(\frac{c}{2}\right)\left(\frac{d}{2}\right)$$

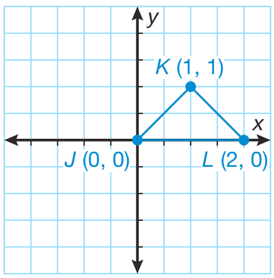
$$A = \frac{cd}{8}$$

Therefore, the surface area of the wading pool is one-eighth the surface area of the swimming pool.

Lesson Practice

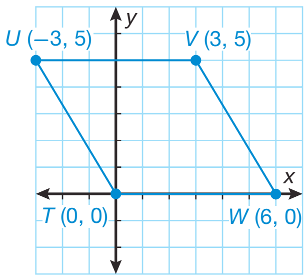
a. Place a right triangle with leg lengths of 2 and 6 units on the coordinate plane so that its legs are on the x-axis and y-axis. Label the vertices with their respective coordinates.
(Ex 1)

b. Prove that $\triangle JKL$ is an isosceles triangle.
(Ex 2)



c. Place a right triangle with leg lengths of a and b units on the coordinate plane. Label the vertices with their coordinates.
(Ex 3)

d. Prove that figure $TUVW$ is a parallelogram.
(Ex 4)

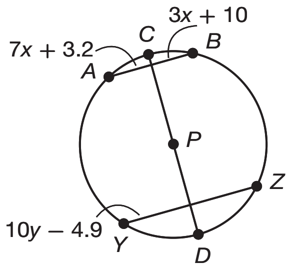


e. **Traffic Signs** A yield sign is an equilateral triangle. Draw a yield sign on the coordinate plane, using the origin as a vertex. If the length of one side is m units, find the coordinates of the other vertices in terms of m.
(Ex 5)

Practice Distributed and Integrated

1. (45) **Coordinate Geometry** An isosceles triangle has vertices $K(0, 0)$, $L(-2, 2)$, and $M(x, y)$. Find the coordinates of one possible position of M.

2. (44) Similar pentagons *EFGHI* and *QRSTU* are regular and have a similarity ratio of 5:3. Find the value of x if $FG = 30$ and $TU = x - 11$.

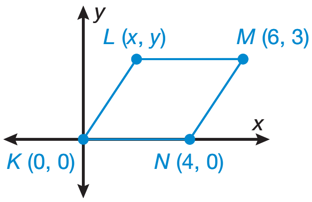


3. (43) If $\overline{CD}$ is a diameter of circle P in the diagram, and it is perpendicular to $\overline{AB}$, what is the value of x?

4. (43) In the circle at right if $YZ = 30.2$, what is the value of y?

5. (42) Find the distance from the line $y = \frac{5}{4}x - 4$ to $(-1, 5)$.

6. (45) *KLMN* is a parallelogram. Find the coordinates of $L(x, y)$.

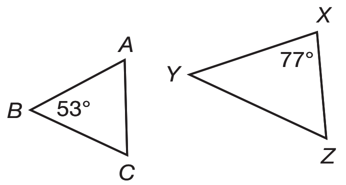


7. (35) What is the arc length of the minor arc bounded by a 60° central angle in a circle with a radius of 30 centimeters? Use 3.14 for π.

8. (34) Angela draws diagonals on a quadrilateral and notices that they bisect each other. Without knowing any other information about the shape, can you classify it as a parallelogram? As a rectangle? Explain.

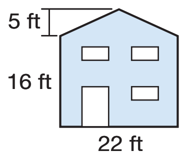
9. (45) **Coordinate Geometry** Parallelogram *JKLM* has vertices $J(0, 0)$, $K(2, 2)$, $L(x, y)$, and $M(5, 0)$. Find (x, y).

10. (41) If $\triangle ABC \sim \triangle XYZ$, what are the measures of $\angle A$ and $\angle Y$?



11. (41) **Analyze** The side lengths of rectangle *A* are 16.4 inches and 10.8 inches. The side lengths of rectangle *B* are 10.25 inches and 6.75 inches. Determine whether the two rectangles are similar. If so, find the similarity ratio. If not, explain why.

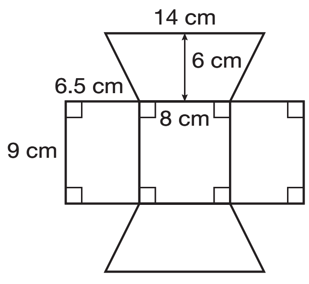
12. (40) **Architecture** How much veneer siding is required to cover the front of this house? Do not include the 3-by-7-foot door or the three 6-by-3-foot windows.



13. (40) **Packaging** The cardboard pattern for a gift box is shown at right. The two trapezoids are congruent. The two end rectangles are also congruent.

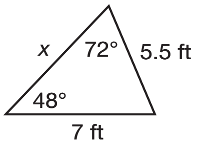
a. Calculate the area of card stock used for the box.

b. If the box is cut from a square sheet with side lengths of 21 centimeters each, how much card stock is wasted?



14. (44) Lauren and her grandmother have tables that are similar rectangles. The ratio of their corresponding sides is 2:3. Lauren's table is 5 feet wide and 6 feet long, and her grandmother's is the larger table. What is the perimeter of her grandmother's table?

15. (39) **Construction** The measurements in the diagram were taken from the attic space in a new home. What is the range of possible lengths of the unknown side? Explain.



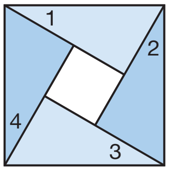
16. (44) Given $\triangle LMN \sim \triangle UVW$, with $LM = 10$, and $UV = 4$, what is the ratio of their corresponding sides?

17. (Inv 3) What is the sum of the measures of the interior angles of a heptagon? If it is a regular heptagon, what is the measure of each interior angle to the nearest hundredth degree?

18. (37) Find the line parallel to $2y + 4 = 6x - 5$ that passes through the origin. Write it in slope-intercept form.

19. (37) **Justify** If $y = 2x + 3$ and $y = 2x + k$ is parallel to it, what are all the possible values of k? Why does changing the value of k not make the lines intersect?

20. (36) **Design** The math faculty of Pythagoras College uses this logo. All four triangles are right triangles, the whole logo is a square, and the inner quadrilateral is also a square. Prove that the right triangles are all congruent.



21. (28) **Analyze** In $\triangle ABC$, $AB = (x + 3)$, $m\angle ABC = (16x + 8)°$, and $BC = (4x - 8)$. In $\triangle DEF$, $DE = (2x - 1)$, $m\angle DEF = (18x)°$, and $EF = (3x - 6)$. Is it possible for $\triangle ABC \cong \triangle DEF$? Explain.

22. (33) A triangle has side lengths that are 100, 240, and 265 units long. Classify the triangle by side lengths and by angles.

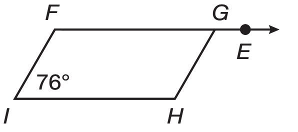
23. (32) Draw an equilateral triangle using a protractor and straightedge. Locate and label its orthocenter and centroid. What do you notice about the two points?

24. (Inv 3) What is the sum of the interior angle measures of a convex, irregular octagon?

25. (30) **Error Analysis** Mick has two triangles, $\triangle MNO$ and $\triangle DEF$. He knows that $m\angle MNO = m\angle DEF$, $m\angle NOM = m\angle EFD$, and $NO = DF$. Mick concludes that $\triangle MNO \cong \triangle DEF$. Is he correct? Explain.

26. (28) For a circle with center O and diameters $\overline{AB}$ and $\overline{CD}$, prove $\triangle AOC \cong \triangle BOD$.

27. (34) In the parallelogram shown, what is the measure of $\angle IFG$? $m\angle FGH$? $m\angle HGE$?



28. (45) Write a coordinate proof showing that the midpoint of a hypotenuse of a right triangle is equidistant from the three vertices.

29. (26) **Algebra** If $m\widehat{AB} = (2x + 10)°$ and $m\widehat{BC} = (4x + 5)°$ are non-overlapping adjacent arcs, $m\widehat{XY} = (x + 19)°$ and $m\widehat{YZ} = (2x + 11)°$ are non-overlapping adjacent arcs and $\widehat{AC}$ and $\widehat{XZ}$ are congruent, what is the measure of each arc?

30. (10) **Multiple Choice** What is the conclusion of this conjecture?

If a polygon is regular, then it is equilateral and equiangular.

A A polygon is equilateral.
B A polygon is regular.
C A polygon is equilateral and equiangular.
D A polygon is equiangular.

LESSON **46**

Triangle Similarity: AA, SSS, SAS

Warm Up

1. (41) **Vocabulary** The ratio of two corresponding linear measurements in a pair of similar figures is the ____________.

2. (30) If two angles and a nonincluded side of one triangle are congruent to two angles and the corresponding nonincluded side of another triangle, then the two triangles are congruent by ________________________.

3. (30) **Multiple Choice** If two angles and the corresponding included sides of two triangles are congruent, then which triangle congruency postulate or theorem applies?
   A SSS Postulate **B** SAS Postulate
   C AAS Theorem **D** ASA Postulate

New Concepts

Two triangles are similar if all their corresponding angles are congruent. Since the sum of any triangle's angles is 180°, only two angles are required to prove that two triangles are similar.

Reading Math

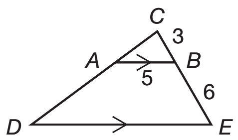
The symbol ~ is used to show that two polygons are similar. For example, $\triangle XYZ \sim \triangle KLM$ means, "triangle *XYZ* is similar to triangle *KLM*."

Postulate 21: Angle-Angle (AA) Triangle Similarity Postulate

If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.

Example 1 Using the AA Similarity Postulate

Show that the two triangles are similar if $\overline{AB} \parallel \overline{DE}$. Then, find *DE*.



SOLUTION

Statements	Reasons
1. $\overline{AB} \parallel \overline{DE}$	1. Given
2. $m\angle ABC = m\angle DEC$	2. Corresponding Angles Postulate
3. $m\angle BAC = m\angle EDC$	3. Corresponding Angles Postulate
4. $\triangle ABC \sim \triangle DEC$	4. AA Similarity Postulate

Since the two triangles are similar, the ratios of the lengths of corresponding sides are equal.

$$\frac{AB}{DE} = \frac{BC}{EC}$$
$$\frac{5}{DE} = \frac{3}{9}$$
$$3(DE) = 45$$
$$DE = 15$$

Math Reasoning

Write Are all pairs of congruent triangles also similar triangles? Explain.

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Exploration Understanding AA Similarity

In this exploration, you will construct similar triangles and observe the properties of each.

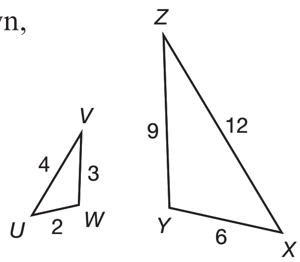
1. Draw two different line segments on a sheet of paper. Make sure the segments each have a different length.
2. At each end of the first line segment, measure acute angles and draw the rays out to create a triangle.
3. On your second line segment, measure the same two angles with a protractor and draw a second triangle.
4. Measure the unknown angle of each of the triangles you have drawn. What is the relationship between these two angles? What is the relationship between these two triangles?
5. Measure the side lengths of the larger triangle and one side of the small triangle. Now, use what you know about the triangles to predict the side lengths of the smaller triangle without using a ruler. Then, measure the lengths. Were your predictions correct?

It is not always necessary to know a triangle's angle measures to determine similarity. Another way to determine similarity is to verify that the lengths of all the corresponding sides of both triangles are related in the same ratio.

Theorem 46-1: SSS Similarity Theorem
If the lengths of the sides of a triangle are proportional to the lengths of the sides of another triangle, then the triangles are similar.

Example 2 Using the SSS Similarity Theorem

Given the two triangles with lengths as shown, show that they are similar triangles.



SOLUTION

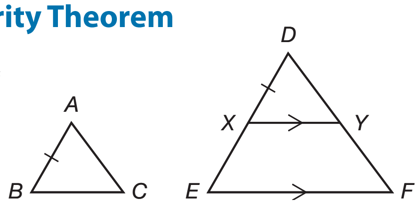
Statements	Reasons
1. $\frac{UW}{XY} = \frac{2}{6} = \frac{1}{3}$	1. Similarity ratio for $\overline{UW}:\overline{XY}$.
2. $\frac{WV}{YZ} = \frac{3}{9} = \frac{1}{3}$	2. Similarity ratio for $\overline{WV}:\overline{YZ}$.
3. $\frac{VU}{ZX} = \frac{4}{12} = \frac{1}{3}$	3. Similarity ratio for $\overline{VU}:\overline{ZX}$.
4. $\triangle UWV \sim \triangle XYZ$	4. SSS Similarity Theorem

Hint

This example demonstrates a simple way to find out if all the side pairs of two triangles are in the same proportion. If the ratio of each pair of sides reduces to the same fraction, they are proportional.

Example 3 Proving the SSS Similarity Theorem

Prove Theorem 46-1: If the lengths of the sides of a triangle are proportional to the lengths of the sides of another triangle, then the triangles are similar.



Given: $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$, $\overline{DX} \cong \overline{AB}$, $\overline{XY} \parallel \overline{EF}$

Prove: $\triangle ABC \sim \triangle DEF$

SOLUTION

Statements	Reasons
1. $\overline{DX} \cong \overline{AB}$, $\overline{XY} \parallel \overline{EF}$	1. Given; Parallel Postulate
2. $\angle DXY \cong \angle DEF$	2. Corresponding angles are congruent
3. $\angle D \cong \angle D$	3. Reflexive Property of Congruence
4. $\triangle DXY \sim \triangle DEF$	4. AA Similarity Postulate
5. $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$	5. Given
6. $\frac{DX}{DE} = \frac{XY}{EF} = \frac{YD}{FD}$	6. Similarity ratio from $\triangle DXY \sim \triangle DEF$
7. $DX = AB$	7. Definition of congruent segments
8. $\frac{AB}{DE} = \frac{XY}{EF} = \frac{YD}{FD}$	8. Substitute AB for DX in step 6
9. $\frac{BC}{EF} = \frac{XY}{EF}, \frac{CA}{FD} = \frac{YD}{FD}$	9. Substitute
10. $BC = XY$, $CA = YD$	10. Simplify
11. $\overline{BC} \cong \overline{XY}, \overline{CA} \cong \overline{YD}$	11. Definition of Congruent Segments
12. $\triangle ABC \cong \triangle DXY$	12. SSS Triangle Congruence Postulate
13. $\triangle ABC \sim \triangle DEF$	13. Transitive Property of Similarity

Caution

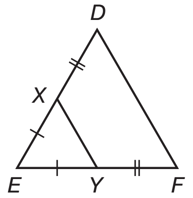
To apply SAS similarity, one angle pair has to be congruent, but the side pairs only have to be proportional. Do not confuse this with SAS congruence, where the two pairs of sides must be congruent.

One final way to prove triangle similarity is the SAS Similarity Theorem. You will notice that it is similar to one of the congruence postulates you have learned about.

Theorem 46-2: SAS Similarity Theorem
If two sides of one triangle are proportional to two sides of another triangle and the included angles are congruent, then the triangles are similar.

Example 4 Proving Similarity

a. Prove that $\triangle EXY \sim \triangle EDF$.



SOLUTION

By the Reflexive Property, $\angle XEY \cong \angle DEF$. It is given in the diagram, $\overline{EX} \cong \overline{EY}$ and $\overline{XD} \cong \overline{YF}$. The ratio of EX to ED can be given by $\frac{EX}{EX + XD}$. By substituting the congruent segments, it can be rewritten as $\frac{EY}{EY + YF}$, which is also the ratio of EY to EF. So the triangles have two proportional sides and one congruent angle. By the SAS Similarity Theorem, they are similar triangles.

b. If $EX = 6$, $ED = 11$, and $XY = 7$, find DF.

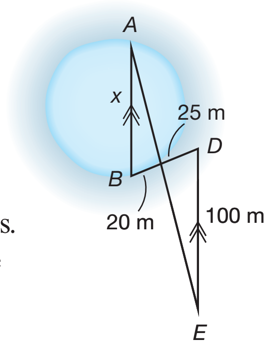
SOLUTION

Use the similarity ratio given by $EX : ED$ and a proportion.

$\frac{EX}{ED} = \frac{XY}{DF}$	Definition of similar triangles
$\frac{6}{11} = \frac{7}{DF}$	Substitute
$11 \cdot 7 = 6 \cdot DF$	Cross product
$DF = \frac{5}{6}$	Solve

Example 5 Application: Land Surveying

A surveyor needs to find the distance across a lake. The surveyor makes some measurements as shown. Find the distance across the lake.



SOLUTION

First, determine if the triangles are similar. The angles at C are congruent since they are vertical angles. AB and DE are parallel lines, so $\angle ABC$ and $\angle EDC$ are congruent. Therefore, by the AA Similarity Postulate, $\triangle ABC \sim \triangle EDC$. Now, find the missing value using a proportion.

$$\frac{x}{100} = \frac{20}{25}$$
$$25x = 2000$$
$$x = 80$$

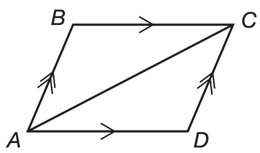
The distance across the lake is 80 meters.

Math Reasoning

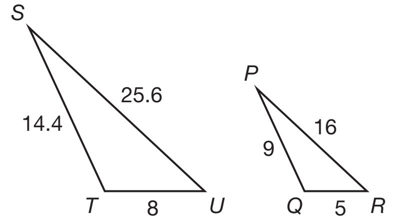
Write Why might someone use this method to find the distance across the lake?

Lesson Practice

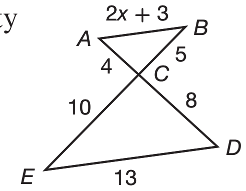
a. Given the two triangles shown, prove they are similar using the AA Similarity Postulate.
(Ex 1)



b. Given the two triangles shown, use SSS similarity to prove that they are similar.
(Ex 2)



c. Given the two triangles shown, use SAS similarity to prove that they are similar. Find the value of x.
(Ex 4)



d. Laura wants to find out how tall a tree is. She notices that the tree makes a shadow on the ground. The top of the shadow of the treehouse is 25 feet away from the base of the tree. Laura is 5 feet 8 inches tall and she casts a shadow that is 6 feet 2 inches long. How tall is the tree, to the nearest foot?
(Ex 5)

Practice Distributed and Integrated

1. Find the distance from $(-1, 7)$ to the line $x = 3$.
(42)

* **2.** **Flying** Two aircraft depart from an airport at (10, 12). The first aircraft travels to an airport at $M(-220, 80)$, and the second aircraft travels to an airport at $N(100, -400)$. If each unit on the grid represents one mile, what is the distance to the nearest mile between the two aircraft after they both land?
(9)

3. **Analyze** The side lengths of rectangle A are 18.3 inches and 24.6 inches. The side lengths of rectangle B are 24.4 inches and 32.8 inches. Determine whether the two rectangles are similar. If so, write the similarity ratio.
(41)

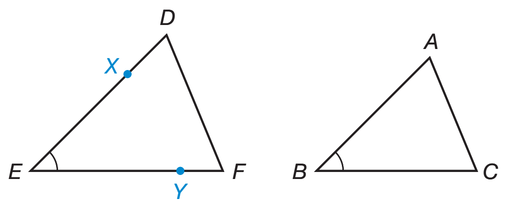
4. **Multiple Choice** Which of the following lines is parallel to $y = 7$?
(37)

A $y = 2x$ **B** $y = -\frac{1}{7}x$

C $y + 7 = 0$ **D** $y = \frac{7}{x}$

5. What is the central angle measure of a regular octagon?
(Inv 3)

* **6.** Write a two-column proof to prove the SAS Similarity Theorem: If two sides of one triangle are proportional to two sides of another triangle and the included angles are congruent, then the triangles are similar.
(27)

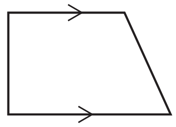


Given: $\angle B \cong \angle E, \frac{AB}{DE} = \frac{BC}{EF}$

Prove: $\triangle ABC \sim \triangle DEF$

Hint: Assume that $AB < DE$ and choose point X on $\overline{DE}$ so that $\overline{EX} \cong \overline{BA}$. Then choose point Y on $\overline{EF}$ so that $\angle EXY \cong \angle EDF$. Show that $\triangle XEY \sim \triangle DEF$ and that $\triangle ABC \cong \triangle XEY$.

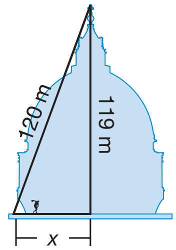
7. (34) **Error Analysis** A student drew the figure at right and called it a parallelogram. Explain where the student erred.



*** 8.** (46) **Landscaping** A gardener wants all of the triangular gardens in a yard to be similar shapes. The first garden has sides that are 3, 4, and 6 feet long. If the second garden has to have sides of 12 and 16 feet corresponding to the 3- and 4-foot sides of the other garden, what is the length of the third side?

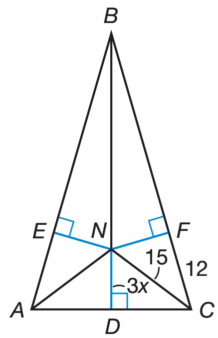
9. (44) Given $\triangle JKL \sim \triangle EFG$, with $JL = 25$ and $EG = 15$, what is the ratio of their corresponding sides?

10. (29, 33) **Architecture** The Dome of St. Peters in Vatican City is 119 meters tall. If you are standing 120 meters away from the zenith of the dome, how far are you from standing directly underneath it?



11. (20) **Algebra** Suppose the conjunction, "$x^2 < 9$ and $x^2 > 4$" is true. Write a disjunction of two statements, involving x but not x^2, which must be true.

12. (38) **Algebra** Find the value of x that makes N the incenter of this triangle.

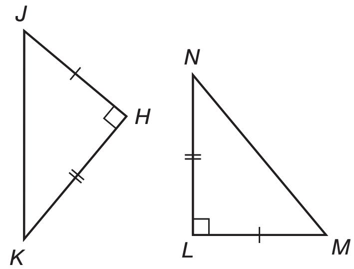


***13.** (45) A square on the coordinate plane has side lengths that are each 6 units long. A vertex is at the origin. Find the coordinates of each vertex.

14. (43) **Write** Explain the difference between a tangent line and a secant line with respect to circles.

15. (29) The legs in a right triangle are 9 inches and 12 inches long. What is the length of the hypotenuse?

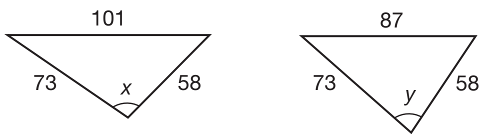
16. (36) Use the LL Congruence Theorem to prove that $\triangle HJK \cong \triangle LMN$.



***17.** (46) **Algebra** Given that $\triangle ABC \sim \triangle DEF$, $AB = 12$, $DE = 16$, and $EF = 20$, what is the length of BC?

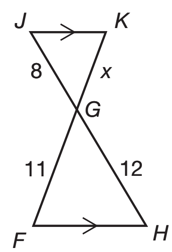
***18.** (45) An isosceles triangle on the coordinate plane has side lengths of 8, 5, and 5 units, respectively. The long side is along the x-axis. Find the coordinates of each vertex.

19. (Inv 4) **Justify** In the two triangles shown, which is greater, x or y? Explain.



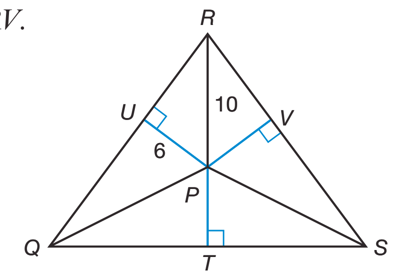
***20.** (46) **Construction** A house has triangular roof structures, or gables, which are similar to the gables of the attached garage. If the base of the house gable is 25 feet and the base of the garage gable is 40 feet, what is the similarity ratio between the two lengths?

***21.** (46) Explain why the triangles are similar, then find the value of x.



22. (7) **Wordplay** Formulate a conjecture that describes the following pattern. *pop, noon, level, redder, racecar.*

23. Using the diagram at right, P is the incenter of $\triangle QRS$. Find RV.
(38)

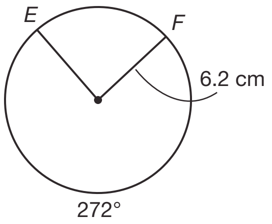


24. How many sides does a convex polygon have, if each interior angle is equal to 179°?
(Inv 3)

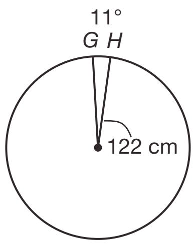
***25.** **Generalize** Line m is given by $ax + by = j$. Line n is given by $-bx + ay = k$. Is n perpendicular to m?
(37)

Find the lengths of the minor arcs given. Round to the nearest hundredth of a centimeter.

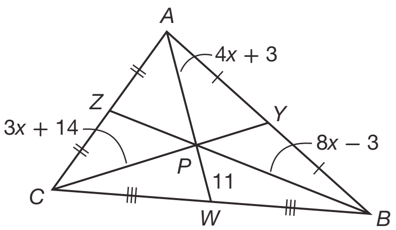
26. $\overparen{EF}$
(35)



27. $\overparen{GH}$
(35)



28. **Algebra** Find the length of $\overline{CY}$.
(32)



***29.** In triangles RST and UVW, $\angle T$ and $\angle W$ are right angles, $\overline{RT} \cong \overline{UW}$, and $\overline{ST} \cong \overline{VW}$. Prove that $\triangle RST \cong \triangle UVW$.
(36)

***30.** **Write** Suppose you know $\triangle ABC \sim \triangle DEF$, and you are given the lengths of all three sides of one triangle and the length of one side of the other. Explain how you would find the lengths of all six sides and the ratio of the corresponding sides.
(44)

LESSON 47

Circles and Inscribed Angles

Warm Up

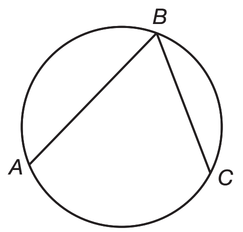
1. (3) **Vocabulary** The set of all points between the sides of an angle is the __________ of the angle. (***interior, exterior, measure***)

2. (26) **Multiple Choice** A central angle separates a circle into two arcs called
   A the greater arc and the lesser arc.
   B the major arc and the minor arc.
   C the larger arc and the smaller arc.
   D the semi arc and the full arc.

3. (26) An arc with endpoints that lie on the diameter of a circle is called a ______________.

New Concepts

Lessons 23 and 26 introduce circles. This lesson also addresses circles and introduces inscribed angles of circles. Recall that a central angle is an angle with the center of a circle as its vertex. Another kind of angle found in circles is the inscribed angle. An **inscribed angle** is an angle whose vertex is on a circle and whose sides contain chords of the circle. In the diagram, $\angle ABC$ is an inscribed angle.



The arc formed by an inscribed angle is the **intercepted arc** of that angle. In the diagram, $\overset{\frown}{AC}$ is the intercepted arc of $\angle ABC$.

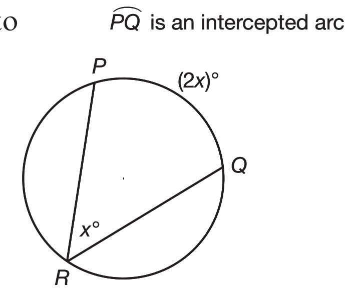
Math Reasoning

Justify A triangle is inscribed in a circle. Use Theorem 47-1 to explain why the sum of the measures of the angles in the triangle is 180°.

Theorem 47-1

The measure of an inscribed angle is equal to half the measure of its intercepted arc.

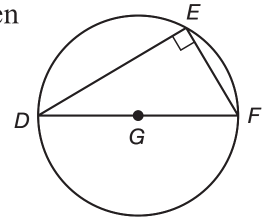
$$m\angle PRQ = \left(\frac{1}{2}\right)m\overset{\frown}{PQ}$$



Theorem 47-2

If an inscribed angle intercepts a semicircle, then it is a right angle.

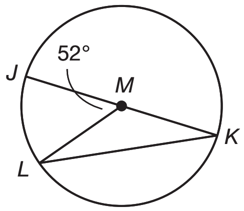
$\angle DEF$ intersects the semicircle, so $m\angle DEF = 90°$.



Online Connection
www.SaxonMathResources.com

Example 1 Proving and Applying Inscribed Angle Theorems

Use $\odot M$ to answer each question.



a. Name the inscribed angle.

SOLUTION

The inscribed angle is $\angle JKL$.

> **Math Language**
>
> Recall that minor arcs are labeled with 2 points, and major arcs are labeled with 3 points.

b. Name the arc intercepted by $\angle JKL$.

SOLUTION

$\angle JKL$ intercepts the minor arc $\overset{\frown}{JL}$.

c. If $m\angle JML = 52°$, find $m\angle JKL$.

SOLUTION

$\angle JML$ is a central angle, so $m\angle JML = m\overset{\frown}{JL}$. By Theorem 47-1, the measure of inscribed angle $\angle JKL$ is half the measure of $\overset{\frown}{JL}$.

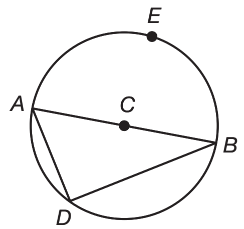
$m\angle JKL = \frac{1}{2}(52°)$

$m\angle JKL = 26°$

d. Prove Theorem 47-2.

Given: $\overline{AB}$ is a diameter of $\odot C$

Prove: $m\angle ADB = 90°$

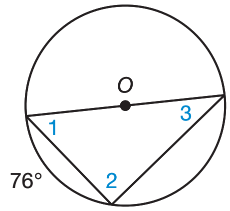


SOLUTION

Statements	Reasons
1. $\overline{AB}$ is a diameter	1. Given
2. $\angle ACB = 180°$	2. Protractor Postulate
3. $m\overset{\frown}{AEB} = 180°$	3. Definition of the measure of an arc
4. $m\angle ADB = 90°$	4. Theorem 47-1

Example 2 Finding Angle Measures in Inscribed Triangles

Find the measure of $\angle 1$, $\angle 2$, and $\angle 3$.



SOLUTION

The arc intercepted by $\angle 3$ measures 76°.

$m\angle 3 = \frac{1}{2}(76°)$ Theorem 47-1

$m\angle 3 = 38°$ Simplify.

Because $\angle 2$ is an inscribed angle that intercepts a semicircle, it measures 90°, by Theorem 47-2.

You can use the Triangle Angle Sum Theorem (Theorem 18-1) to find $m\angle 1$.

$m\angle 1 + m\angle 2 + m\angle 3 = 180°$ Triangle Angle Sum Theorem

$m\angle 1 + 90° + 38° = 180°$ Substitute.

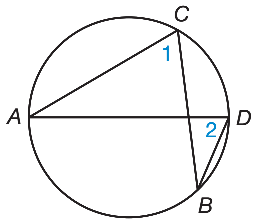
$m\angle 1 = 52°$ Solve.

More than one inscribed angle can intercept the same arc. Since both of these inscribed angles measure one-half what the arc does, they have the same measure, and are congruent.

Theorem 47-3

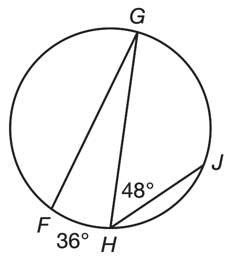
If two inscribed angles intercept the same arc, then they are congruent.

$\angle 1 \cong \angle 2$



Example 3 Finding Measures of Arcs and Inscribed Angles

a. Find the measures of $\angle FGH$ and of $\widehat{GJ}$.



SOLUTION

$\angle FGH$ is an inscribed angle with intercepted arc $\widehat{FH}$. Use Theorem 47-1.

$m\angle FGH = \left(\frac{1}{2}\right)m\widehat{FH}$ Theorem 47-1

$m\angle FGH = \left(\frac{1}{2}\right)36°$ Substitute.

$m\angle FGH = 18°$ Solve.

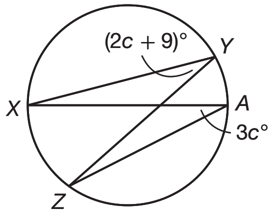
$\widehat{GJ}$ is the intercepted arc of $\angle GHJ$. Use Theorem 47-1.

$m\angle GHJ = \left(\frac{1}{2}\right)m\widehat{GJ}$ Theorem 47-1

$48° = \left(\frac{1}{2}\right)m\widehat{GJ}$ Substitute.

$m\widehat{GJ} = 96°$ Solve.

b. Find the measure of $\angle XYZ$.



SOLUTION

By Theorem 47-3, we know that $\angle XYZ \cong \angle XAZ$.

$\angle XYZ \cong \angle XAZ$ Theorem 47-3

$2c + 9 = 3c$ Substitute.

$c = 9$ Solve.

Substituting $c = 9$ into the expression for $\angle XYZ$ yields $m\angle XYZ = 27°$.

Math Reasoning

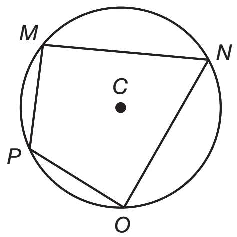
Analyze In Example 3b, could you find the value of c if you were given the measure of $\widehat{XZ}$? Explain.

Theorem 47-4

If a quadrilateral is inscribed in a circle, then it has supplementary opposite angles.

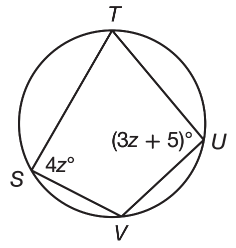
$\angle M + \angle O = 180°$

$\angle P + \angle N = 180°$



Example 4 Finding Angle Measures in Inscribed Quadrilaterals

Find the measure of $\angle U$.



SOLUTION

By Theorem 47-4, $\angle S$ is supplementary to $\angle U$.

$m\angle S + m\angle U = 180°$	Theorem 47-4
$4z + 3z + 5 = 180°$	Substitute.
$z = 25$	Solve for *z*.

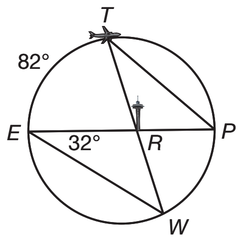
Next, find the measure of $\angle U$.

$m\angle U = 3z + 5$	Given
$m\angle U = 3(25) + 5$	Substitute.
$m\angle U = 80$	Simplify.

The measure of $\angle U$ is 80°.

Example 5 Application: Air Traffic Control

A circular radar screen in an air traffic control tower shows aircraft flight paths. The control tower is labeled *R*. One aircraft must fly from point *T* to the control tower, and then to its destination at point *P*. Find $m\angle TRP$.



SOLUTION

$\angle WEP \cong \angle WTP$	Theorem 47-3
$m\angle WEP = m\angle WTP$	Definition of Congruence
$m\angle WEP = 32°$	Given
$m\angle WTP = 32°$	Transitive Property of Equality
$m\angle TPE = \frac{1}{2}(82) = 41°$	Theorem 47-1
$m\angle WTP + m\angle TPE + m\angle TRP = 180°$	Triangle Angle Sum Theorem
$32° + 41° + m\angle TRP = 180°$	Substitution Property of Equality
$m\angle TRP = 107°$	Solve

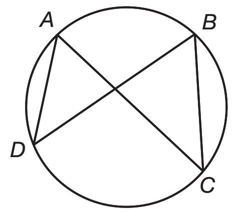
The measure of $\angle TRP$ is 107°.

Math Reasoning

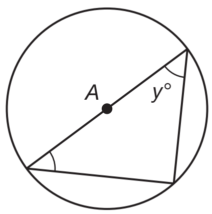
Formulate Write and solve an equation to find the sum of the measures of arcs $\overset{\frown}{EW} + \overset{\frown}{TP}$.

Lesson Practice

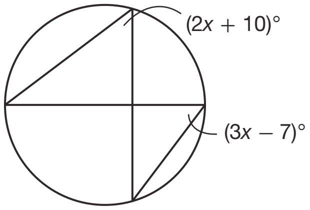
a. Prove Theorem 47-3.
(Ex 1)
Given: Inscribed angles $\angle ADB$ and $\angle ACB$
Prove: $\angle ADB \cong \angle ACB$



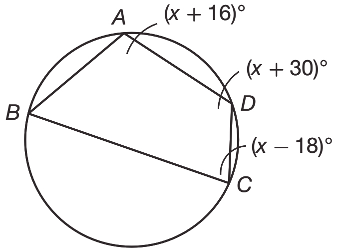
b. Find the value of *y* in the triangle inscribed in $\odot A$.
(Ex 2)



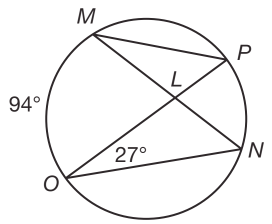
c. Find the value of x.
(Ex 3)



d. Find the measure of $\angle A$.
(Ex 4)

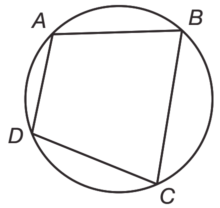


e. **Air Traffic Control** A radar screen in an air traffic control tower shows flight paths. The control tower is labeled L. Points M, L, and P mark the flight path of a commercial jet. Find the measure of $\angle MLP$.
(Ex 5)



Practice Distributed and Integrated

* **1.** Write a two-column proof, proving that if a quadrilateral is inscribed in a circle, then it has supplementary opposite angles.
(27)
Given: $ABCD$ is inscribed in a circle.
Prove: $\angle A$ is supplementary to $\angle C$.



2. **Electrical Wire** An electrician cuts a 45-inch cable cord into three pieces, with a ratio of 2:3:4. What are the lengths of the three pieces of cable?
(41)

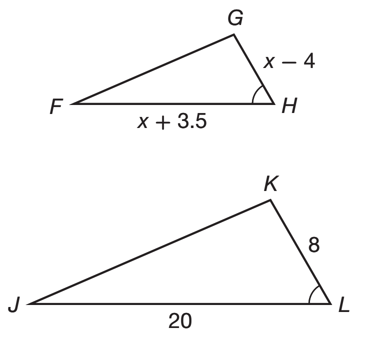
* **3.** **Formulate** Quadrilateral $ABCD$ is inscribed in a circle. Write two equations that show the relationships of the angles of the quadrilateral.
(47)

4. **Justify** Ariel is fencing off a triangular area with some caution tape. One side of the triangle must be 14 feet and a second side must be 22 feet. If the roll of tape is 40 feet long, can he use it to fence off the whole triangular area? Explain.
(39)

5. **Bicycling** Norma started her cycling trip at (0, 7). Eduardo started his trip at (0, 0). They crossed paths at (4, 2). Draw their routes on a coordinate plane.
(45)

6. **Analyze** A composite figure is made up of a square and an equilateral triangle. If the figure has five sides, what is the relationship between the perimeter P and the square side length s? Explain.
(40)

* **7.** (46) **Algebra** Find the value(s) of x that make $\angle FGH \sim \angle JKL$.

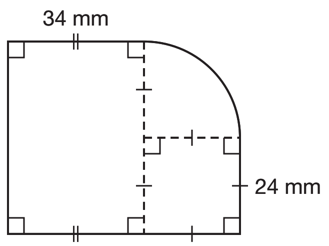


8. (44) **Hardware** If one socket wrench has a $\frac{5}{8}$-inch diameter and the other has a $\frac{3}{4}$-inch diameter, what is the ratio of the diameter of the bigger wrench to that of the smaller wrench?

9. (30) If in $\triangle ABC$, $AB = 7$, $m\angle ABC = 55°$, and $m\angle BCA = 60°$, and in $\triangle DEF$, $m\angle DEF = 60°$, $m\angle DFE = 55°$, and $FD = 10$, is $\triangle ABC \cong \triangle DEF$?

10. (40) **a. Multi-Step** Determine the figure's perimeter. Express your answer in terms of π.

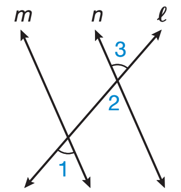
b. Multi-Step Determine the figure's area. Express your answer in terms of π.



***11.** (47) **Analyze** If a triangle is inscribed in a circle such that one edge of the triangle goes through the center of the circle, what statement can be made about the measure of one of the angles in the triangle?

12. (29) Give an example of a Pythagorean triple that is not a multiple of 3, 4, 5.

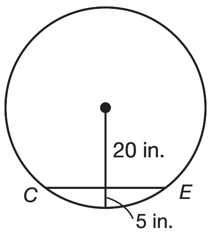
13. (12) There are two theorems or postulates you could use to prove that lines m and n in this figure are parallel. Which are they? What other facts are you using?



14. (43) A pipe with a diameter of 2.2 centimeters is being used for a drainage system. If a cross section of the pipe reveals the water level creates a chord 2 centimeters long, how close to the center of the pipe is the water level to the nearest hundredth of a centimeter?

15. (Inv 3) What is the measure of each interior angle in a regular heptagon? Round your answer to the nearest degree.

16. (43) Find the length of the chord CE.



17. (17) Consider the following statement.

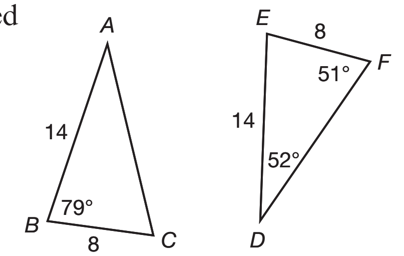
If the month is April, then it is spring in the Northern Hemisphere.

a. Write the negation of this statement's hypothesis.

b. Write the negation of the statement's conclusion.

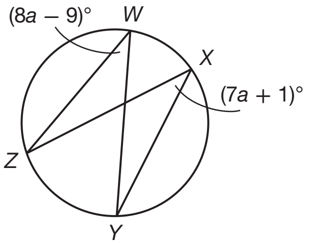
c. Write the contrapositive of the statement.

18. (Inv 4) **Error Analysis** Rosalba was studying the given triangles and determined that $\overline{DF}$ must be longer than $\overline{AC}$. Is she correct? Explain.



19. (37) What is the relationship between the lines $y = \frac{-1}{2}x + b$ and $2x - y = -\sqrt{b}$?

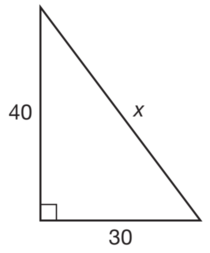
*20. **Algebra** Find the measure of $\angle ZWY$.
(47)



21. What is the value of y in the proportion $\frac{2}{y-3} = \frac{4}{y}$?
(41)

22. **Generalize** A square centered at the origin of the coordinate plane has a side length, k. Find expressions for the coordinates of each vertex.
(45)

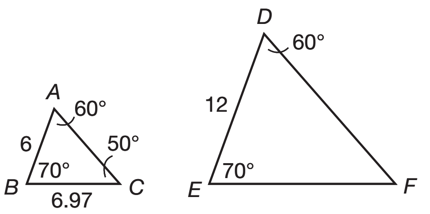
23. Find the unknown length of the side in the triangle shown.
(29)



*24. Use a ruler and a protractor to find the approximate circumcenter of a triangle with vertices at (–5, –2), (2, 3), and (–2, 4).
(38)

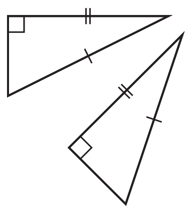
25. If in $\triangle ABC$ and $\triangle XYZ$, $m\angle ACB = m\angle XYZ$, $AC = XY$, and $CB = XZ$, is it true that $\triangle ACB \cong \triangle XYZ$? How do you know?
(28)

*26. **Baking** Tomas is making a layered cake out of 2 individual triangular cakes. The cakes need to be similar triangles so Tomas can stack the smaller one on top of the larger cake. Prove that the two pieces of cake shown are similar. What is the measure of EF?
(46)



27. The ratio of the angle measures in a triangle is 3:5:10. What are the measures of the angles?
(41)

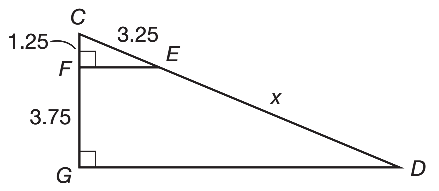
28. Which congruence theorem applies to these triangles?
(36)



*29. **Multiple Choice** The measure of an inscribed angle is ______________ the measure of the intercepted arc.
(47)

A half **B** one third
C one fourth **D** twice

*30. **Carpentry** Gwen is building a truss for her roof. She needs to find the length of segment DE. Use similar triangles to help Gwen find the measurement she needs.
(46)



LESSON
48 Indirect Proofs

Warm Up

1. (31) **Vocabulary** A style of proof that uses boxes and arrows to show the structure of the proof is called a ______________.

2. (14) Is this statement true or false?
A counter example is an example that can help prove a statement true.

3. (31) A proof can be written in a variety of ways. Which is not a method for writing a proof?
A algebraic proof **B** flowchart proof
C paragraph proof **D** flow proof

New Concepts

Direct reasoning is the process of reasoning that begins with a true hypothesis and builds a logical argument to show that a conclusion is true. In some cases it is not possible to prove a statement directly, so an **indirect proof** must be used. An indirect proof is a proof in which the statement to be proved is assumed to be false and a contradiction is shown. This is also called **proof by contradiction.**

> **Math Reasoning**
>
> **Write** What is the primary difference between direct reasoning and the use of an indirect proof?

Follow these three steps to write an indirect proof.

1. Assume the conclusion is false.
2. Show that the assumption you made is contradicted by a theorem, a postulate, a definition, or the given information.
3. State that the assumption must be false, so the conclusion must be true.

Example 1 Writing an Indirect Proof

Prove Theorem 4-1: If two lines intersect, then they intersect at exactly one point.

SOLUTION

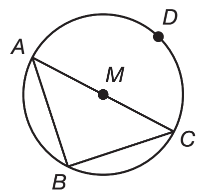
Since this is an indirect proof, we start by assuming that the statement is not true. In other words, it must be possible for two lines to intersect at more than one point.

Assume that the lines m and n intersect at both points, A and B. Now we must show that this contradicts another theorem or postulate. It contradicts Postulate 5, which states that through any 2 points, there exists exactly one line. Since it is not possible for two lines to pass through both point A and point B, the assumption we have made is contradicted, and Theorem 4-1 must be correct.

In an indirect proof, it is often helpful to draw a diagram, just as you would for a 2-column, paragraph, or flowchart proof. A diagram is helpful in determining what assumptions should be made to prove the statement, and in finding the postulate, theorem, or definition that contradicts the assumption.

Example 2 Writing an Indirect Proof

Use the diagram to prove Theorem 47-2:
If an inscribed angle intercepts a semicircle, then it is a right angle.



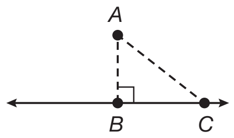
Given: $\overline{AC}$ is a diameter of $\odot M$.
Prove: $m\angle ABC \neq 90°$

SOLUTION

Assume that $m\angle ABC \neq 90°$. By Theorem 47-1, this implies that $\widehat{ADC} \neq 180°$, since the arc is twice the measure of the inscribed angle. It is given that $\overline{AC}$ is a diameter. Since $\overline{AC}$ goes through the center of the circle, $\angle AMC$ is the central angle that intercepts $\widehat{ADC}$. Since $\angle AMC$ is a straight angle, its measure is 180°. An arc's measure is equal to the measure of its central angle, so the measure of $\widehat{ADC}$ must be 180°. This contradicts our assumption.

Example 3 Writing an Indirect Proof

Use the diagram to prove Theorem 42-1: The perpendicular segment from a point to a line is the shortest segment from the point to the line.



Given: $\overline{AB} \perp \overleftrightarrow{BC}$
Prove: $\overline{AB}$ is the shortest segment from A to $\overleftrightarrow{BC}$.

SOLUTION

First, assume that there is another segment from A to $\overleftrightarrow{BC}$ that is shorter than $\overline{AB}$. In the diagram, this segment is shown as $\overline{AC}$. Our assumption is that $AC < AB$.

$\triangle ABC$ is a right triangle, so by the Pythagorean Theorem, it must be true that $AB^2 + BC^2 = AC^2$. Since $AB > AC$, and both AB and AC are greater than 0, by squaring both sides of the inequality we know that $AB^2 > AC^2$. Using the Subtraction Property of Equality, subtract AB^2 from both sides. Then $BC^2 = AC^2 - AB^2$. Since $AB^2 > AC^2$, $AC^2 - AB^2 < 0$. Substituting shows that $BC^2 < 0$. However, the length of BC must be greater than 0, so this contradicts the definition of a line segment. Therefore, AC is not less than AB, and the theorem is true.

Lesson Practice

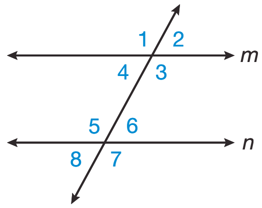
a. *(Ex 1)* State the assumption you would make to start an indirect proof to show that $m\angle X = m\angle Y$.

b. *(Ex 1)* State the assumption you would make to start an indirect proof to show that $\overleftrightarrow{AB} \perp \overleftrightarrow{CB}$.

c. *(Ex 2)* An isosceles triangle has at least two congruent sides. To prove this statement indirectly, assume an isosceles triangle does not have at least two congruent sides. What case needs to be explored to find a contradiction?

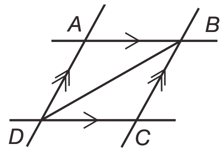
d. *(Ex 2)* Use an indirect proof to prove that a triangle can have at most one right angle.

e. *(Ex 3)* Use an indirect proof to show that $\angle 4 \cong \angle 6$, if $m \parallel n$.



Practice Distributed and Integrated

1. *(16)* **Verify** Choose two points on the line $y = 3x - 4$ and use them to verify that the value $m = 3$ from the equation is actually the slope of the line.



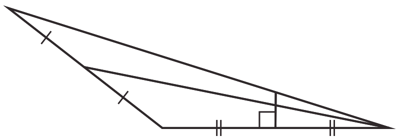
2. *(30)* In the diagram shown, prove that $AD = BC$.

3. *(42)* Find the distance from $(-6, 4)$ to the line $y = x$.

4. *(39)* **Floor Plans** In Thuy's room, the distance from the door D to the closet C is 4 feet, and the distance from the door D to the window W is 5 feet. The distance from the closet C to the window W is 6 feet. On a floor plan, she draws $\triangle CDW$. Order the angles from the least to the greatest measure.

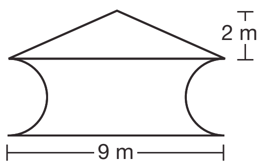
*** 5.** *(47)* **Justify** If the measures of the angles in a quadrilateral are 132°, 90°, 48°, and 90°, can the quadrilateral be inscribed in a circle? Explain.

6. *(32)* **Error Analysis** Arturo wanted to find the orthocenter of the triangle shown. Explain the error Arturo made.



7. *(41)* Given $\triangle MNP \sim \triangle HJK$, with $MN = 3$ and $HJ = 66$, what is the ratio of the corresponding sides?

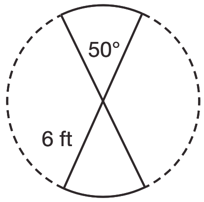
8. *(40)* **Landscaping** A lawn for an ornamental garden is designed as shown. Determine the area of grass needed for the lawn, to the nearest square meter. The semicircular regions have radius lengths of 2.5 meters each.



9. *(41)* Solve the proportion for g. $\frac{6}{16} = \frac{g}{12}$

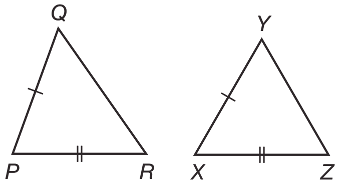
***10.** (45) Triangle STU has vertices $S(0, 0)$ and $T(h, k)$. Find the coordinates of vertex U such that the triangle is isosceles.

11. (35) **Gardening** Oksana needs to fertilize her gardens for the winter, so she needs to find out how many square feet both gardens occupy. If the two gardens are sectors of a circular area, what is the area to be fertilized, to the nearest tenth?



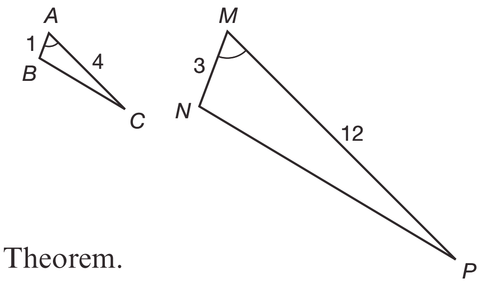
12. (28) If in $\triangle ABC$ and $\triangle DEF$, $AB = DE$ and $m\angle ABC = m\angle EDF$, what other piece of information is needed to show that $\triangle ABC \cong \triangle EDF$ by the SAS postulate?

***13.** (48) Use an indirect proof to prove the Converse of the Hinge Theorem. Refer to $\triangle PQR$ and $\triangle XYZ$ in the diagram.
Given: $\overline{PQ} \cong \overline{XY}$, $\overline{PR} \cong \overline{XZ}$, $QR > YZ$
Prove: $m\angle P > m\angle X$



14. (43) Find the length of a circle's radius given a 12-centimeter chord that is 5 centimeters away from the center of the circle.

15. (46) Verify that $\angle ABC \sim \angle MNP$.



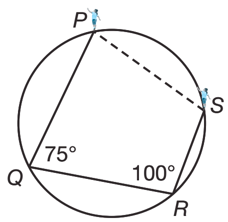
16. (34) **Multiple Choice** The diagonals of parallelogram $RSTU$ intersect at P. Which of the following is true?
A $m\angle RSP = m\angle UTP$ **B** $RP + SP = RT$
C $m\angle UPT = m\angle RPU$ **D** $2TP = RT$

17. (31) Write a flowchart proof of the Right Angle Congruence Theorem.
Given: $\angle 1$ and $\angle 2$ are right angles.
Prove: $\angle 1 \cong \angle 2$

***18.** (48) Write an indirect proof showing that if two lines intersect, then there exists exactly one plane that contains them.

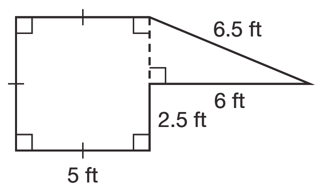
19. (Inv 4) **Outdoor Cooking** Two pairs of tongs are being designed for a barbecue kit, each being of the same length. One is to be used to pick up larger items while the other is to be used to pick up smaller items. Which should be set with a smaller maximum spread angle? Explain.

***20.** (47) **Skating** Brianna skates in straight lines across a circular rink until she reaches a wall. She starts at P, turns 75° at Q, and turns 100° at R. How many degrees must Brianna turn at S to return to her starting point?



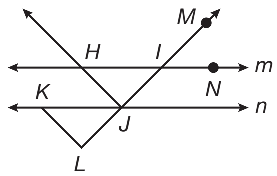
21. (43) A diameter intersects an 8-unit long chord at right angles. What is the length of each of the two chord segments cut by the diameter?

22. (40) **Multi-Step** Determine the perimeter and area of this figure.



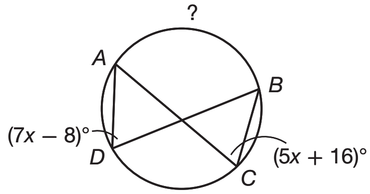
xy^2 ***23.** (Inv 3) **Algebra** In an n-sided convex polygon, the sum of the interior angles is $2x + 80$. In a $2n$-sided convex polygon, the sum of the interior angles is $5x + 20$. Determine the value of n and x.

***24.** Use an indirect proof to prove the following.
(48)
Given: $\triangle HIJ \cong \triangle KJL$
Prove: $\angle KJL \cong \angle MIN$



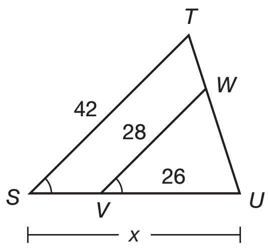
25. Triangles ABC and DEF are right triangles with right angles B and E, respectively. If $m\angle C \cong m\angle F$, $AC = DF = 25$, and $EF = 7$, what is AB?
(36)

26. **Algebra** Find the measure of $\overset{\frown}{AB}$.
(47)



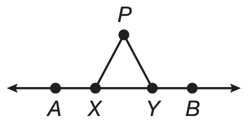
27. **Write** Explain how the method for finding arc length is similar to the formula for the circumference of a circle.
(35)

28. Explain why the triangles STU and VWU are similar, then find the missing length.
(46)



29. **Model** Fold a standard 8-by-11-inch sheet of paper along one diagonal. Carefully cut or tear along the diagonal to make two triangles. Explain why the hypothesis of the Leg-Leg Congruence Theorem is true for these two resulting right triangles.
(36)

***30.** Write an indirect proof showing that through a line and a point not on the line, there exists exactly one perpendicular line to the given line.
(48)
Given: P not on $\overleftrightarrow{AB}$
Prove: There exists exactly one perpendicular line through P to $\overleftrightarrow{AB}$.



LESSON 49

Introduction to Solids

Warm Up

1. (15) **Vocabulary** A polygon in which all sides are congruent is a(n) ________________.

2. (15) What is the name of a polygon with 7 sides?

3. (15) What is the name of a polygon with 10 sides?

4. (13) **Multiple Choice** An equiangular triangle has three angles that are _________.

   A congruent **B** obtuse
   C vertical **D** corresponding

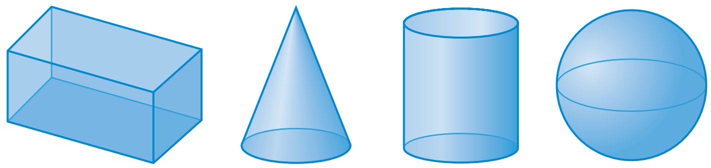
New Concepts

The figures discussed in previous lessons are two-dimensional figures. This lesson introduces three-dimensional figures called **solids**. Solids can have flat or curved surfaces.

Math Reasoning

Write What are some common objects that are polyhedrons? Spheres?

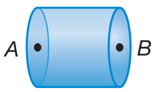
The surfaces of some solids are polygons. Any closed three-dimensional figure formed by four or more polygons that intersect only at their edges is called a **polyhedron**. Some other solids have circular bases or curved sides. A **cone** is a three-dimensional figure with a circular base and a curved lateral surface that comes to a point. A **cylinder** is a three-dimensional figure with two parallel circular bases and a curved lateral surface that connects the bases. A **sphere** is the set of points in space that are a fixed distance from a given point, called the center of the sphere.



Example 1 Classifying Solids

Classify each of the three-dimensional solids shown.

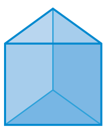
a.



SOLUTION

The figure has two parallel circular bases, and a curved lateral surface. Therefore, the solid is a cylinder.

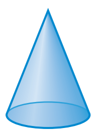
b.



SOLUTION

The figure is made up of five polygons that meet at their edges. Therefore, the figure is a polyhedron.

c.



SOLUTION

The figure has a circular base and a curved lateral surface that comes to a point. Therefore, the figure is a cone.

Online Connection
www.SaxonMathResources.com

Hint

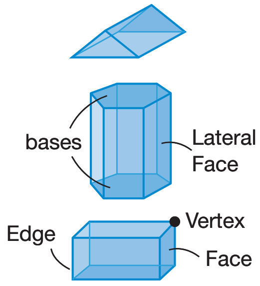
One example of a common polyhedron is a cereal box. It is composed of six rectangles joined at the edges. The seams of the box are the edges of the polyhedron, and each rectangle is a face of the polyhedron. The corners of the box are the vertices.

Math Language

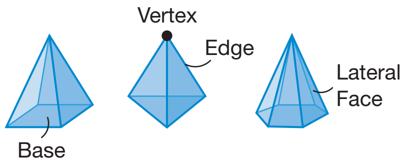
An edge of a prism or pyramid that is not an edge of a base is a **lateral edge**.

Each flat surface of a polyhedron is called a **face of the polyhedron**. The segment that is the intersection of two faces of a solid is the **edge of a three-dimensional figure**. The **vertex of a three-dimensional figure** is the point of intersection of three or more faces of the figure.

A **prism** is a polyhedron formed by two parallel congruent polygonal bases connected by lateral faces that are parallelograms. The **base of a prism** is one of the two congruent parallel faces of the prism. A face of a prism that is not a base is called a **lateral face**.



A **pyramid** is a polyhedron formed by a polygonal base and triangular lateral faces that meet at a common vertex. The faces of a pyramid all share a common vertex. The base is the side of the pyramid that does not share a single vertex with all of the other sides.

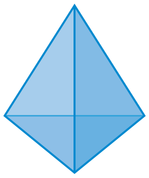


Prisms and pyramids are named by the shape of their bases. For example, a prism with a triangle for a base is called a triangular prism. A pyramid with a hexagon for a base would be called a hexagonal pyramid. A **cube** is the special name for a prism with six square faces.

Example 2 Classifying Polyhedra

Classify each polyhedron.

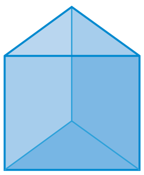
a.



SOLUTION

The polyhedron has one base and the triangular faces meet at a common vertex. Therefore, the polyhedron is a pyramid. Since the base is a triangle, the polyhedron is a triangular pyramid.

b.



SOLUTION

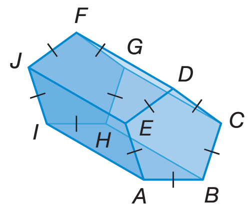
The polyhedron has two parallel bases and the lateral faces are parallelograms. Therefore, the polyhedron is a prism. Since the bases are triangles, the polyhedron is a triangular prism.

A polyhedron is regular if all of its faces are congruent, regular polygons. A pyramid is regular if its base is a regular polygon and its lateral faces are congruent isosceles triangles. A prism is regular if its base is regular and its faces are rectangles. A cube is both a regular polyhedron and a regular rectangular prism. A triangular prism with equilateral bases is a regular prism but is not a regular polyhedron, since its faces are not congruent to its bases.

A **diagonal of a polyhedron** is a segment whose endpoints are the vertices of two different faces of a polyhedron.

Example 3 Describing Characteristics of Solids

Classify the polyhedron in the diagram, assuming all the angles of each pentagon are congruent. Is it a regular polyhedron? How many edges, vertices, and faces does it have? Name one diagonal segment of the polyhedron.



SOLUTION

The figure has two parallel pentagonal bases. Therefore, the polyhedron is a pentagonal prism. The sides of the bases are all congruent, and it is given that the angles are congruent, so it is a regular prism. Since the lateral faces are not congruent to the pentagonal bases, it is not a regular polyhedron. It has 7 faces, 15 edges, and 10 vertices. One diagonal is the segment $\overline{BF}$.

> **Math Reasoning**
>
> **Generalize** A pentagonal pyramid does not have a diagonal. Is this true of all pyramids? Explain.

A unique relationship exists among the number of faces, vertices, and edges of any polyhedron.

Euler's Formula
For any polyhedron with V vertices, E edges and F faces, $V - E + F = 2.$

Example 4 Using Euler's Formula

How many faces does a polyhedron with 12 vertices and 18 edges have?

SOLUTION

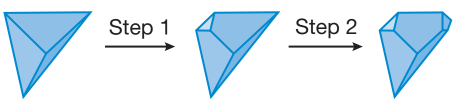
Substitute $V = 12$ and $E = 18$ and solve for F.

$$V - E + F = 2$$
$$12 - 18 + F = 2$$
$$F = 8$$

The polyhedron has 8 faces.

Example 5 Application: Diamond Cutting

Diamonds are cut to change them from a rough stone into a gemstone. The figure below shows two steps in cutting a particular diamond.



If each of the other vertices is cut in the next steps, what is the number of faces, vertices, and edges of the diamond in Step 4? Verify your answer.

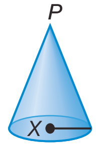
> **Math Reasoning**
>
> **Analyze** When a vertex of the pyramid in this example is cut off, why does a triangular face form at the vertex?

SOLUTION

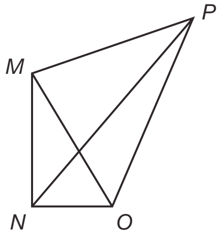
At the start the diamond has 4 faces, 4 vertices, and 6 edges. After cutting in Step 1, the diamond has 5 faces, 6 vertices, and 9 edges. After Step 2, the diamond has 6 faces, 8 vertices, and 12 edges. Since this pattern continues, after Step 3, the diamond will have 7 faces, 10 vertices, and 15 edges. After Step 4, the diamond will have 8 faces, 12 vertices, and 18 edges. Euler's Formula can verify the relationship among the faces, vertices, and edges: $12 - 18 + 8 = 2$.

Lesson Practice

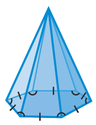
a. Classify the solid. Name its vertices, edges, and bases.
(Ex 1)



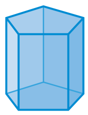
b. Classify the solid. How many vertices, edges, and bases does it have?
(Ex 1, 3)



c. Classify the polyhedron. Determine whether it is a regular polyhedron.
(Ex 2, 3)

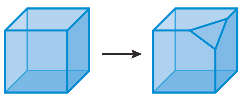


d. Classify the polyhedron. Determine whether it is a regular polyhedron.
(Ex 2, 3)



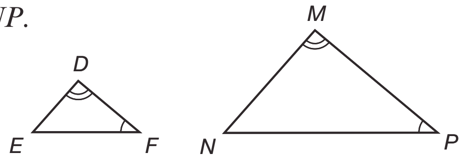
e. How many edges does a polyhedron with 14 vertices and 9 faces have?
(Ex 4)

f. **Gemstones** A gemstone is cut in the shape of a cube. Each vertex of the cube is then cut so that there is a triangular facet at each vertex. What is the number of faces, vertices and edges when the first four vertices of the cube are removed? Verify the results with Euler's Formula.
(Ex 5)



Practice Distributed and Integrated

1. **Verify** Using the information shown, verify that $\triangle DEF \sim \triangle MNP$.
(46)

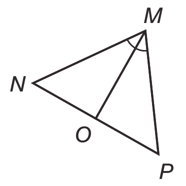


* **2.** State the assumption you would make to start an indirect proof if asked to prove the following statement.
(48)

If a polygon is a hexagon, then the sum of the interior angles is 720°.

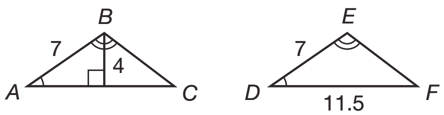
*3. **Multiple Choice** Which statement contradicts the fact that $\triangle ABC$ is an equilateral triangle?
(48)
A All angle measures of $\triangle ABC$ are equal.
B The altitude of $\triangle ABC$ is not a median.
C All sides lengths of $\triangle ABC$ are equal.
D All angles of $\triangle ABC$ are acute angles.

4. **Algebra** Using the diagram at the right, find the length of $\overline{NO}$ in terms of x if $MP = x + 3$, $OP = x$, and $MN = 28$.
(38)



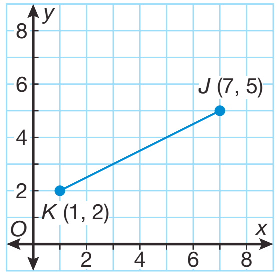
*5. **Generalize** Describe the resulting figures when any prism is cut parallel to its base.
(49)

6. Using information from the diagram, determine the area of both triangles.
(30)



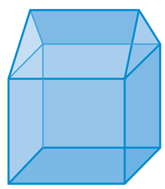
*7. **Multi-Step** A corner garden has vertices $Q(0, 0)$, $R(0, 2d)$, and $S(2c, 0)$. A brick walkway runs from point Q to the midpoint of RS. What is the length of the walkway?
(45)

8. Determine the midpoint M of the line segment $\overline{JK}$ with endpoints $J(7, 5)$ and $K(1, 2)$.
(11)

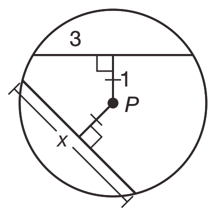


9. What is the included side of $\angle LMO$ and $\angle OMN$?
(28)

*10. **Construction** A construction company wants to pre-cut all the pieces for the barn in the diagram. Each face, including the floor, is cut from a special fiberboard. Metal strips will be used along each edge, and a connector pipe placed at each vertex. How many fiberboards, metal strips, and connector pipes are needed to construct the barn?
(49)



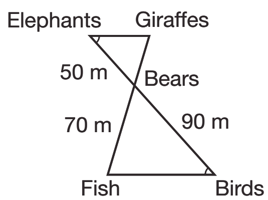
11. In $\odot P$, determine the value of x.
(43)



*12. If you are trying to prove indirectly that the altitude of an equilateral triangle is also a median, what assumption should you make to start the proof?
(48)

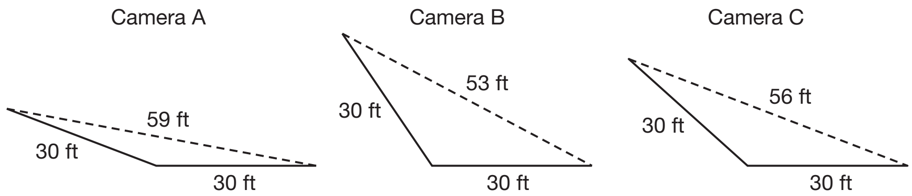
13. **Carpentry** Susanna wants to increase the perimeter of her garden shed by a factor of 1.5. If the shed is 10 feet wide by 12 feet long already, what will be the perimeter of the new garden shed?
(44)

14. **Zoos** The habitats of the animals at a local zoo is shown in the diagram. The zookeeper wants to keep the mammals grouped nearby each other. How far are the bears from the giraffes, to the nearest meter?
(41)



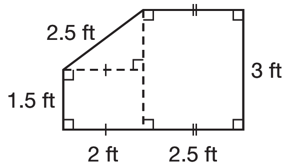
15. (41) The noncongruent angle in an isosceles triangle measures 76°. What are the measures of the three angles in a triangle that is similar to this triangle?

16. (Inv 4) **Security Cameras** A surveillance camera company needs to set up cameras that pan across three areas as indicated in the given diagram. Which camera pivots through the largest angle? … the smallest angle? Explain.



***17.** (44) **Analyze** $\triangle FGH$ and $\triangle KLM$ are similar scalene triangles. How many of their six sides do you need numerical values for in order to find all the other side lengths and the perimeters of both triangles? Explain.

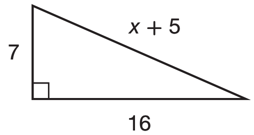
18. (40) **a.** Determine this figure's perimeter.
b. Determine its area.



19. (42) Find the closest point on the line $y = -x - 4$ to (4, 4).

***20.** (49) **Verify** Is it possible to have a solid with 7 faces, 12 edges, and 10 vertices? Explain.

21. (29, 33) **Error Analysis** Below are two students' solutions for finding the length of the hypotenuse. Determine which is incorrect and explain where the error was made.



Leonardo's Solution

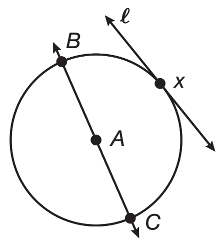
$16^2 + 7^2 = (x + 5)^2$
$16^2 + 7^2 = x^2 + 5^2$
$256 + 49 = x^2 + 25$
$280 = x^2$
$x \approx 16.7$
$x + 5 \approx 16.7$
$x \approx 11.7$

Florence's Solution

Let $x + 5 = c$
$16^2 + 7^2 = c^2$
$256 + 49 = c^2$
$305 = c^2$
$c \approx 17.5$
$x + 5 \approx 17.5$
$x \approx 12.5$

***22.** (49) **Probability** Explain why a cube is the only prism used in fair probability experiments.

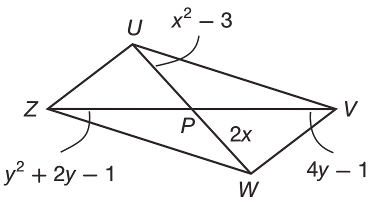
23. (43) Identify each line or segment that intersects the circle at right.



24. (47) **Write** What conclusion can you make about three inscribed angles that intercept the same arc?

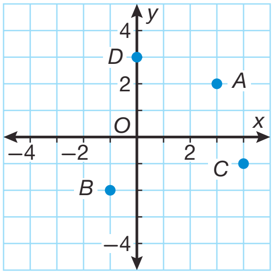
25. (11) What are the coordinates of the midpoint of (2, −2) and (−7, 3)?

26. **Algebra** *UVWZ* is a parallelogram. Find each length.
(34)



a. *UW*
b. *VP*
c. *WP*
d. *ZV*

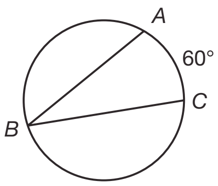
Use the diagram to answer the next two questions.



27. **Multiple Choice** Which distance is closest to 5 units?
(9)
A *AB*
B *CD*
C *BC*
D *AD*

28. **Multiple Choice** What is the length to the nearest hundredth of $\overline{AB}$?
(9)
A 11.49
B 8.12
C 5.66
D 5.83

29. Find the measure of $\angle ABC$.
(47)



30. **Interior Design** The kitchen sink, countertop, and refrigerator are the vertices of a triangle in a kitchen. If the distance from the refrigerator to the countertop is 9 feet and from the countertop to the sink is 8 feet, what range of values would the distance from the sink to the fridge have?
(39)

LESSON 50

Geometric Mean

Warm Up

1. (41) **Vocabulary** In a proportion, the extremes are the two values at the edges of the proportion, and the ________ are the two values that are in the center of the proportion.

2. (Inv 2) In a right triangle, the two sides of the triangle that include the right angle are the ______.

3. (41) In the proportion $5:8 = 10:16$, the means are ____ and ____.

4. (41) For a proportion such as $3:6 = 5:10$, the product of the means will equal the product of the __________.

New Concepts

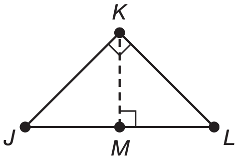
When an altitude is drawn from the vertex of a right triangle's 90° angle to its hypotenuse, it splits the triangle into two right triangles that exhibit a useful relationship.

Caution

Theorem 50-1 is true only if the altitude of the right triangle has an endpoint on its hypotenuse. An altitude to either one of the triangle's legs will not exhibit this relationship.

Theorem 50-1
If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to each other and to the original triangle.

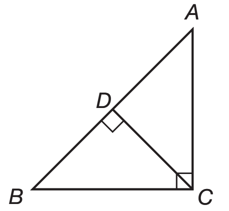
In $\triangle JKL$, for example, $\triangle JMK$ is similar to $\triangle LMK$, and both $\triangle JMK$ and $\triangle LMK$ are similar to $\triangle JKL$.



Example 1 Proving Theorem 50-1

Given: $\overline{DC}$ is an altitude of $\triangle ABC$.

Prove: $\triangle ABC \sim \triangle CBD$, $\triangle ABC \sim \triangle ACD$, and $\triangle ACD \sim \triangle CBD$



SOLUTION

In $\triangle ABC$, $\overline{CD} \perp \overline{AB}$ by the definition of an altitude. All right angles are congruent, so $\angle BCA \cong \angle CDA$, $\angle BDC \cong \angle CDA$, and $\angle BCA \cong \angle BDC$.

By the Reflexive Property, $\angle B \cong \angle B$. This is sufficient to show that $\triangle ABC \sim \triangle CBD$, by the AA Similarity Postulate.

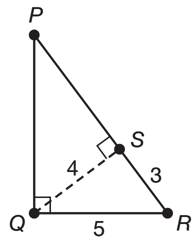
Again, by the Reflexive Property $\angle A \cong \angle A$, so $\triangle ABC \sim \triangle ACD$, by the AA Similarity Postulate.

By the Transitive Property of Similarity, $\triangle ACD \sim \triangle CBD$ since both triangles are similar to $\triangle ABC$.

Online Connection
www.SaxonMathResources.com

Example 2 Identifying Similar Right Triangles

Find PS and PQ.



SOLUTION

Since $\overline{QS}$ is a segment that is perpendicular to one side of the triangle with one endpoint on a vertex of the triangle, it is an altitude of $\triangle PQR$. By Theorem 50-1, $\triangle PQR \sim \triangle PSQ \sim \triangle QSR$. Set up a proportion to solve for the missing sides.

$$\frac{SQ}{SR} = \frac{PQ}{QR} = \frac{PS}{QS}$$

$$\frac{4}{3} = \frac{PQ}{5} = \frac{PS}{4}$$

$$PQ = 6.\overline{6}$$

$$PS = 5.\overline{3}$$

> **Math Reasoning**
>
> **Write** Take the cross product of the definition of the geometric mean and solve for x. What is another way to state the geometric mean of a and b, according to the formula you have found?

Sometimes, the means of a proportion are equal to one another. This is a special kind of proportion that can be used to find the geometric mean of two numbers. The **geometric mean** for positive numbers a and b, is the positive number x such that $\frac{a}{x} = \frac{x}{b}$.

Example 3 Finding Geometric Mean

a. Find the geometric mean of 3 and 12.

SOLUTION

Using the definition of geometric mean, you can obtain the following algebraic expression, where x represents the geometric mean.

$$\frac{3}{x} = \frac{x}{12}$$

$$x(x) = 12(3)$$

$$x^2 = 36$$

$$x = \sqrt{36}$$

$$x = 6$$

b. Find the geometric mean of 2 and 9 to the nearest tenth.

SOLUTION

Using the definition of geometric mean, you can obtain the following algebraic expression, where x represents the geometric mean.

$$\frac{2}{x} = \frac{x}{9}$$

$$x(x) = 2(9)$$

$$x^2 = 18$$

$$x \approx 4.2$$

> **Math Reasoning**
>
> **Formulate** Write the answer to part b of Example 3 in simplified radical form.

Two corollaries to Theorem 50-1 use geometric means to relate the segments formed by the altitude of a right triangle to its hypotenuse.

Corollary 50-1-1

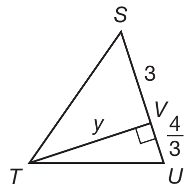
If the altitude is drawn to the hypotenuse of a right triangle, then the length of the altitude is the geometric mean between the segments of the hypotenuse.

Corollary 50-1-2

If the altitude is drawn to the hypotenuse of a right triangle, then the length of a leg is the geometric mean between the hypotenuse and the segment of the hypotenuse that is closer to that leg.

Example 4 Using Geometric Mean with Right Triangles

a. Given the triangle *STU*, find the missing value, *y*.



SOLUTION

Since *TV* is an altitude, by Corollary 50-1-1, *y* is the geometric mean of the segments of the hypotenuse, which are 3 and $\frac{4}{3}$. Using the definition of geometric mean, you can obtain the following algebraic expression.

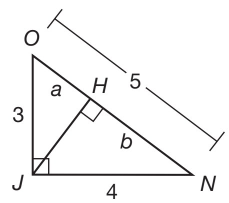
$$\frac{3}{y} = \frac{y}{\frac{4}{3}}$$

$$y^2 = 3\left(\frac{4}{3}\right)$$

$$y^2 = 4$$

$$y = 2$$

b. Given the triangle, find the missing values *a* and *b*.



SOLUTION

Since *JH* is an altitude, there are two relationships that can be derived from Corollary 50-1-2.

$$\frac{a}{3} = \frac{3}{5} \qquad \frac{b}{4} = \frac{4}{5}$$

$$5(a) = 3(3) \qquad 5(b) = 4(4)$$

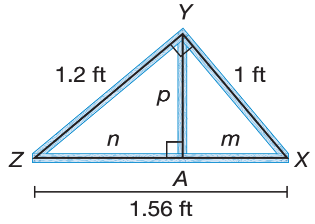
$$5a = 9 \quad \text{and} \quad 5b = 16$$

$$a = \frac{9}{5} \qquad b = \frac{16}{5}$$

$$a = 1.8 \qquad b = 3.2$$

Example 5 Real World Application

Jayden is building a truss for a shed, shown in the diagram. Jayden needs to find the lengths of the truss brace $\overline{AY}$ and the lengths of $\overline{XA}$ and $\overline{ZA}$.



SOLUTION

Since $\overline{AY}$ is an altitude to the triangle, then

$$\frac{n}{1.2} = \frac{1.2}{1.56} \qquad\qquad \frac{m}{1} = \frac{1}{1.56}$$

$$1.56n = 1.44 \qquad \text{and} \qquad 1.56m = 1$$

$$n = \frac{1.44}{1.56} \qquad\qquad m = \frac{1}{1.56}$$

$$n \approx 0.92 \qquad\qquad m \approx 0.64$$

These are the lengths of $\overline{XA}$ and $\overline{ZA}$. To find the length of the truss brace $\overline{AY}$, apply Corollary 50-1-1.

$$\frac{m}{p} = \frac{p}{n}$$

$$\frac{0.64}{p} \approx \frac{p}{0.92}$$

$$p^2 \approx 0.5888$$

$$p \approx 0.77$$

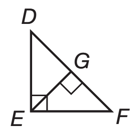
So, Jayden needs a brace that is 0.77 feet long, which will divide the truss into two pieces that are 0.64 feet long and 0.92 feet long, respectively.

Hint

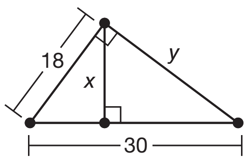
If you find it difficult to remember Corollaries 50-1-1 and 50-1-2, use Theorem 50-1 to find the lengths of the segments. The corollaries are useful, but not essential.

Lesson Practice

a. Name the similar triangles. *(Ex 1)*



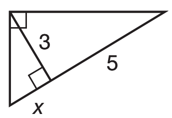
b. Find the values of x and y. *(Ex 2)*



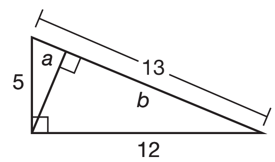
c. Find the geometric mean between 4 and 11 to the nearest tenth. *(Ex 3)*

d. Find the geometric mean between 2 and 16 in simplified radical form. *(Ex 3)*

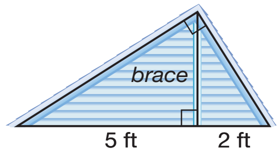
e. Find the value of x. *(Ex 4)*



f. Find the values of a and b to the nearest tenth. *(Ex 4)*



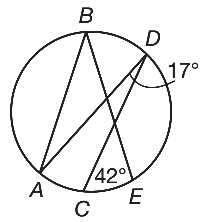
g. *(Ex 5)* To support an old roof, a brace must be installed at the altitude. Find the length of the brace to the nearest tenth of a foot.



Practice Distributed and Integrated

* **1.** *(49)* What is the name of a figure that has 5 vertices and 5 faces?

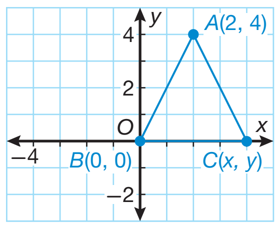
2. *(47)* **Interior Design** A wallpaper design is made up of repeating circles that contain inscribed angles as shown. If m∠*ADC* = 17°, and m$\overset{\frown}{CE}$ = 42°, find m∠*ABE*.



3. *(34)* **Multi-Step** Parallelogram *EFGH* has three vertices at *E*(−4, −2), *F*(0, 6), and *G*(3, 7). Find the coordinates of *H*.

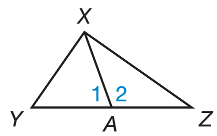
4. *(43)* **Algebra** If a chord that is perpendicular to a 6-inch long radius cuts the radius into two equal lengths, what is the length of the chord?

5. *(45)* Given that △*ABC* is an isosceles triangle, find (*x*, *y*).



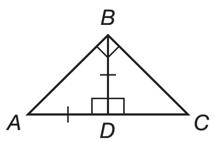
* **6.** *(50)* **Write** Explain how the mean of two numbers is different from their geometric mean.

* **7.** *(48)* On the diagram, m∠1 ≠ m∠2. Prove $\overline{XA}$ is not an altitude of △*XYZ*.



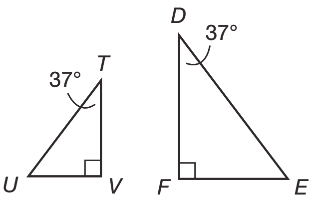
* **8.** *(50)* **Algebra** Find the geometric mean of *x* and *y*.

9. *(30)* Using the diagram given, complete the proof that △*ABD* ≅ △*BCD*.



Statements	Reasons
1. m∠*ADB* = 90°, m∠*BDC* = 90°, m∠*ABC* = 90°, *AD* = *DB*	1. Given
2.	2. Acute angles in a right triangle are complementary
3. m∠*ABD* + m∠*DBC* = 90°	3.
4. m∠*DAB* + m∠*ABD* = m∠*ABD* + m∠*DBC*	4. Substitution Property of Equality
5. m∠*DAB* + m∠*ABD* − m∠*ABD* = m∠*ABD* + m∠*DBC* − m∠*ABD*	5. Subtraction Property of Equality
6.	6. Simplify
7.	7. Definition of congruent angles
8.	8. All right angles are congruent
9. △*ABD* ≅ △*BCD*	9.

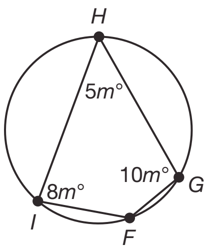
10. Name the angles that are congruent. If these triangles are similar, explain why.
(46)



11. **Algebra** The sum of the interior angles of a convex polygon is 5400°. How many sides does the polygon have?
(Inv 3)

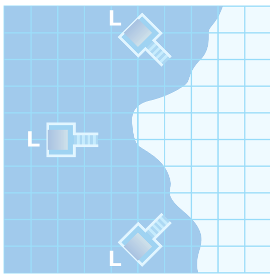
***12.** **Analyze** $\triangle XYZ$ and $\triangle MNO$ are similar equilateral triangles. How many of their six sides do you need numerical values for in order to find all the other side lengths and the perimeters of both triangles? Explain.
(44)

13. **Algebra** Find the measure of $\angle G$ in the figure shown.
(47)



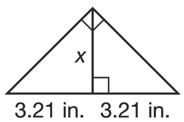
14. Given the line equation $y = -\frac{2}{5}x + 4$ and $H(3, -3)$, find the point on the line that is closest to H.
(42)

15. **Lifeguarding** A beach inlet has three lifeguards on duty at all times. They are spaced around the inlet so that they are all equidistant from the diving board. Copy this diagram to find the approximate location of the diving board based on the location of the lifeguards.
(38)



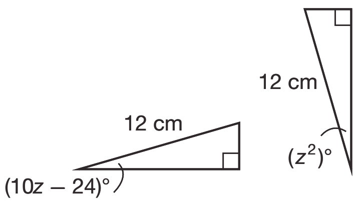
16. **Justify** Explain why a triangle cannot have side lengths measuring 51, 13, and 31.
(39)

***17.** **Carpentry** A builder is making a triangular eave as shown. What is the length of the brace in the center?
(50)



***18.** **Multiple Choice** Which of the following *does not* describe a polyhedron?
(49)
A 6 vertices, 9 edges, 5 faces **B** 6 vertices, 10 edges, 6 faces
C 8 vertices, 10 edges, 6 faces **D** 8 vertices, 12 edges, 6 faces

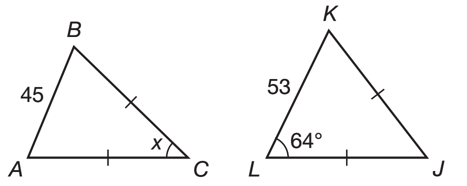
***19.** **Algebra** Suppose you can use the Hypotenuse-Angle Congruence Theorem to prove that these two triangles are congruent. What could the value of z be?
(36)



20. **Landscaping** City planners have decided to build a new park that is in the shape of a triangle. Along one side of the park will be a 12-foot fence. It will meet at right angles with another fence that is 5 feet long and will run along a second side. What length will be the third side of the park?
(29)

21. A transversal crossing two lines, makes a right angle with one of the lines. When does it make a right angle with the other?
(Inv 1)

22. (Inv 4) **Justify** Give a possible value for the angle at vertex C if you know that $AB < LK$. Explain your answer.



23. (45) A square on the coordinate plane has side lengths of 10 units. The midpoint of the bottom side is at the origin. Find the coordinates of the vertices.

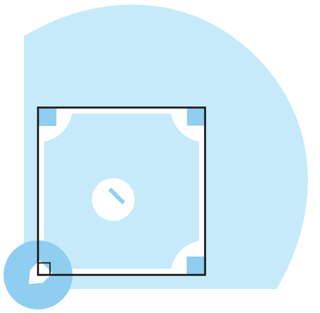
24. (37) **Multiple Choice** Which of the following are perpendicular lines?

i) $x + y = 1$ ii) $-2x - y = 3$ iii) $2y - x = 5$

A i and ii **B** i and iii

C ii and iii **D** None is perpendicular to either of the other lines.

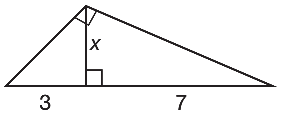
25. (29) **Baseball** The distance between consecutive bases in a baseball diamond is 90 feet. How far does the catcher have to throw the ball to get it from home plate to second base?



26. (35) **Analyze** Find the measure of an arc so that the associated sector has one-tenth the area of the circle.

***27.** (49) **Error Analysis** Nikki counted 48 edges, 24 vertices, and 22 faces on a polyhedron. Is it possible for Nikki to be correct? Explain.

***28.** (50) Find the value of x.



29. (28) **Algebra** If in $\triangle ABC$ and $\triangle DEF$, $AB = DE$, $m\angle ABC = m\angle EDF$, $BC = DF$, $AC = (20x - 10)$, and $EF = (15x + 15)$, what is the value of x?

30. (48) Which statement, if true, would contradict the fact that $\triangle JKL \sim \triangle PQR$?

A $JK = rPQ$ for some factor r

B $JK \neq PR$

C $\angle J \neq \angle R$

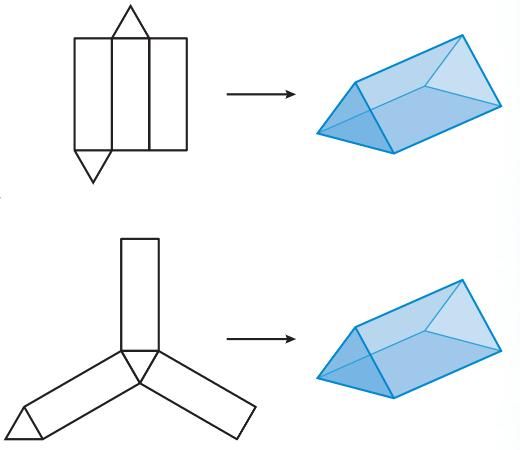
D None of the angles in $\triangle JKL$ have the same measure as any of the angles in $\triangle PQR$.

INVESTIGATION

5

Nets

Recall that a polyhedron is a 3-dimensional solid with polygonal faces. A **net** is a diagram of the faces of a three-dimensional figure, arranged so that the diagram can be folded to form the three-dimensional figure.



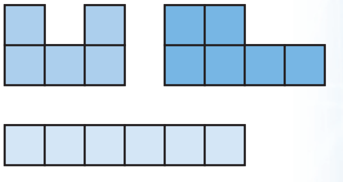
The diagram shows two possible nets of a triangular prism. As shown, a given polyhedron can have more than one net.

Math Reasoning

Model Draw a third net for the triangular prism depicted in the diagram.

1. **Model** Use graphing paper to make a net of a cube. How many squares will comprise the net of the cube? Sketch two possible ways to draw a net for a cube.

Choose one net and draw it on graph paper. Make each side of the cube's faces 5 units long. Cut the net out and fold it along each edge. Fold the net up into the shape of a cube and use tape to secure the edges of the cube.



2. **Analyze** Explain why the nets shown cannot make a cube when folded up.

3. **Model** Draw two more possible nets for a cube.

Next, make a regular tetrahedron. A tetrahedron has four faces that are congruent equilateral triangles. Draw an equilateral triangle on grid paper. Make the length of each side 5 units and use a protractor to ensure that each of the triangle's angles measures 60°. To complete the net, draw three more congruent triangles, each sharing one side with the original triangle.

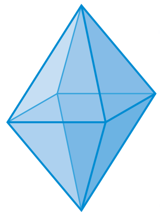
4. Cut out the tetrahedron and fold it up. What kind of solid is it? Classify it based on its faces and its base.

5. Is a cube a regular polyhedron?

6. Are either of the solids made in problem **1** or **4** prisms?

7. Is the tetrahedron a regular polyhedron?

The solid shown in the diagram is a regular octahedron. Think about unfolding one of the pyramids that comprise this shape to create a net. Working in groups with other students, draw two possible nets for the octahedron. Then, pick another solid from the following list, draw a net, and construct it: hexagonal pyramid, pentagonal prism, pentagonal pyramid, rectangular prism, and rectangular pyramid.



Math Reasoning

Generalize Which is the only pyramid that is a regular polyhedron? Which is the only prism that is a regular polyhedron?

8. Are there more than two ways to draw the net of the octahedron?

9. **Write** Describe one method that can be used to make the net of any regular pyramid.

10. **Write** Describe one method that can be used to make the net of any regular prism.

11. Is the last solid that was made a regular polyhedron? Why or why not?

Investigation Practice

a. **Generalize** How many different nets are there for a regular triangular pyramid? Sketch some nets and look for a pattern.

b. Using congruent squares, draw 5 different nets of a cube.

c. Draw a diagram using the connected faces of a cube that cannot be folded into a cube.

d. Draw another net for an octahedron (a regular solid with 8 equilateral-triangle faces).

e. Draw a net for a pentagonal prism.